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Quantum coherent effects in the hydrodynamics of low-temperature 1D gases

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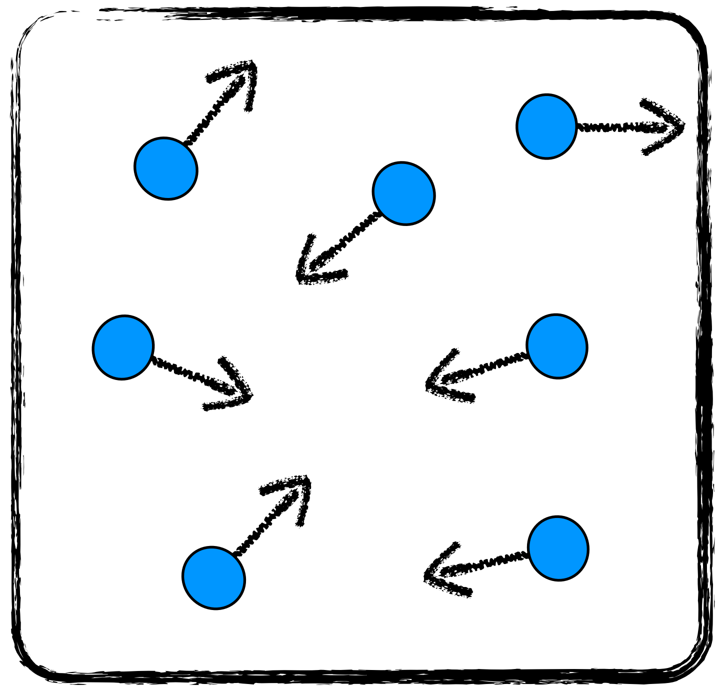
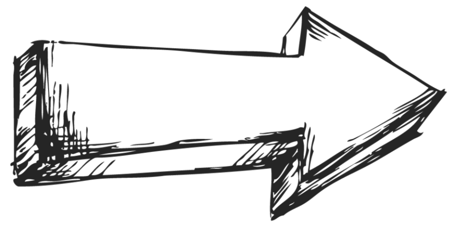
Invitation to nonequilibrium physics



$|\psi\rangle$

Statistical Mechanics



Equilibrium Physics:

Statistical Ensembles ✓

$$\hat{\rho} \propto e^{-\sum_n \beta_n \hat{Q}_n}$$

Conserved quantities

*describe many-body **quantum systems**

⚠ **exponential complexity ~** e^N

1 spin: $|\uparrow\rangle$ or $|\downarrow\rangle$ states

N spins on a lattice $\sim 2^N$ states

$2^{20} = 1\ 048\ 576$

Invitation to nonequilibrium physics

Nonequilibrium Physics: There is no unique framework for nonequilibrium processes

Quantum quench:

- Prepare the system at equilibrium of $\hat{H}(V_0)$
- Sudden change of $\hat{H}(V_0) \rightarrow \hat{H}(V_1)$

$$|\mathrm{GS}\rangle \longmapsto |\Psi_t\rangle = e^{-it\hat{H}}|\mathrm{GS}\rangle$$

exponential complexity

How ?

general mechanisms

↓

Effective descriptions

- No microscopic laws
- Universality

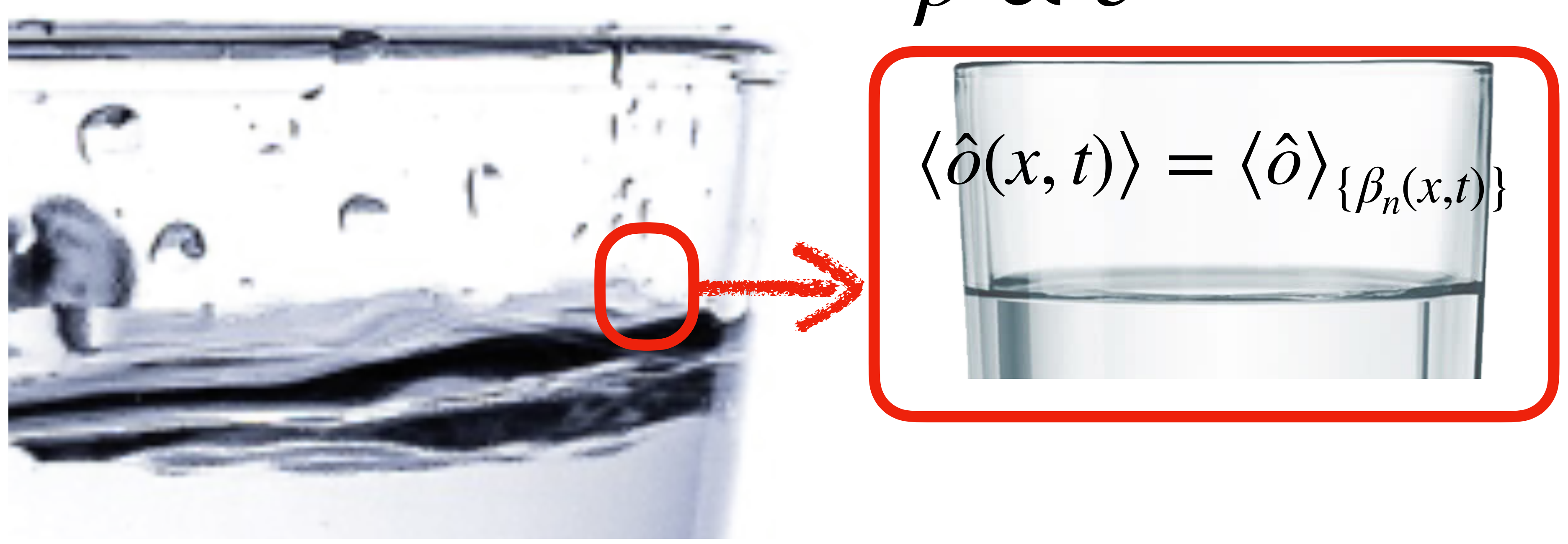
Emergent Hydrodynamics

Look for an **effective description** at large space & time scales

I. Scales separation hypothesis

Assume local stat phys ensembles

$$\hat{\rho} \propto e^{-\sum_n \beta_n(x,t)\hat{Q}_n}$$



II. Local charges

$$\langle \hat{Q}_n \rangle = \int dx \, \mathrm{q}_n(x,t)$$

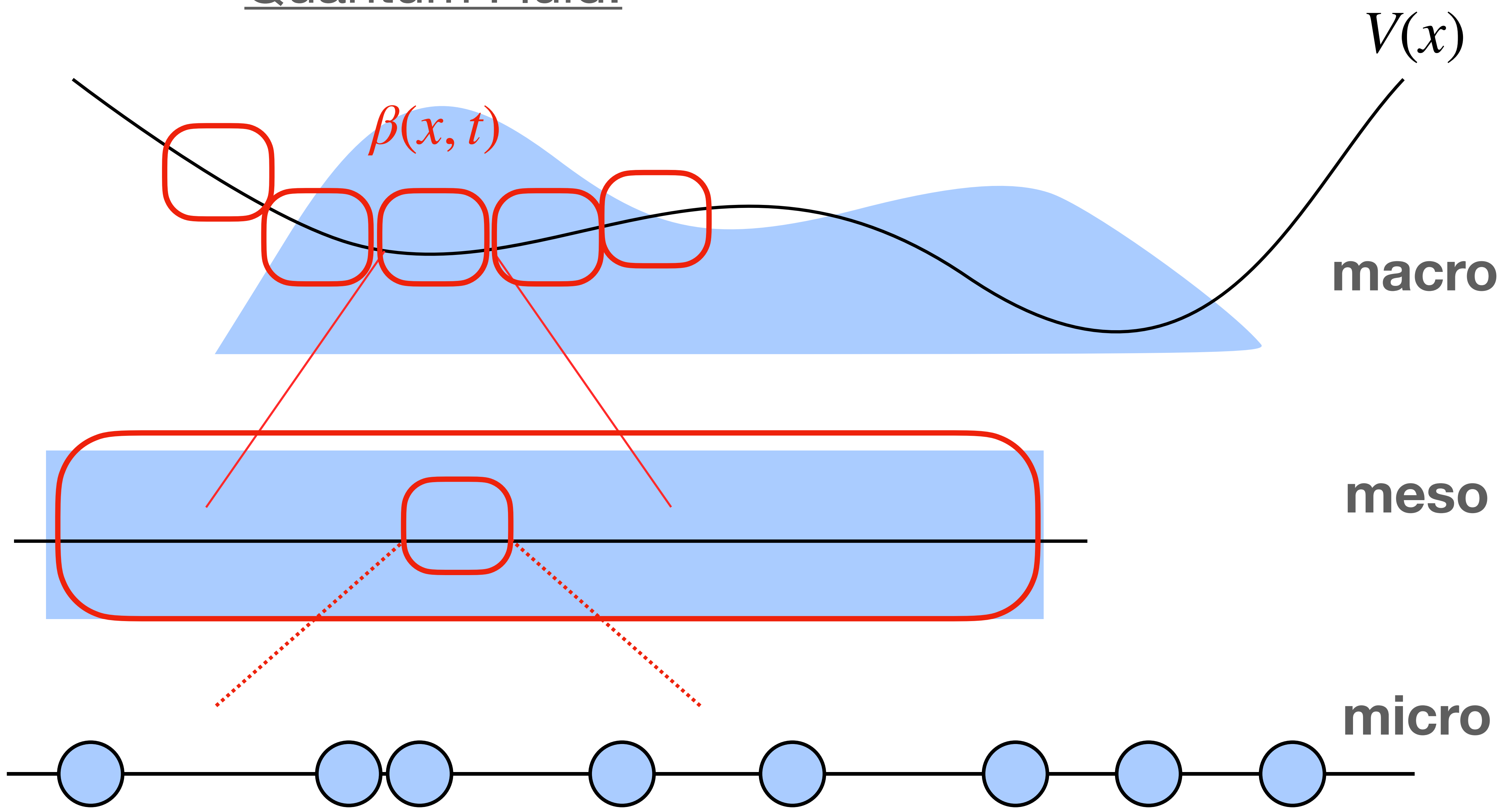
III. Current (*need eq of state*)

$$\mathrm{j}_n = \mathbb{F}[\{\mathrm{q}\}, t]$$

IV. Continuity equation

$$\partial_t \mathrm{q}_n + \partial_x \mathrm{j}_n = 0$$

Quantum Fluid:



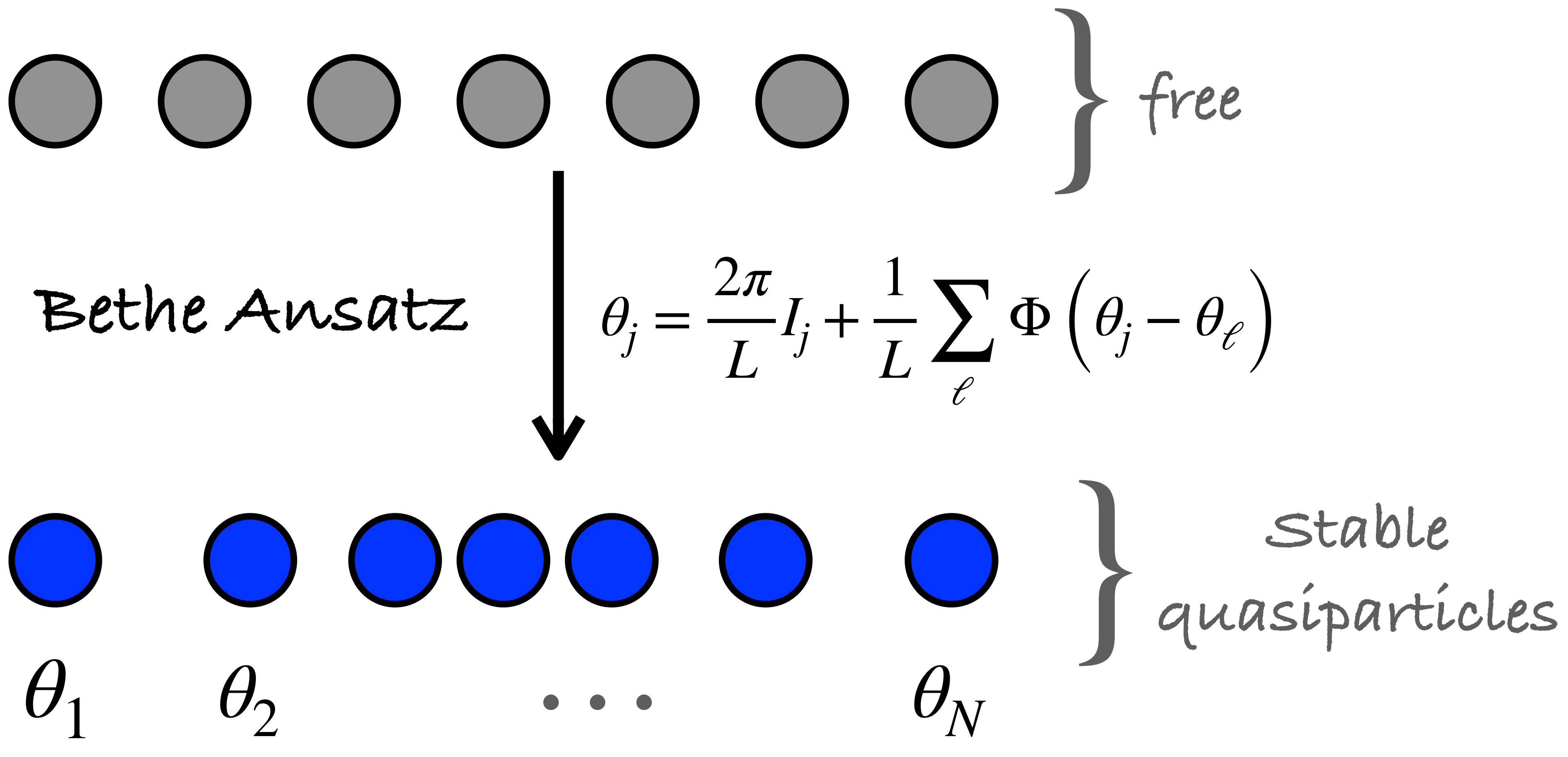
Integrable models

⚠️ **Importance of exact solutions** ↔ infer general mechanisms on nonequilibrium physics

