1 Cluster method for the Ising model

One considers the Ising model with the Hamiltonian

$$H = -J \sum_{\langle i,j \rangle}^{N} S_i S_j \tag{1}$$

where S_i is the spin variable $\{-1, 1\}$.

The given code is written in C++ for a square lattice and can be used with a Metropolis dynamics or a Wolf cluster dynamics. Many parameters can be set at the start of the simulation : L linear size of the simulation box, b number of iterations for the warmup, a number of iterations at equilibrium.

The class structure of the code is suitable for adapting the code in various situations. The output file of the simulation is a trajectory file, where energy and magnetization are written each five steps. For the cluster method, a second file stores the size of the cluster along the simulation.

1. Compile the code, and run by using the parameters used by default. Compile with the option -Ofast and run again. Compare the two execution times.
2. Write a python script for displaying the normalized energy histogram.
3. By using the reweighting method, compute the normalized energy histograms for a sequence of inverse temperatures going from $\beta - 0.2$ to $\beta + 0.2$ and display all histograms on the same figure. Comment the results.
4. By using the reweighting compute the mean energy, the specific heat, the mean magnetization, the susceptibility and the Binder parameter for the same range of temperature.
5. Run a second simulation for $\beta = 0.44$ and compute the same quantities and display the results of the two simulations on the same figures.
6. By choosing the inverse temperature $\beta = 0.4406$, run 4 simulation with different system size $L = 24, 36, 48, 64$. Determine the maximum of the specific heat and of the susceptibilty for each size. Could you obtain rough estimate of the critical exponents ?