Odd dynamics in a chiral active bath

Cory Hargus, MIT & Université Paris Cité (MSC)

with Federico Ghimenti, Julien Tailleur, and Frédéric van Wijland

March 7, 2025

A thermal bath 🛀

Small particles **move as though alive** in water.

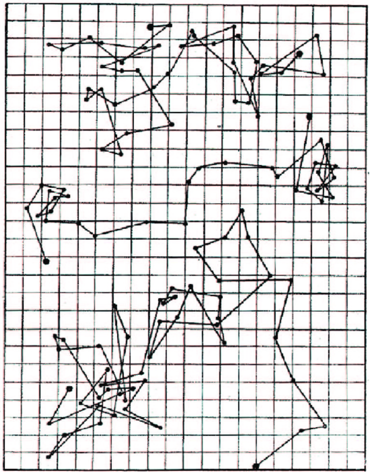
(*Brown, 1827*)

Grains of pollen

very evidently in motion

not limited to organic bodies

Fig. 6.



A thermal bath

Small particles **move as though alive** in water.

(*Brown, 1827*)

Grains of pollen

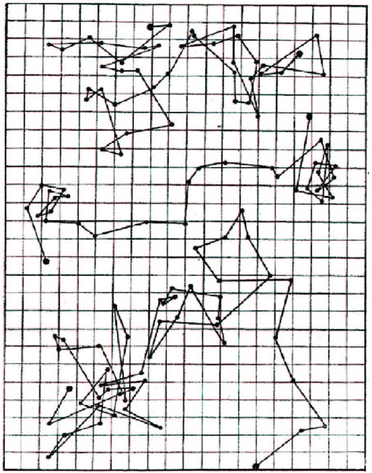
very evidently in motion

not limited to organic bodies

Brownian motion is a consequence of many **collisions**.

(*Einstein 1905, Perrin 1909*)

Fig. 6.



A thermal bath 🛀

Small particles **move as though alive** in water.

(*Brown, 1827*)

Grains of pollen

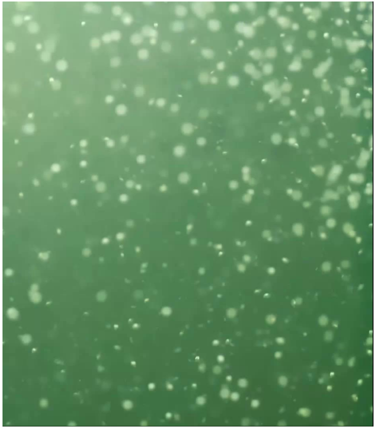
very evidently in motion

not limited to organic bodies

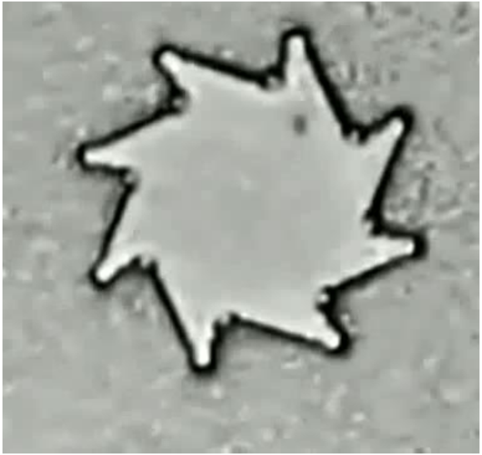
Brownian motion is a consequence of many **collisions**.

(*Einstein 1905, Perrin 1909*)

...but what if the bath really is alive?