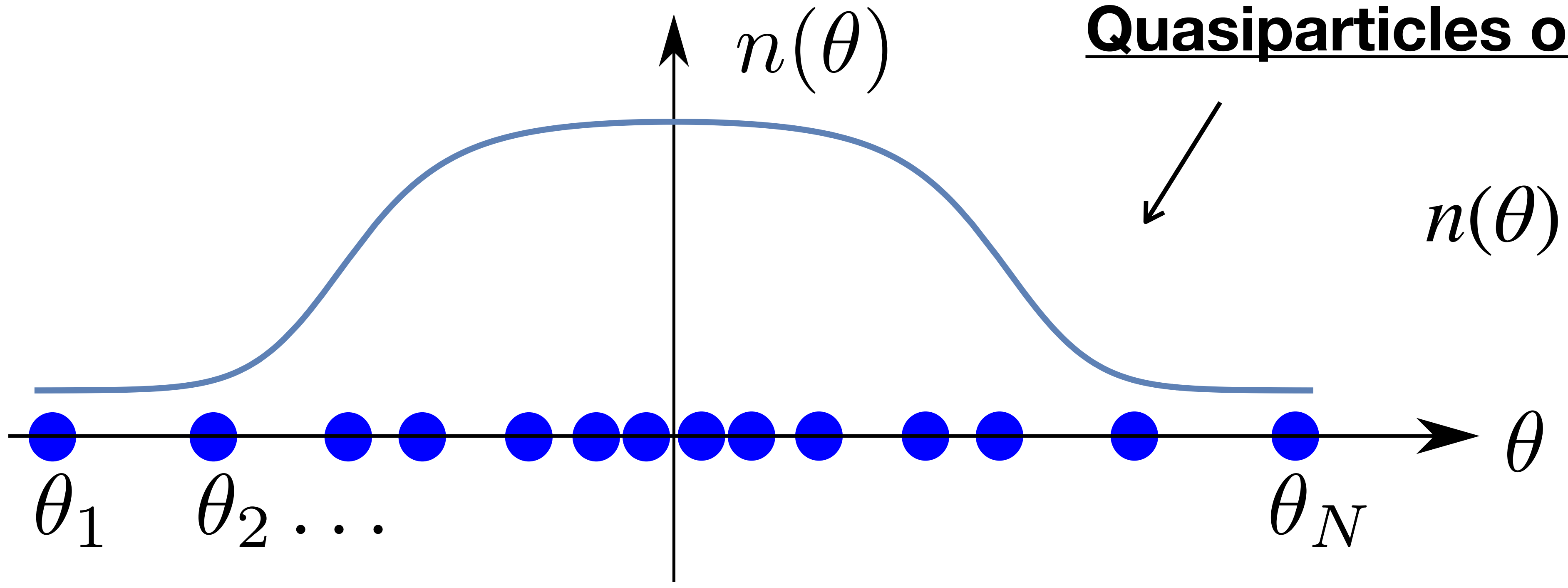
❖ Diffusive transport

❖ Ballistic transport → **Integrable models**

- Models featuring ∞ cons. laws $[\hat{H}, \hat{Q}_n] = 0$
- Many-body wavefunction: $|\Psi\rangle = |\theta_1, \ldots, \theta_N\rangle$



Thermodynamic limit:



Quasiparticles occupation

$$n(\theta) = \frac{1}{1 + e^{\beta\epsilon(\theta)}}$$

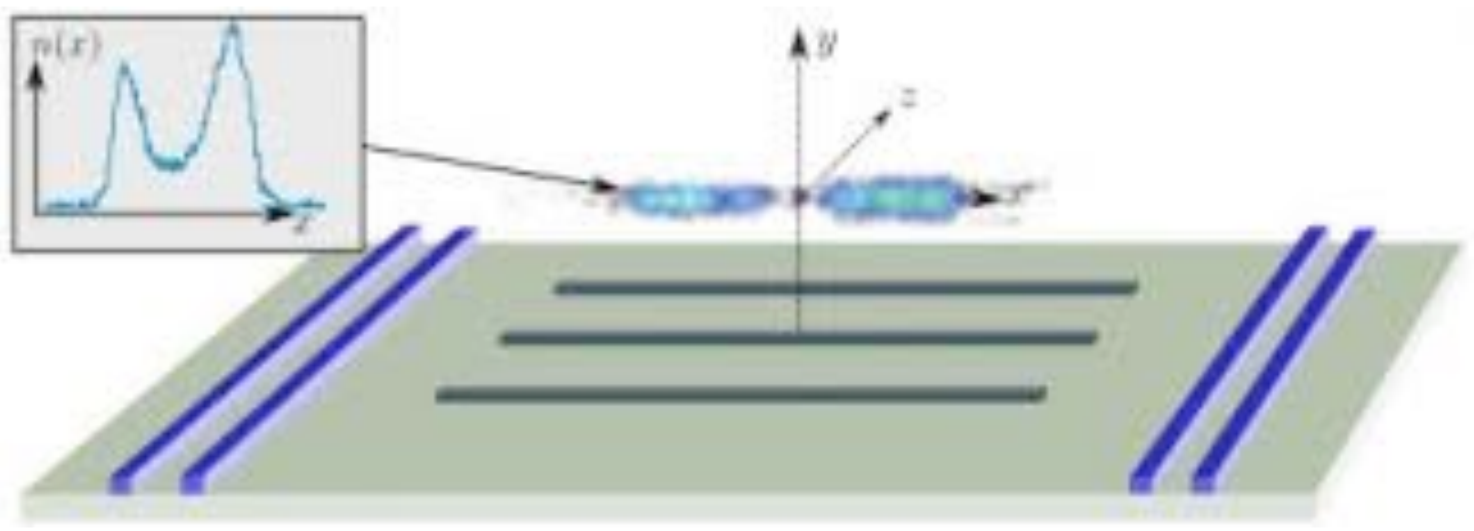
Realizations in Cold-Atom Experiments

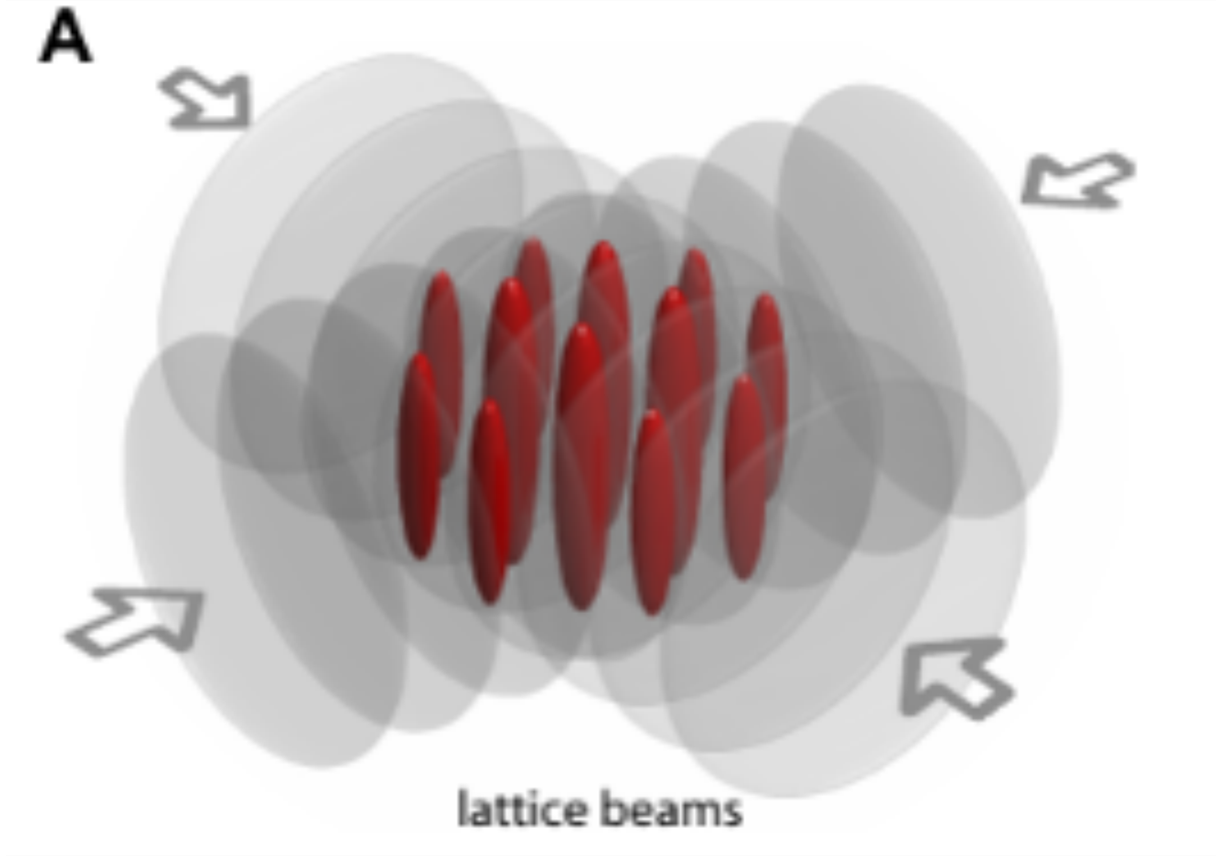
Ultracold atom gases

- Quasi-1D geometries
- tunable interaction and trapping potentials
- temperatures up to nK

- **Atom chips**

  magnetic fields generated by micro-fabricated wire patterns on a chip to trap and manipulate cold atoms very close to the chip surface





- **Optical lattices**

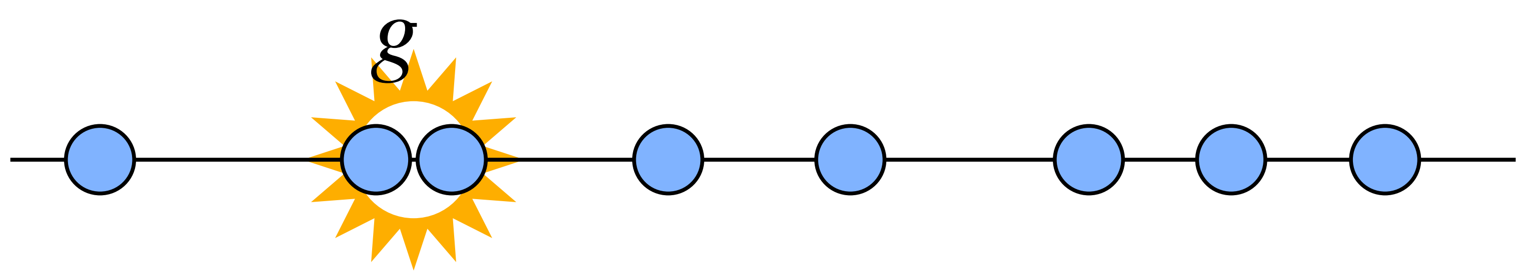
  interfering laser beams used to create periodic potentials that trap neutral atoms at the (anti)nodes of the standing wave. This generates an array of quasi 1D tubes

Reference model:

Lieb-Liniger model [Lieb, Liniger 1963]

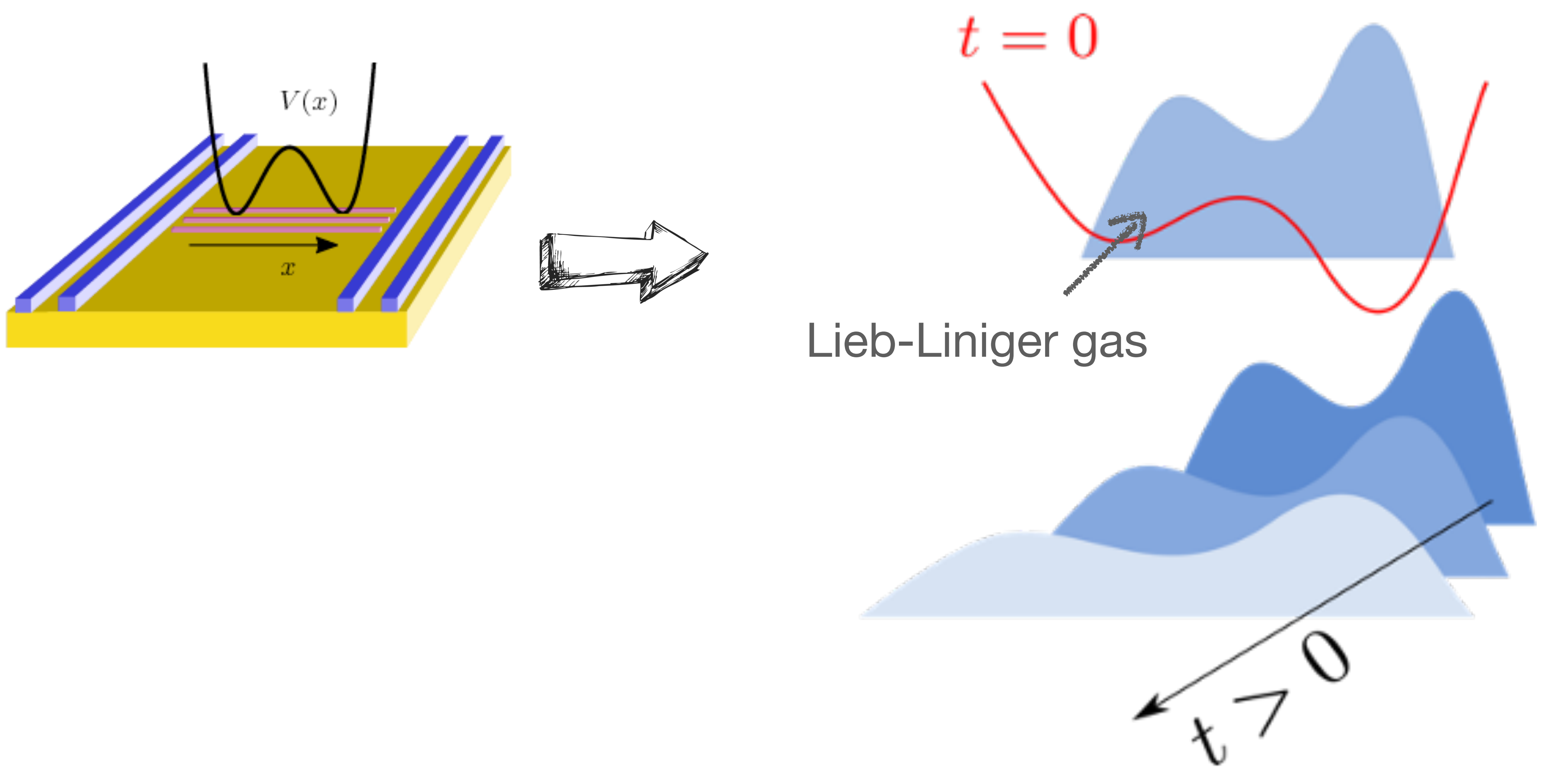
*Describe Bose gases in 1D with contact (repulsive) interactions

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g\sum_{1\leq i<j\leq N}\delta(x_i - x_j) + \sum_{i=1}^{N}V(x_i)$$



✔ **Integrable ($V = 0$)**

Setup of the problem



Protocol:

- Prepare the system at equilibrium with some potential $V_0(x)$

Low temperature: $|\Psi(0)\rangle \approx |\mathbf{GS}\rangle$

- At $t = 0$ sudden change of the confining trap

$$|\text{GS}\rangle \longmapsto |\Psi(t)\rangle = e^{-it\hat{H}}|\text{GS}\rangle$$

nonequilibrium dynamics

Goal:

- Local quantities (e.g. particle density)
- **Quantum coherent effects** (e.g. one-body correlations and entanglement)

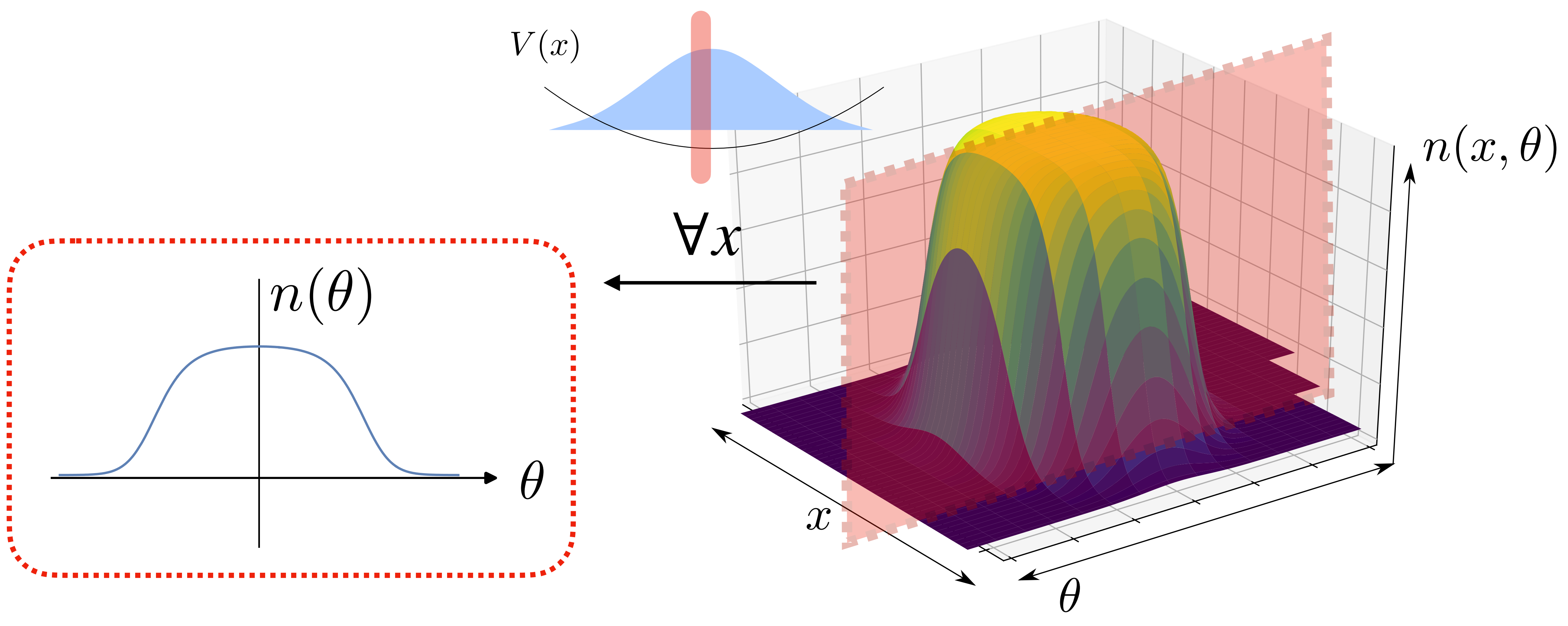
Why?

- **Theory prediction** for some experimental measurm. (beyond current hydrodynamic results)
- Better understand the nonequilibrium dynamics of low-temperature 1D gases

Hydrodynamics of Integrable models

✗ Integrability is lost $V \neq 0 \rightarrow$ **Emergent Hydrodynamics**

⚠ ∞ cons. laws: $\partial_t \mathtt{q}_n + \partial_x \mathtt{j}_n = 0$ not a good strategy!

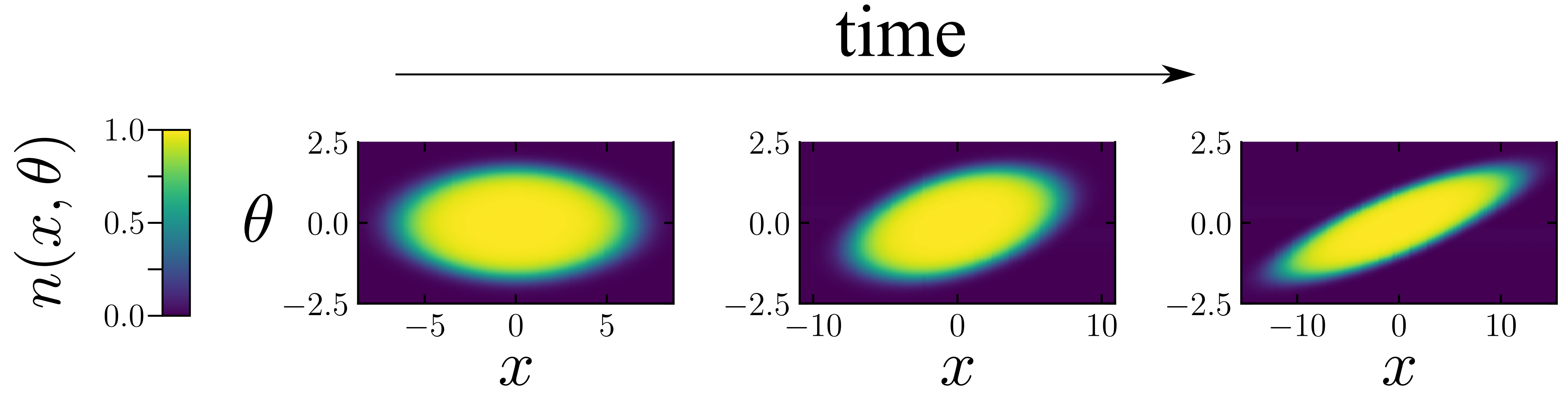


$t = 0$: Phase-space distribution of quasiparticles

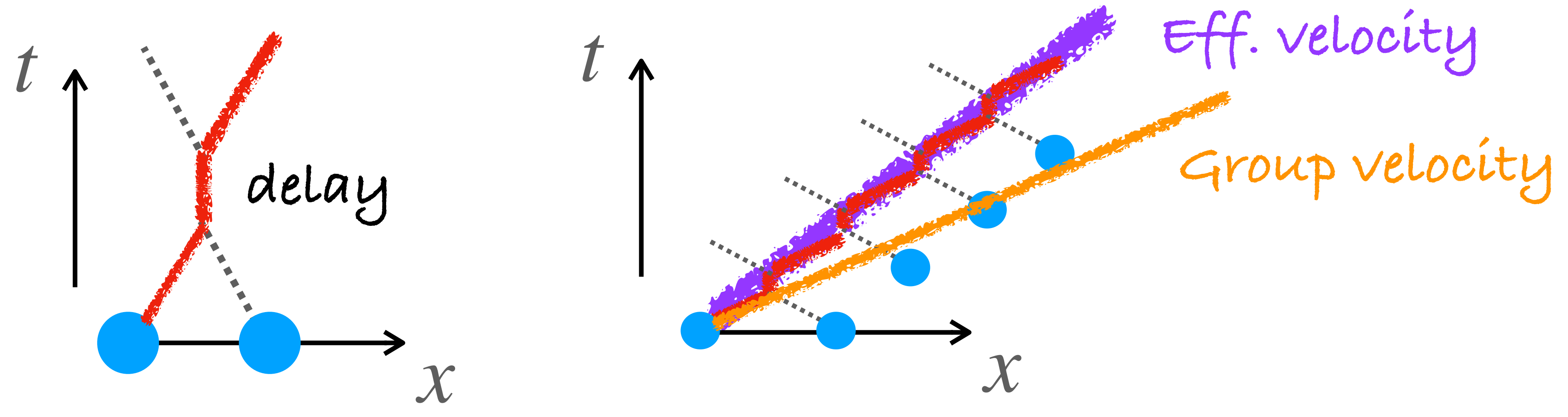
$t > 0$:

Phase-space dynamics → **Euler equation**

$$(\partial_t + v^{\text{eff}}[n]\partial_x) n_t(x,\theta) = 0$$



effective velocity : $v^{\text{eff}}(\theta) = \theta - \int d\theta' \rho_s(\theta') n(\theta') \varphi(\theta - \theta') [v^{\text{eff}}(\theta) - v^{\text{eff}}(\theta')]$



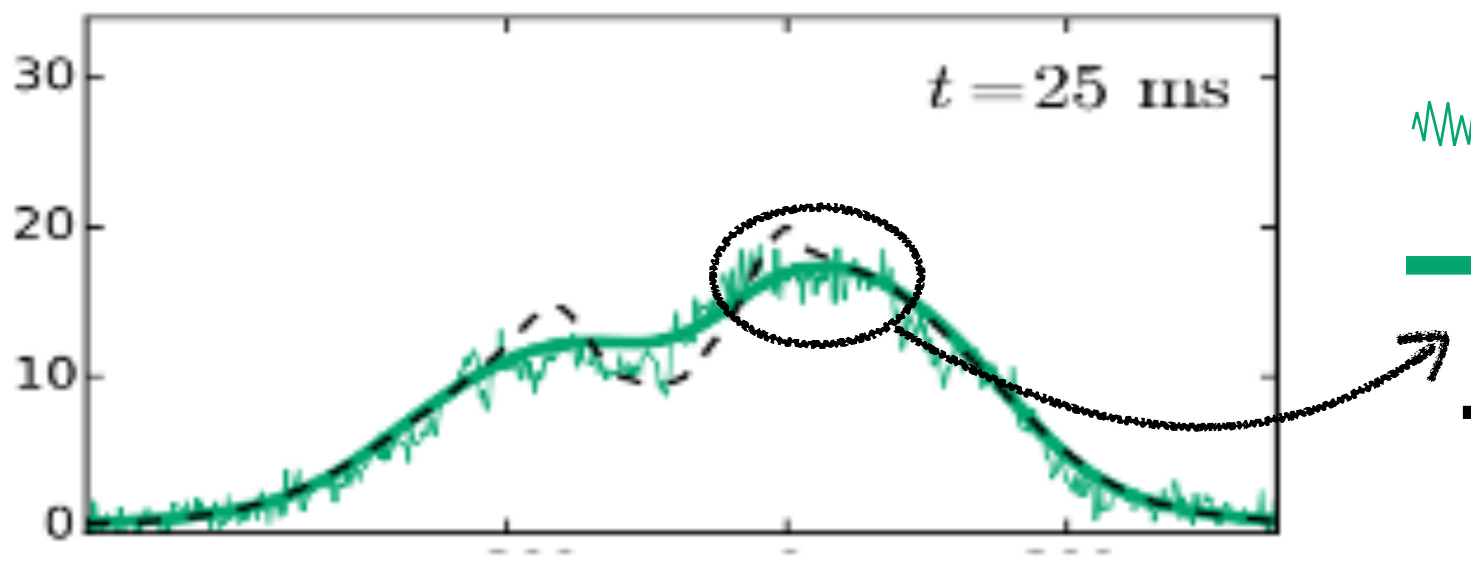
[Bertini et al, 1605.09790]
[C.-Alvaredo et al, 1605.07331]

Missing Quantum Fluctuations

Quantitative results for (mean) charges/currents

[Bouchoule group, 1810.07170]

+many more Weiss, Schmiedmayer, Lev, Nagerl, etc



experimental data

GHD match the experiment ✓

conv. hydrodynamics ✗

Missing fluctuations in the hydrodynamic theory

fluctuations — thermal, quantum, noise, …

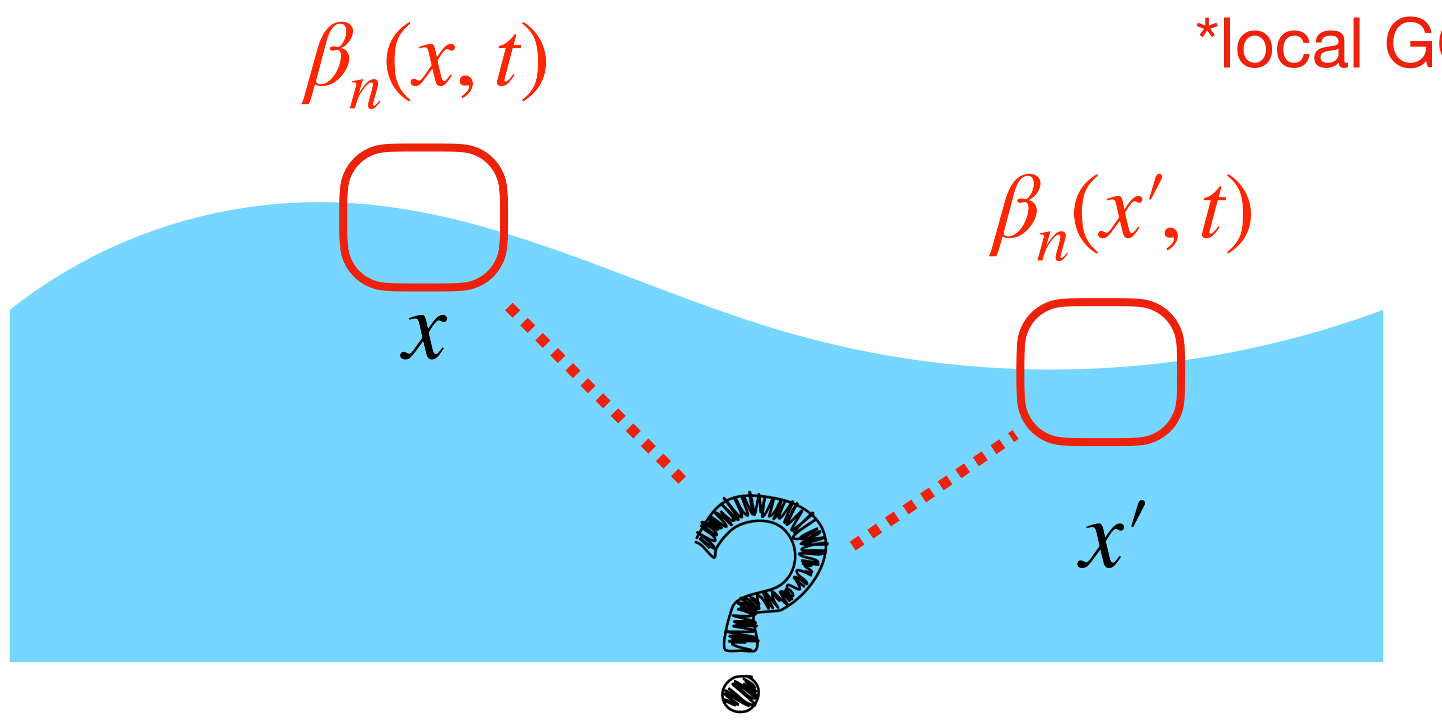
↓

zero-temperature limit

❖ What we miss…

- Zero-temperature entanglement
- Equal-time correlation functions
- Quantum corrections to charge/currents profiles

*local GGE description



*absence of long-range correlations between x and x'

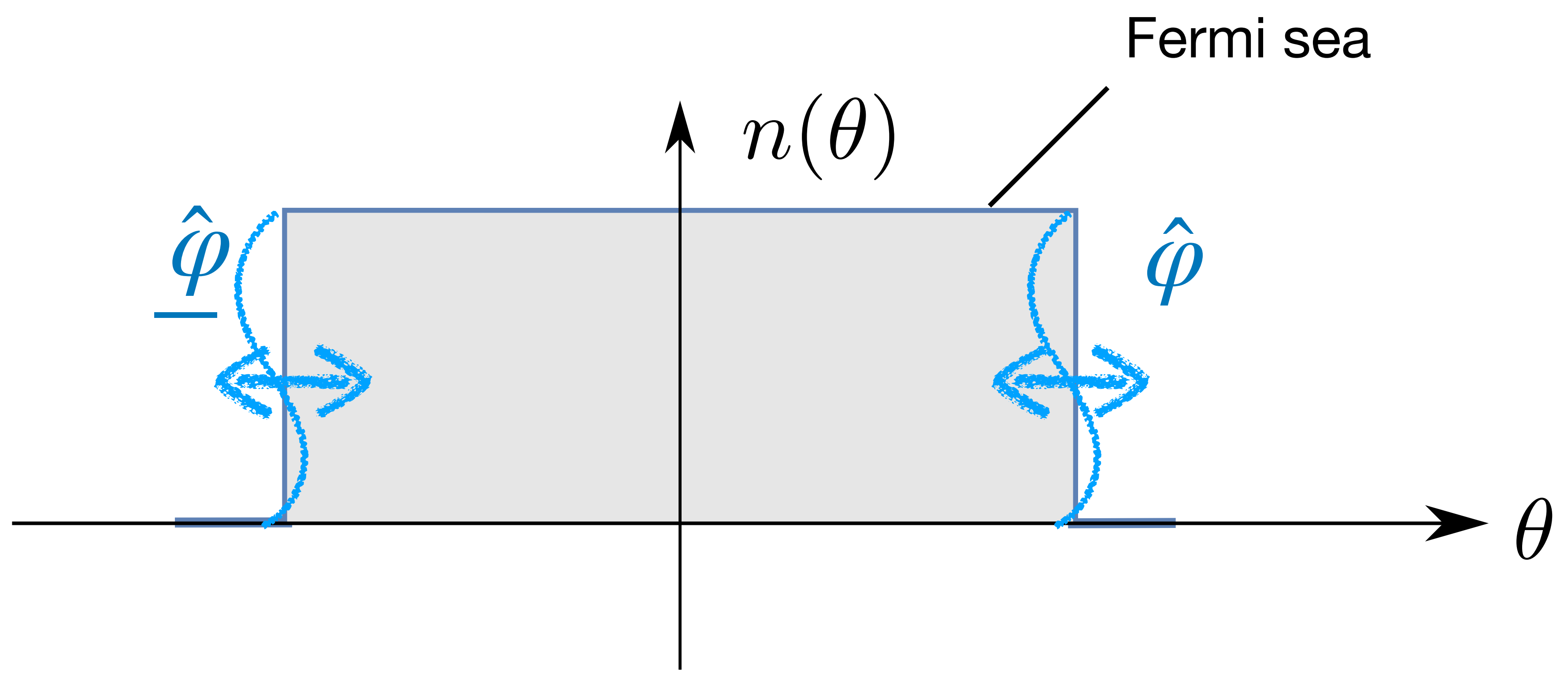
Re-quantization of the theory?