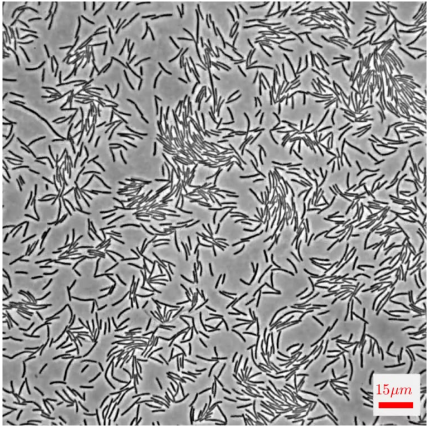


An active bath can drive a ratchet



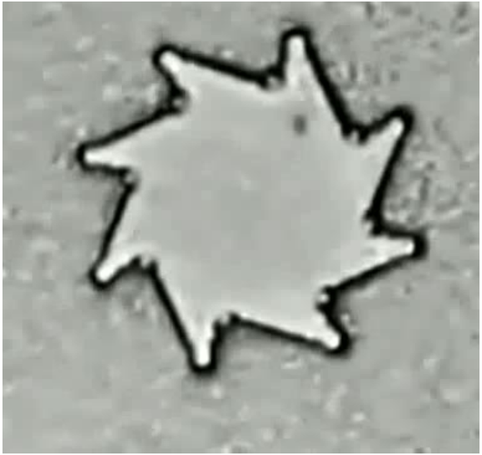
Di Lionardo et al. *PNAS* 2010

Chiral motion has collective benefits



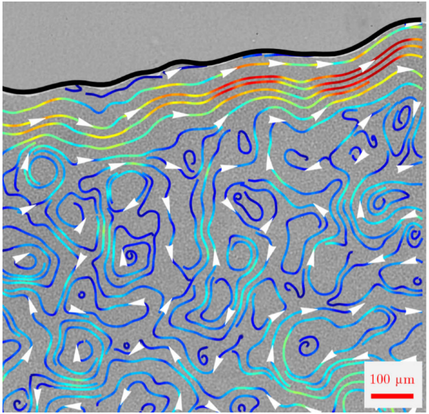
Li, Chaté, et al. *PRX* 2024

An active bath can drive a ratchet



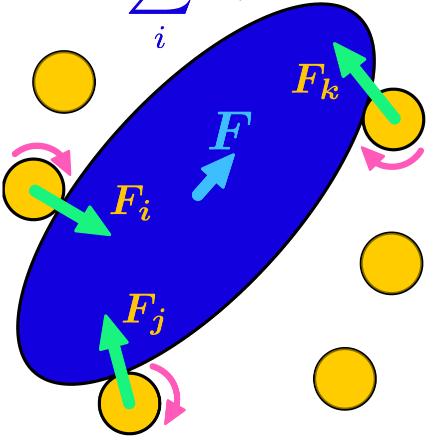
Di Lionardo et al. *PNAS* 2010

Chiral motion has collective benefits

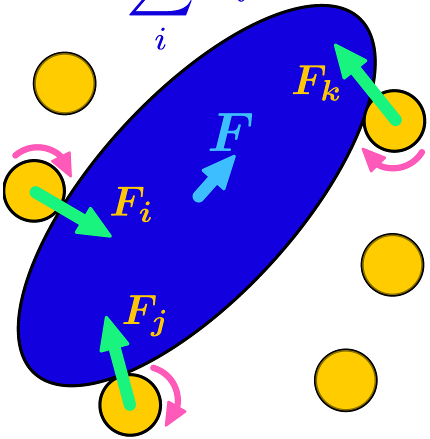


Li, Chaté, et al. *PRX* 2024

$$\boldsymbol{F} = \sum_i \boldsymbol{F}_i$$



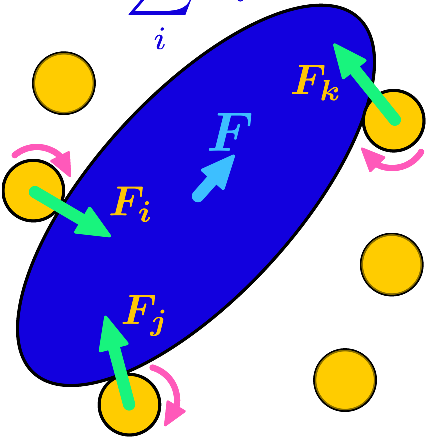
$$\boldsymbol{F} = \sum_i \boldsymbol{F}_i$$