Incorporating Quantum Fluctuations

❖ back to **equilibrium**:

Ground state of integrable models

- Fermionic quasiparticles → **Fermi sea**
- Relevant quantum fluctuations are **particle-hole excitations**
- **effective description** with fluctuating fields at the Fermi edges



‣ Bosonization

$$\hat{\psi}^\dagger = B_0 : \exp\left(\frac{i\hat{\varphi} + i\underline{\hat{\varphi}}}{2\sqrt{K}}\right) : + \dots$$

$K =$ compressibility of the gas

$\hat{\varphi}, \underline{\hat{\varphi}} =$ fluctuating fields with 1D effective Hamiltonian

‣ Tomonaga-Luttinger liquid

GFF

$$\hat{\mathscr{H}} = \int dx \, \frac{v_s}{4\pi K}\left((\partial_x \hat{\varphi})^2 + (\partial_x \underline{\hat{\varphi}})^2\right)$$

❖ $\langle \hat{\varphi}(x)\hat{\varphi}(x')\rangle = \log[-i(x-x')]$: 2pt function fixed by CFT

+ Wick's theorem:

$$g_1(x) = \langle \hat{\psi}^\dagger(x)\hat{\psi}(0)\rangle \sim |x|^{-\frac{1}{2K}}$$

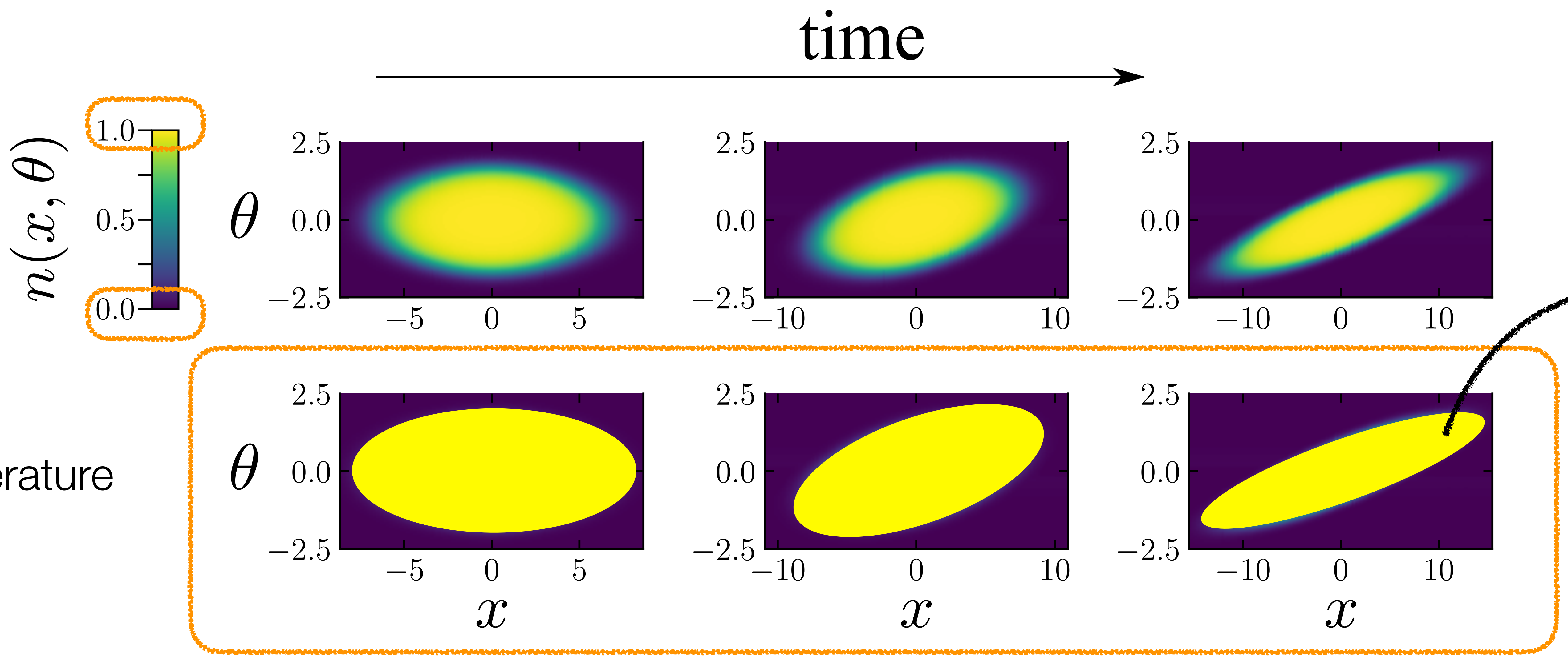
*algebraic decay of correlations

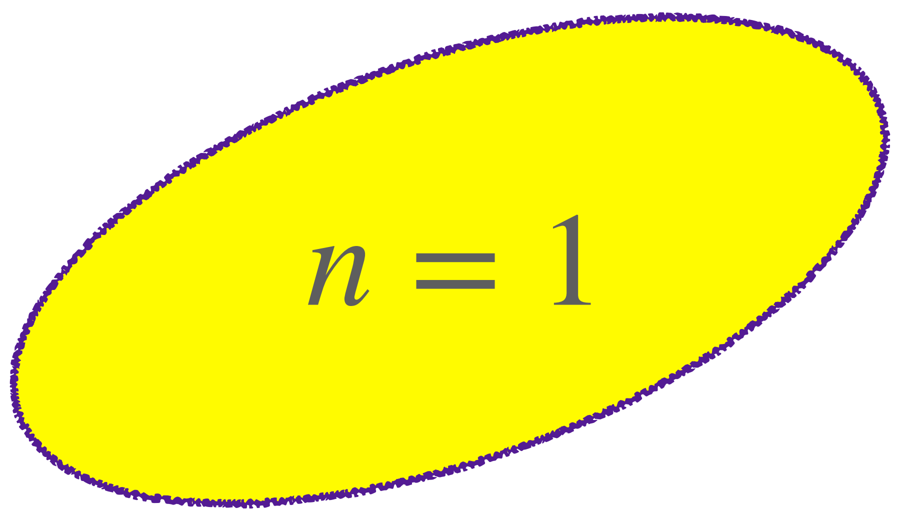
Incorporating Quantum Fluctuations

❖ Out of equilibrium?

Generalized Hydro:

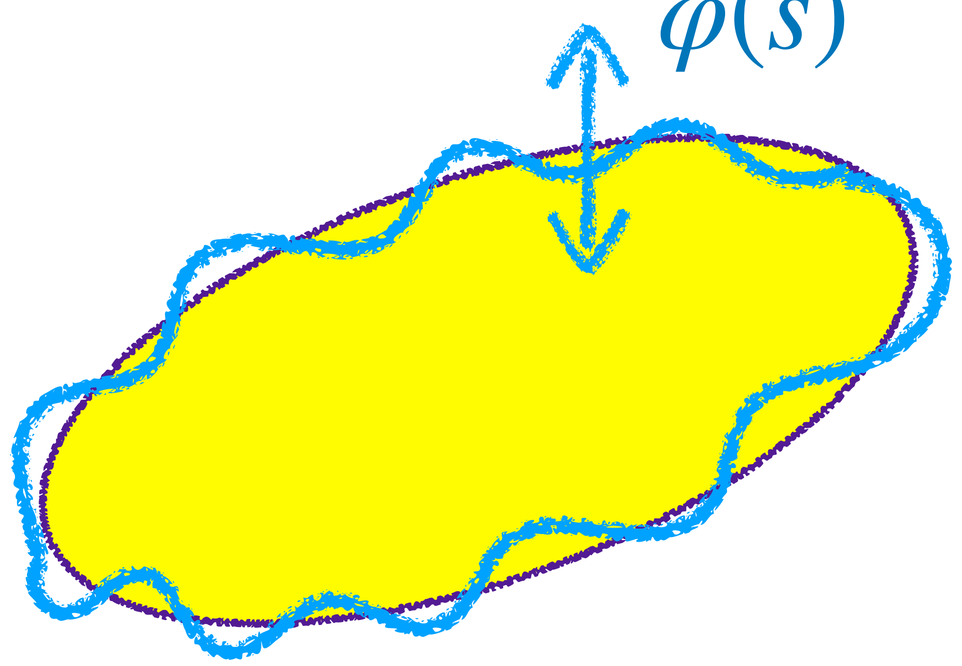
zero-temperature limit



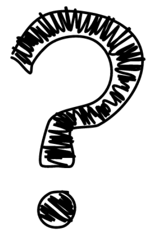


Incompressible droplet in phase space

❖ Quantum Fluctuating Hydrodynamics

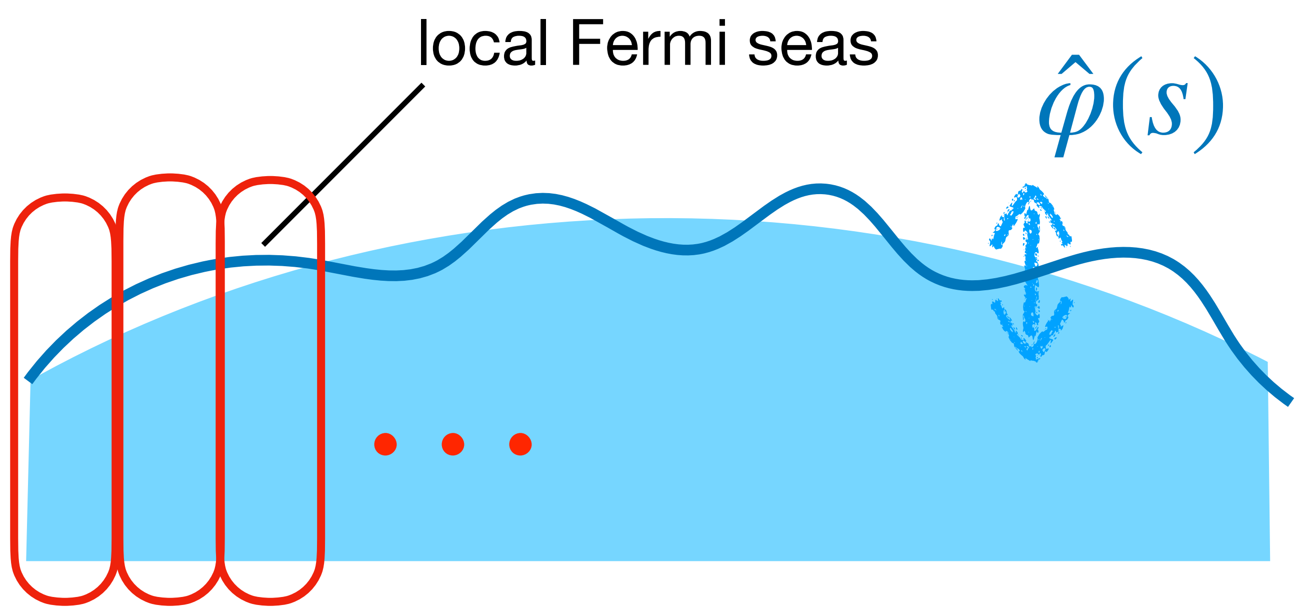


s = Fermi surface $(x_F(s); \theta_F(s))$

How 

Incorporating Quantum Fluctuations

$t = 0$: Ground state of inhomogeneous gas



$K = K[\rho(x)]$ **conformal invariance is broken**

Inhomogeneous GFF

$$\hat{\mathscr{H}} = \int dx \, \frac{v_s(x)}{2\pi} \left(\frac{(\partial_x \hat{\Phi})^2}{K(x)} + K(x)(\pi \hat{\Pi})^2 \right)$$

2D Electrostatic analogy:

$$[d^2\mathbf{x} = dx^2 + v_s^2(x) d\tau^2 \propto ds^2 + d\tau^2]$$

$$\nabla 4K(\mathbf{x}) \nabla G_0^{[\Theta\Theta]}(\mathbf{x}, \mathbf{y}) = 4\pi \delta^{(2)}(\mathbf{x} - \mathbf{y})$$

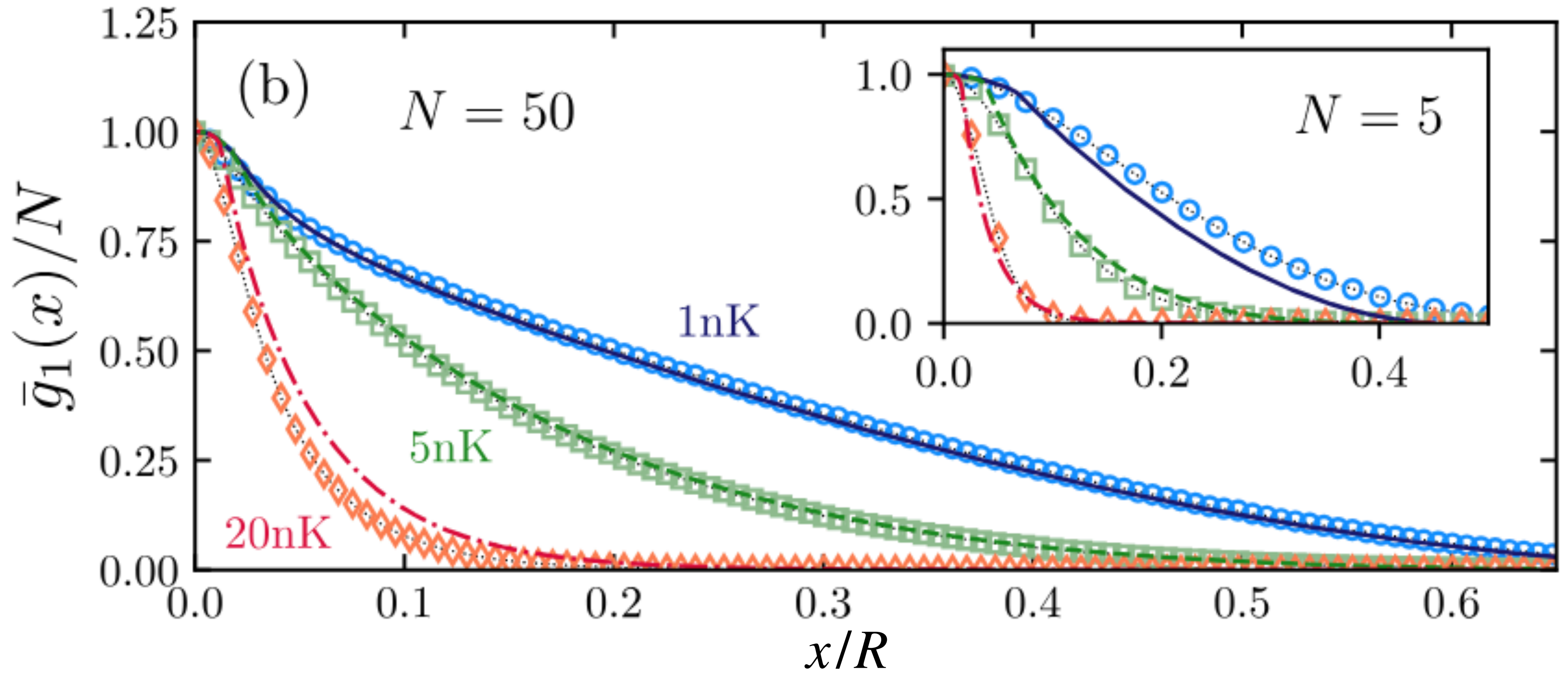
numerical input ✓ **Scopa** et al, 2005.10214

$$C_0 = \begin{pmatrix} G_0^{[\Phi\Phi]} & G_0^{[\Phi\Theta]} \\ G_0^{[\Theta\Phi]} & G_0^{[\Theta\Theta]} \end{pmatrix}$$

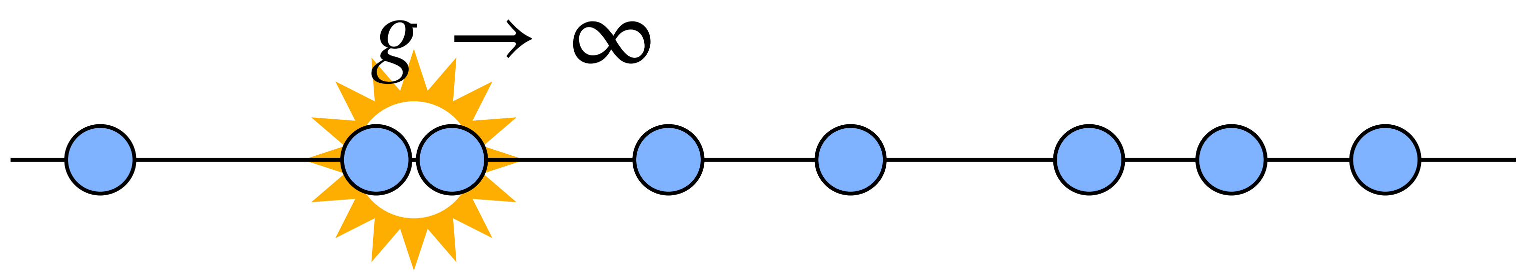
Finite interaction,
Finite (small) temperature

$$g_1(x, y) = e^{-G_0^{[\Theta\Theta]}(s_x, s_y)} \prod_{z=x,y} \frac{|B_0(z)|^2 \rho(z)^{\frac{2K(z)-1}{4K(z)}}}{(v_z)^{\frac{1}{4K(z)}}} e^{\frac{1}{2} G_0^{[\Theta\Theta]}(s_z)}$$

Takacs, Zhang, Calabrese, Dubail, Rigol, **Scopa**, 2409.16929

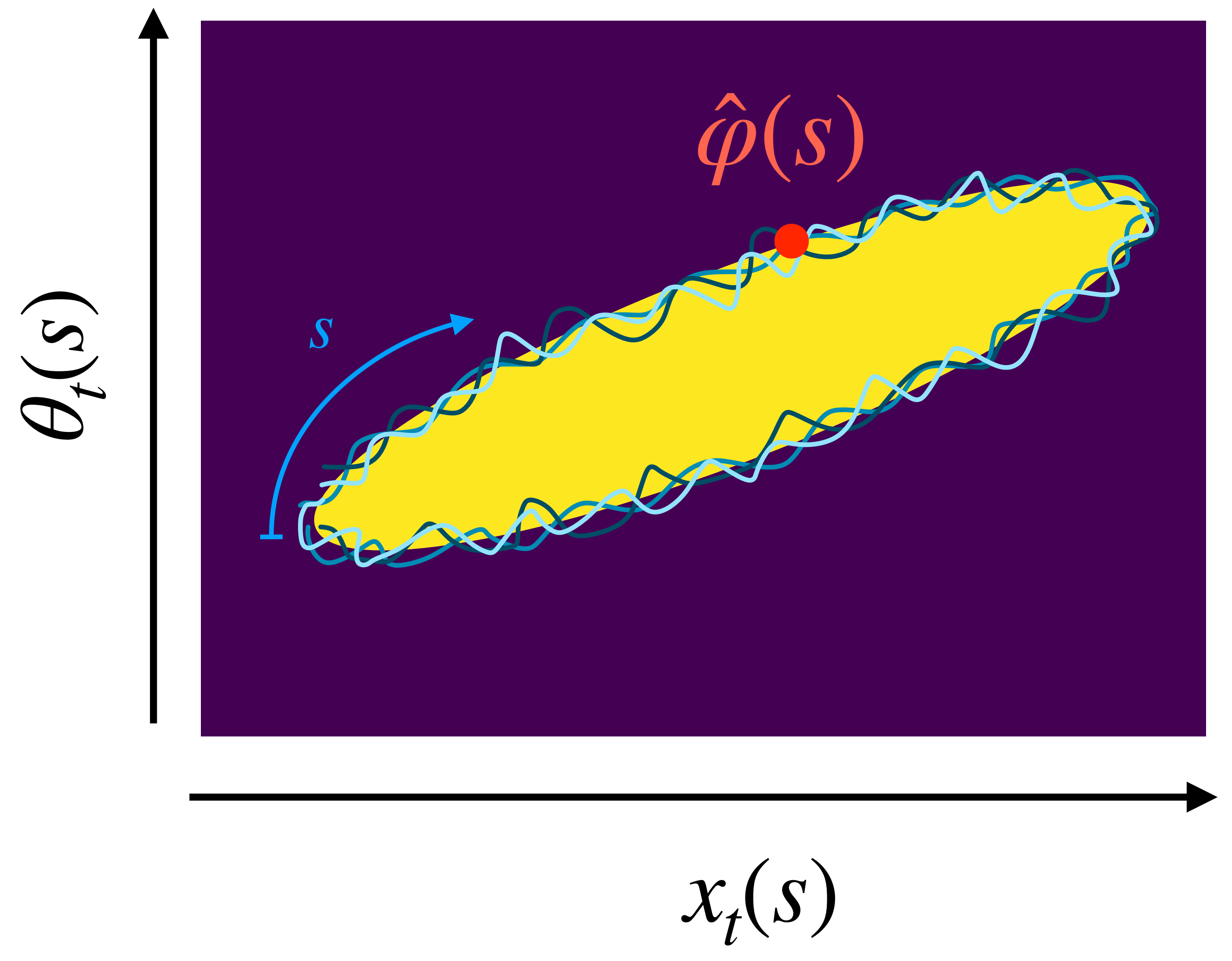


The strong repulsion limit

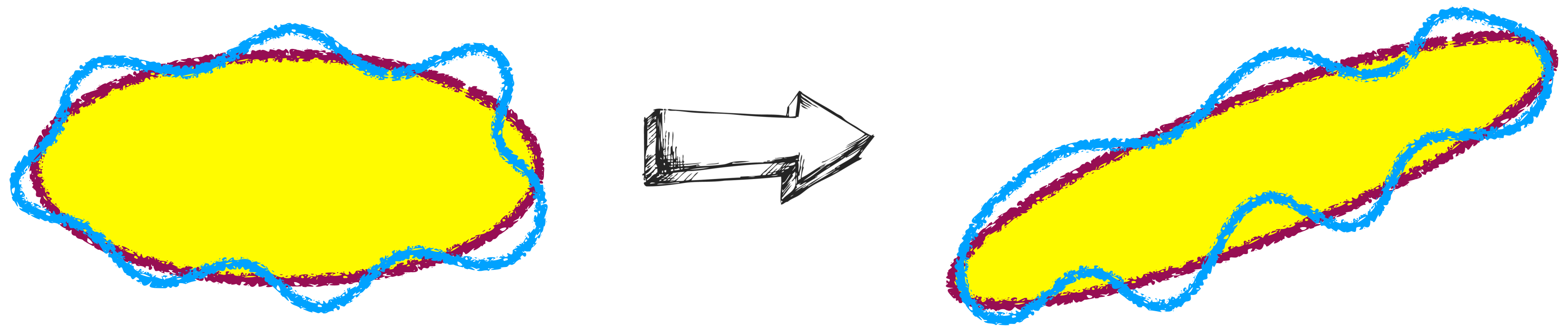


Tonks-Girardeau limit ($K \equiv 1$)

- gas of hardcore bosons
- $\langle \hat{\varphi}(s)\hat{\varphi}(s') \rangle$ = boundary CFT **in s space**
- Gaussian correlations are comovingly transported with the background:



$$\langle \hat{\varphi}(s)\hat{\varphi}(s') \rangle_{t=0} = \langle \hat{\varphi}(s)\hat{\varphi}(s') \rangle_{t>0}$$