- Irreversible ratchet motion

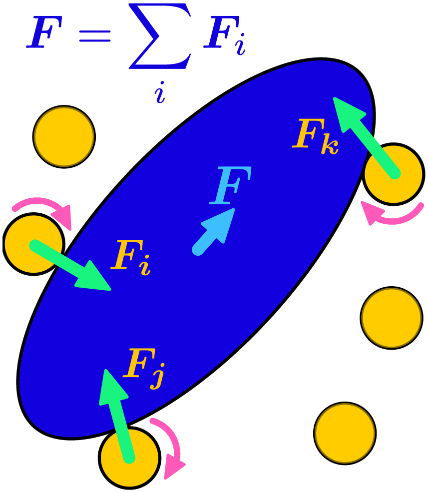
A passive object in a chiral active bath 🌀 + ⚡ + 🛀

$$\boldsymbol{F} = \sum_i \boldsymbol{F}_i$$



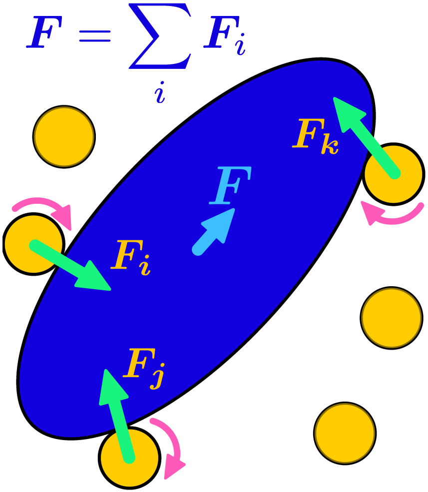
- Irreversible ratchet motion
- Odd diffusion and odd mobility

A passive object in a chiral active bath 🌀 + ⚡ + 🛀



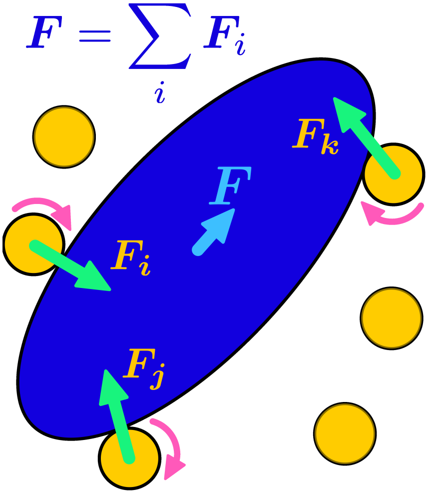
- Irreversible ratchet motion
- Odd diffusion and odd mobility
- Long-range density modulations and flows in the bath

A passive object in a chiral active bath 🌀 + ⚡ + 🛀



- Irreversible ratchet motion
- Odd diffusion and odd mobility
- Long-range density modulations and flows in the bath

A passive object in a chiral active bath 🌀 + ⚡ + 🛀



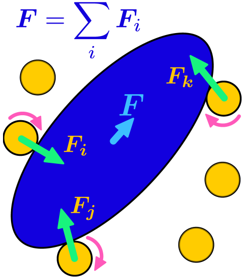
- Irreversible ratchet motion
- Odd diffusion and odd mobility
- Long-range density modulations and flows in the bath

How do the object's shape, size, and mass control these effects?

Langevin dynamics in the adiabatic limit

Bath
(Chiral ABPs)

$$\gamma \dot{\boldsymbol{r}}_i = f_0 \boldsymbol{u}(\theta_i) - \boldsymbol{F}_i$$
$$\dot{\theta}_i = \omega_0 + \sqrt{2D_r}\xi_i$$



Mazur and Oppenheim, *Physica* **50** (1970)
Van Kampen and Oppenheim, *Physica A* **138** (1986)
Solon and Horowitz, *J. Phys. A* **55** (2022)