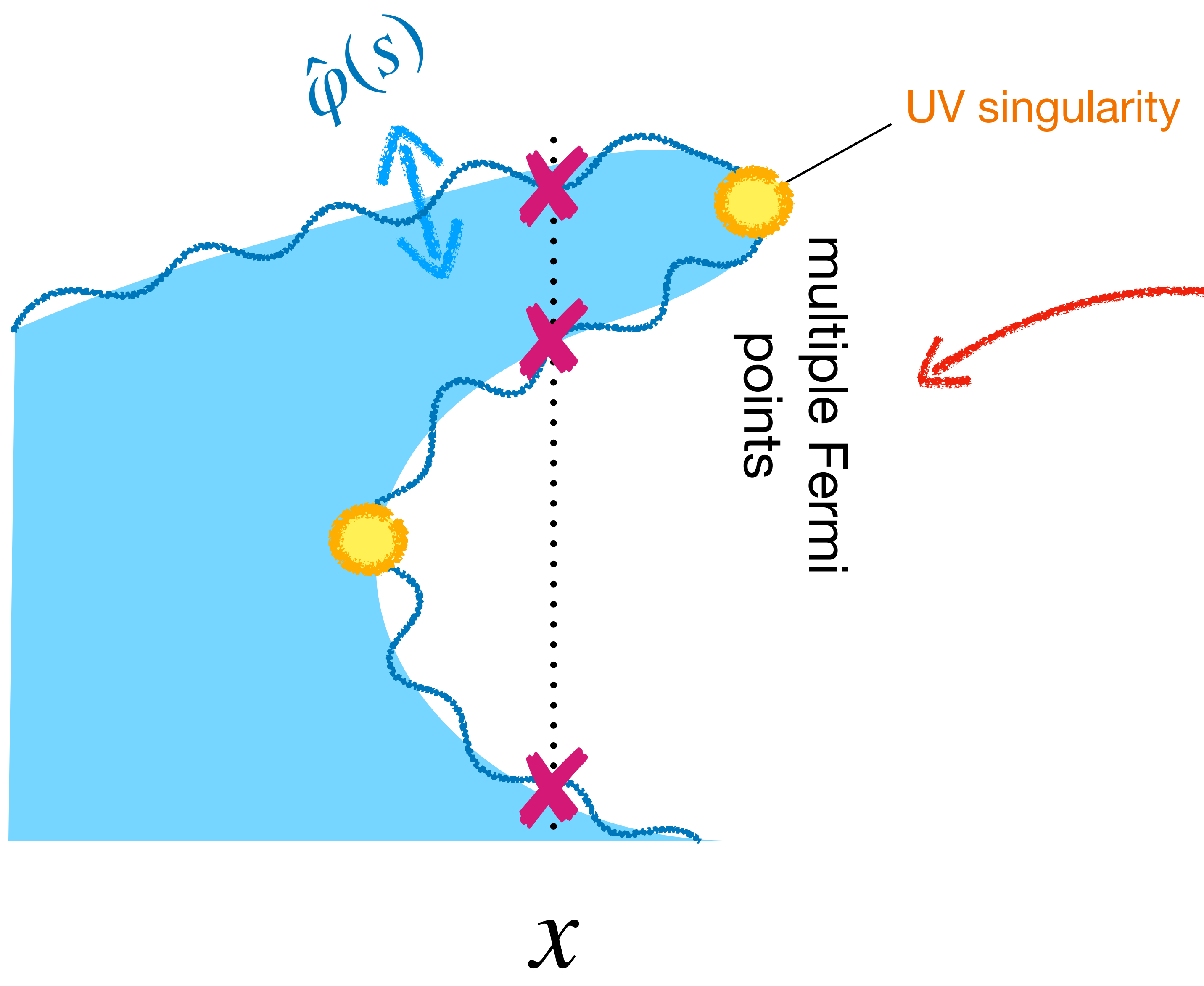


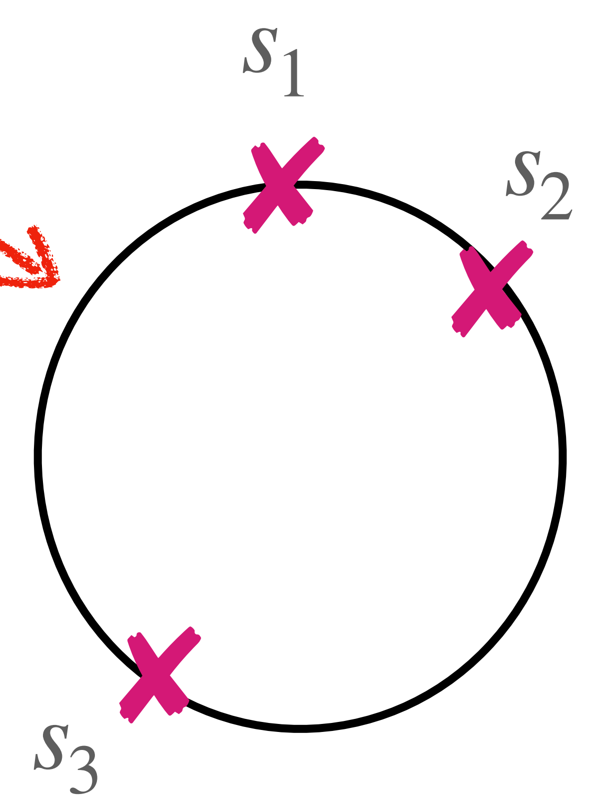
analytical calculations

Bosonization Out of Equilibrium

Scopa et al, 2301.04094

❖ Summarizing:





$\langle \hat{\varphi}(s)\hat{\varphi}(s')\rangle$ = CFT

Non-univ. Amplitude:

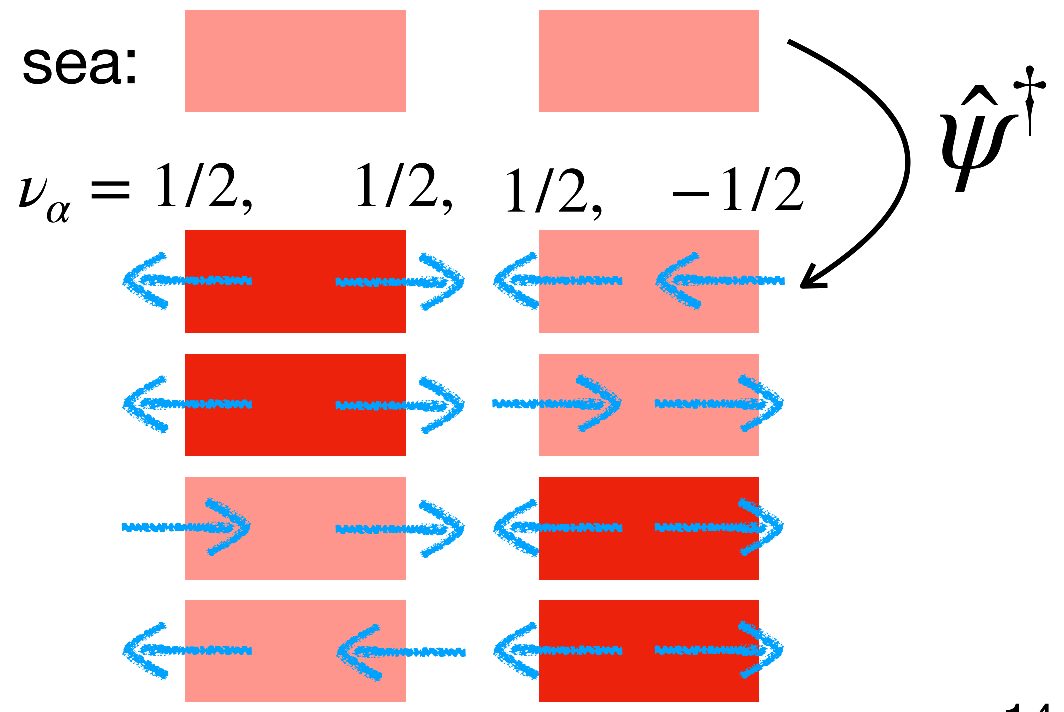
Semiclassical WKB phase

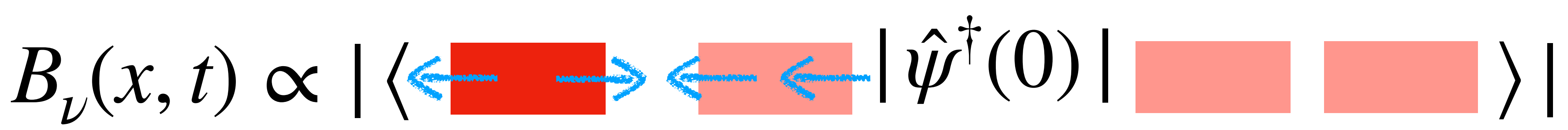
$$\hat{\psi}^\dagger = \sum_{\vec{\nu}} B_{\vec{\nu}}(x,t) \prod_{\alpha} e^{i\nu_\alpha \phi_t(x)} : e^{i\nu_\alpha \hat{\varphi}(s_\alpha)} : + \ldots$$

Sum over configurations

Guassian fields

Split Fermi sea:





One-Body Correlations and Momentum distribution

$$g_1(t;x,y) = \langle \Psi_t | \hat{\psi}^\dagger(x)\hat{\psi}(y) | \Psi_t \rangle \longrightarrow n_t(p) = \iint dxdy\; e^{ip(x-y)} g_1(t;x,y)$$

Scopa et al, 2301.04094

measured in experiments

[Kinoshita et al, Nature **440**, 900 (2006)]

* Any trap quench $V_0(x) \to V_1(x)$

Fully analytical result:

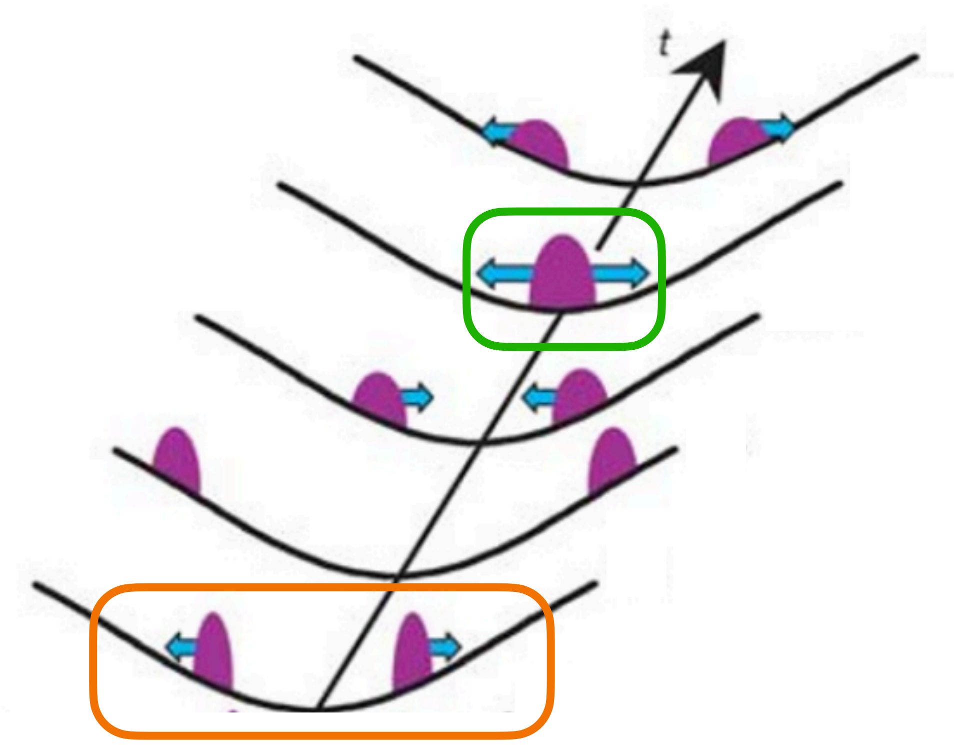
$$g_1(t;x,x') = \sum_{\vec{\nu},\vec{\nu}'} [\mathbf{G}_t(x)]^*_{\vec{\nu}} [\mathbf{F}_t(x,x')]_{\vec{\nu},\vec{\nu}'} [\mathbf{G}_t(x')]_{\vec{\nu}'}$$

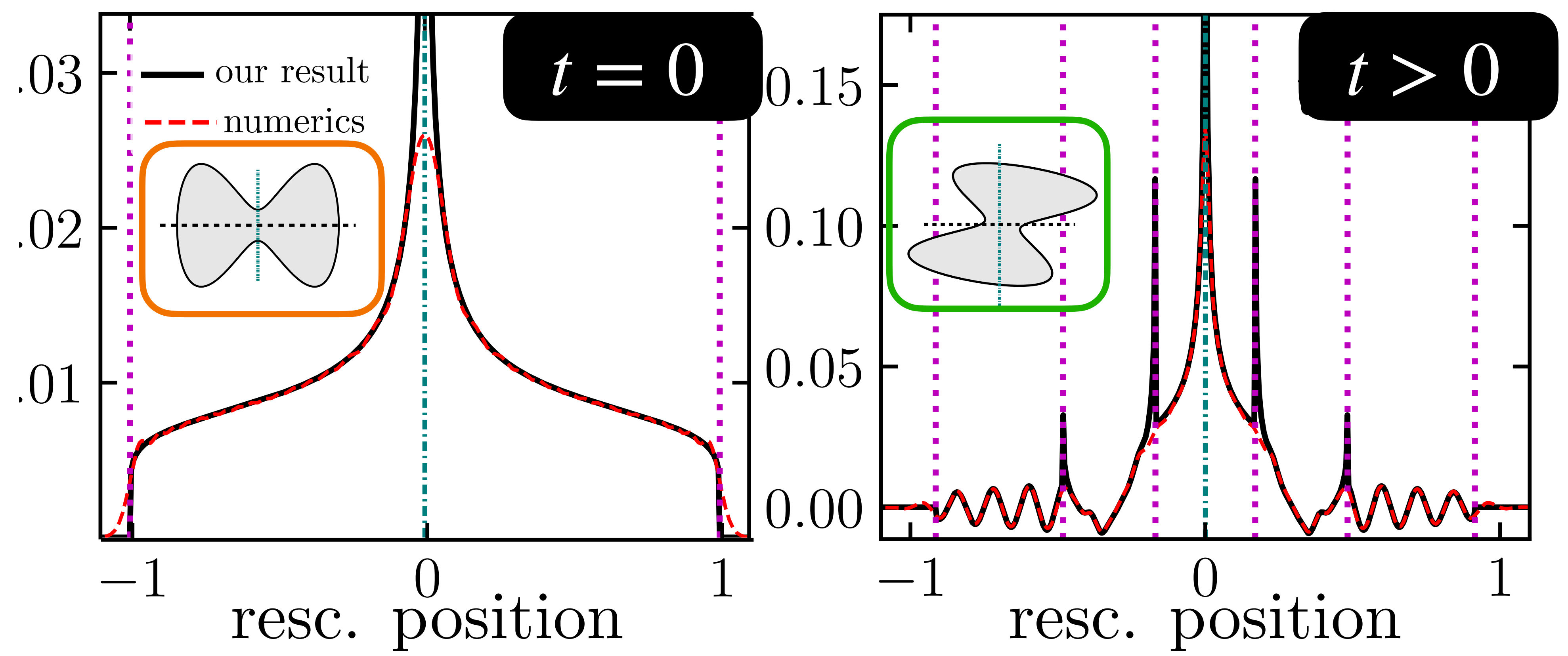
$$[\mathbf{F}_t(x,x')]_{\vec{\nu},\vec{\nu}'} = \frac{\prod_{a<b} |2\sin(\frac{s_a - s_b}{2})|^{\nu_a\nu_b} \prod_{c<d} |2\sin(\frac{s'_c - s'_d}{2})|^{\nu'_c\nu'_d}}{\prod_{j=1}^{2Q} \prod_{j'=1}^{2Q'} |2\sin(\frac{s_j - s'_{j'}}{2})|^{\nu_j\nu'_{j'}}}$$

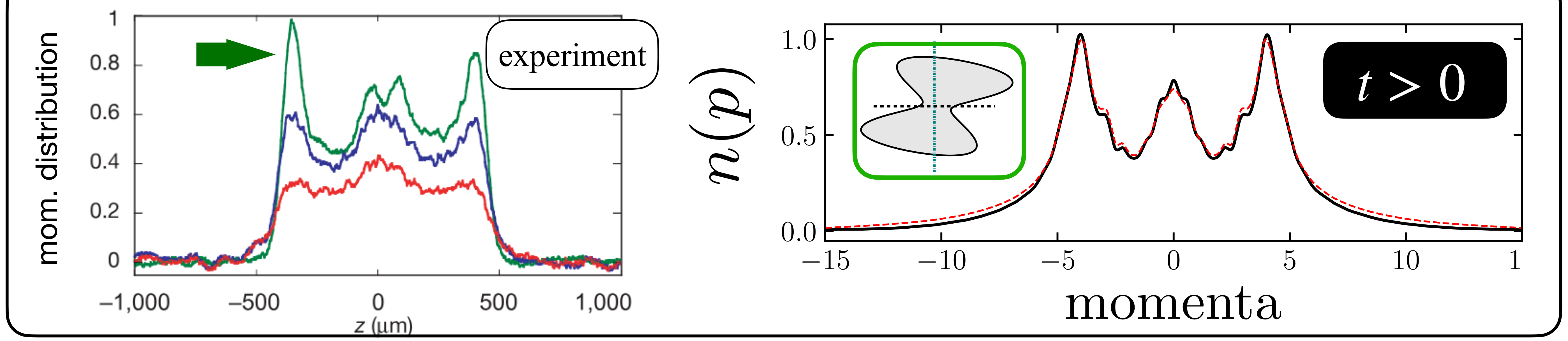
$$[\mathbf{G}_t(x)]_{\vec{\nu}} = \frac{1}{\sqrt{2}} \left(\frac{\mathrm{G}(3/2)^2}{\sqrt{\pi}} \right)^Q \prod_{j=1}^{2Q} \left| \frac{ds_j}{dx} \right|^{1/8} e^{i\nu_j \Phi(s_j)} \prod_{a<b}^{2Q} |\theta_F(s_a) - \theta_F(s_b)|^{\nu_a\nu_b}$$

Ready-to-use:

$$(x, x'; t) \to \{s_a\}, \{s'_a\}$$







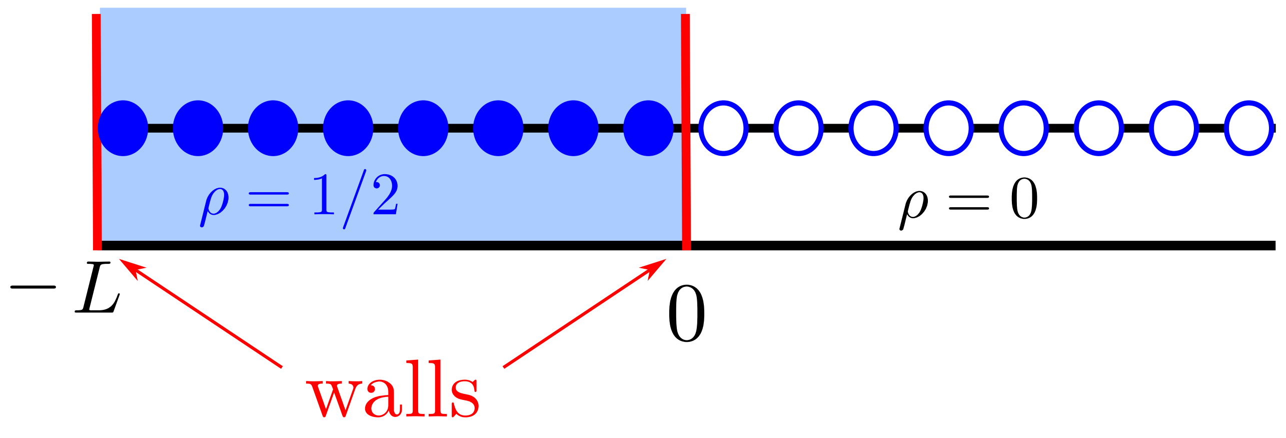
Before: Only harmonic traps

[Forrester-Frankel-Garroni-Witte, 2000]

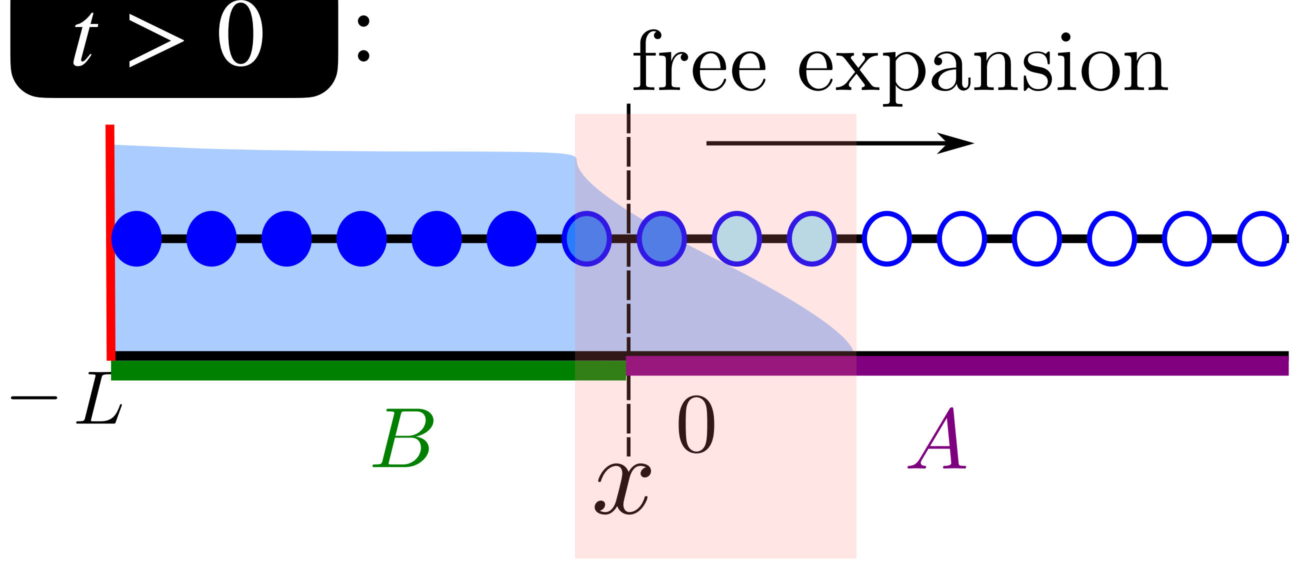
[Minguzzi-Gangardt, 2006]

Entanglement dynamics

$t = 0$:



$t > 0$:



bi-partition of the chain at position x : $S_1(t,x) = -\operatorname{tr}\hat{\rho}_A \log \hat{\rho}_A$

[Calabrese-Cardy, 2004]

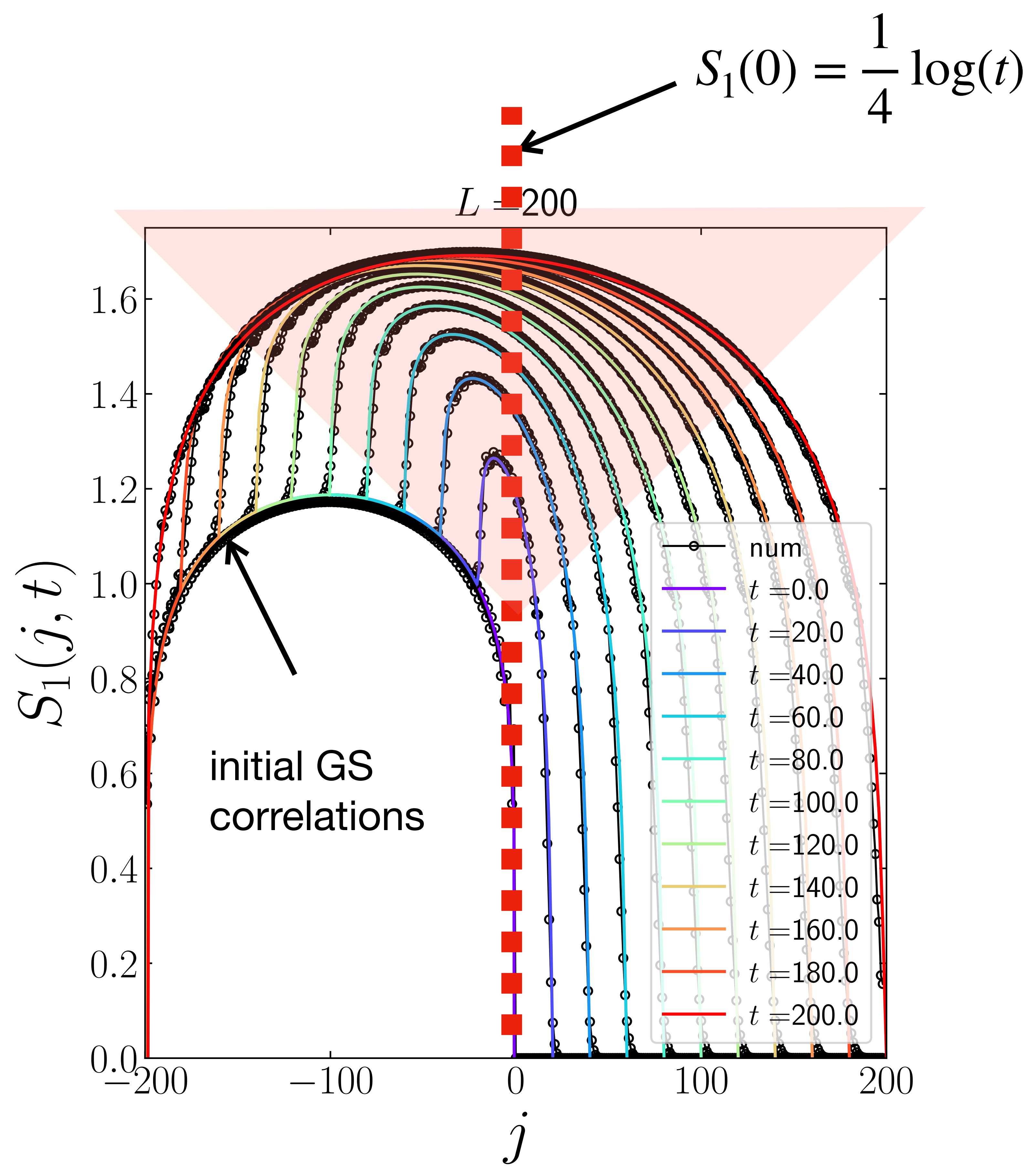
chiral twist fields

$$S_1(t,x) = \lim_{n\to 1} \frac{1}{1-n} \log \left[\left| \prod_\alpha \frac{dx_t(s_\alpha)}{ds} \right|^{-\Delta_n} \langle \prod_\alpha \hat{\tau}_n^+(s_\alpha)\hat{\tau}_n^-(s_{\alpha+1})\rangle \right] + f_{UV}(t,x)$$

Fully analytical result:

$$S_1(|x| \le t) = \frac{1}{6} \log \left(\frac{L}{2\pi} \sqrt{\left| \frac{x}{t}(1 - \frac{x^2}{t^2}) \right|} \left| \sqrt{1 + \sqrt{1 - \frac{x^2}{t^2}}} - \operatorname{sgn}(x) \sqrt{1 - \sqrt{1 - \frac{x^2}{t^2}}} \right| \left| \sin \frac{\pi(x-t)}{2L} \right| \right) + \Upsilon$$

Scopa et al, 2105.05054



Before: CFT-based conjectures

Summary and Conclusions

❖ Study the nonequilibrium dynamics of one-dimensional gases through hydrodynamic methods

- **One-dimensional Bose gas**: quasi-integrable description → **Generalized Hydrodynamics**

✓ Quantitative results for local transport properties (density and currents)

✗ Non-local correlations coming from quantum fluctuations

❖ **Quantum Fluctuating theory** on top of the hydrodynamic background

✓ One-body correlations and Momentum distribution out of equilibrium

✓ Entanglement dynamics

Thank you!