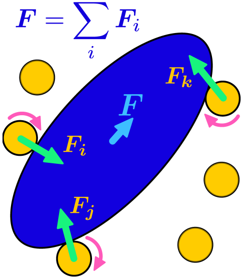
Bath

(Chiral ABPs)

$$\gamma \dot{\boldsymbol{r}}_i = f_0 \boldsymbol{u}(\theta_i) - \boldsymbol{F}_i$$

$$\dot{\theta}_i = \omega_0 + \sqrt{2D_r}\xi_i$$

$$\ell_p = \frac{f_0}{\gamma D_r}, \quad \ell_g = \frac{f_0}{\gamma|\omega_0|}$$



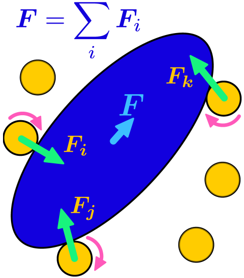
Mazur and Oppenheim, *Physica* **50** (1970)
Van Kampen and Oppenheim, *Physica A* **138** (1986)
Solon and Horowitz, *J. Phys. A* **55** (2022)

Langevin dynamics in the adiabatic limit

Bath
(Chiral ABPs)

$$\gamma \dot{\boldsymbol{r}}_i = f_0 \boldsymbol{u}(\theta_i) - \boldsymbol{F}_i$$
$$\dot{\theta}_i = \omega_0 + \sqrt{2D_r}\xi_i$$

$$\ell_p = \frac{f_0}{\gamma D_r}, \quad \ell_g = \frac{f_0}{\gamma|\omega_0|}$$



Object
(Rigid Newtonian dynamics)

$$\dot{\boldsymbol{R}} = \frac{1}{M}\boldsymbol{P}, \quad \dot{\Theta} = \frac{1}{I}L,$$
$$\dot{\boldsymbol{P}} = \boldsymbol{F} \qquad \dot{L} = \Gamma$$

Mazur and Oppenheim, *Physica* **50** (1970)
Van Kampen and Oppenheim, *Physica A* **138** (1986)
Solon and Horowitz, *J. Phys. A* **55** (2022)

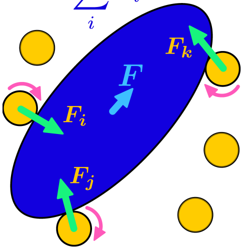
Langevin dynamics in the adiabatic limit

Bath
(Chiral ABPs)

$$\gamma \dot{\boldsymbol{r}}_i = f_0 \boldsymbol{u}(\theta_i) - \boldsymbol{F}_i$$
$$\dot{\theta}_i = \omega_0 + \sqrt{2D_r}\xi_i$$

$$\ell_p = \frac{f_0}{\gamma D_r}, \quad \ell_g = \frac{f_0}{\gamma|\omega_0|}$$

$$\boldsymbol{F} = \sum_i \boldsymbol{F}_i$$



Object
(Rigid Newtonian dynamics)

$$\dot{\boldsymbol{R}} = \frac{1}{M}\boldsymbol{P}, \quad \dot{\Theta} = \frac{1}{I}L,$$
$$\dot{\boldsymbol{P}} = \boldsymbol{F} \qquad \dot{L} = \Gamma$$

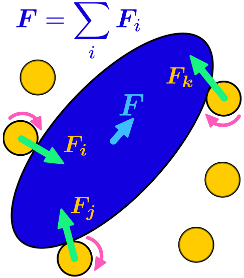
$$\epsilon = \sqrt{\text{“}m\text{”}/M} = \sqrt{\gamma/D_r M}\,.$$

Mazur and Oppenheim, *Physica* **50** (1970)
Van Kampen and Oppenheim, *Physica A* **138** (1986)
Solon and Horowitz, *J. Phys. A* **55** (2022)

Bath
(Chiral ABPs)

$$\gamma \dot{\boldsymbol{r}}_i = f_0 \boldsymbol{u}(\theta_i) - \boldsymbol{F}_i$$
$$\dot{\theta}_i = \omega_0 + \sqrt{2D_r}\xi_i$$

$$\ell_p = \frac{f_0}{\gamma D_r}, \quad \ell_g = \frac{f_0}{\gamma|\omega_0|}$$



Object
(Rigid Newtonian dynamics)

$$\dot{\boldsymbol{R}} = \frac{1}{M}\boldsymbol{P}, \quad \dot{\Theta} = \frac{1}{I}L,$$
$$\dot{\boldsymbol{P}} = \boldsymbol{F} \qquad \dot{L} = \Gamma$$

$$\epsilon = \sqrt{\text{“}m\text{”}/M} = \sqrt{\gamma/D_r M}\,.$$

In the adiabatic limit of $M \to \infty$:

$$\begin{bmatrix} \dot{\boldsymbol{P}} \\ \dot{L} \end{bmatrix} = \begin{bmatrix} \langle \boldsymbol{F} \rangle_{\mathrm{b}} \\ \langle \Gamma \rangle_{\mathrm{b}} \end{bmatrix} - \underbrace{\begin{bmatrix} \zeta_{\boldsymbol{PP}} & \zeta_{\boldsymbol{P}L} \\ \zeta_{L\boldsymbol{P}} & \zeta_{LL} \end{bmatrix}}_{\boldsymbol{\zeta}} \begin{bmatrix} M^{-1}\boldsymbol{P} \\ I^{-1}L \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{\xi}_P(t) \\ \xi_L(t) \end{bmatrix}}_{\boldsymbol{\xi}}$$

Mazur and Oppenheim, *Physica* **50** (1970)
Van Kampen and Oppenheim, *Physica A* **138** (1986)
Solon and Horowitz, *J. Phys. A* **55** (2022)

Langevin dynamics in the adiabatic limit

$$\begin{bmatrix} \dot{\boldsymbol{P}} \\ \dot{L} \end{bmatrix} = \begin{bmatrix} \langle \boldsymbol{F} \rangle_{\mathsf{b}} \\ \langle \Gamma \rangle_{\mathsf{b}} \end{bmatrix} - \underbrace{\begin{bmatrix} \boldsymbol{\zeta}_{\boldsymbol{PP}} & \boldsymbol{\zeta}_{\boldsymbol{P}L} \\ \boldsymbol{\zeta}_{L\boldsymbol{P}} & \zeta_{LL} \end{bmatrix}}_{\boldsymbol{\zeta}} \begin{bmatrix} M^{-1}\boldsymbol{P} \\ I^{-1}L \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{\xi}_P(t) \\ \xi_L(t) \end{bmatrix}}_{\boldsymbol{\xi}},$$

Langevin dynamics in the **adiabatic limit**

$$\begin{bmatrix} \dot{\boldsymbol{P}} \\ \dot{L} \end{bmatrix} = \begin{bmatrix} \langle \boldsymbol{F} \rangle_{\mathsf{b}} \\ \langle \Gamma \rangle_{\mathsf{b}} \end{bmatrix} - \underbrace{\begin{bmatrix} \boldsymbol{\zeta}_{\boldsymbol{PP}} & \boldsymbol{\zeta}_{\boldsymbol{P}L} \\ \boldsymbol{\zeta}_{L\boldsymbol{P}} & \zeta_{LL} \end{bmatrix}}_{\boldsymbol{\zeta}} \begin{bmatrix} M^{-1}\boldsymbol{P} \\ I^{-1}L \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{\xi}_P(t) \\ \xi_L(t) \end{bmatrix}}_{\boldsymbol{\xi}},$$

where $\langle A(t) \rangle_{\mathsf{b}} := \int d\boldsymbol{r}^N A(\boldsymbol{r}^N) \rho_{\mathsf{b}}(\boldsymbol{r}^N | \boldsymbol{R}, \Theta; t)$,

Langevin dynamics in the **adiabatic limit**

$$\begin{bmatrix} \dot{\boldsymbol{P}} \\ \dot{L} \end{bmatrix} = \begin{bmatrix} \langle \boldsymbol{F} \rangle_{\mathsf{b}} \\ \langle \Gamma \rangle_{\mathsf{b}} \end{bmatrix} - \underbrace{\begin{bmatrix} \boldsymbol{\zeta}_{\boldsymbol{PP}} & \boldsymbol{\zeta}_{\boldsymbol{P}L} \\ \boldsymbol{\zeta}_{L\boldsymbol{P}} & \zeta_{LL} \end{bmatrix}}_{\zeta} \begin{bmatrix} M^{-1}\boldsymbol{P} \\ I^{-1}L \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{\xi}_P(t) \\ \xi_L(t) \end{bmatrix}}_{\boldsymbol{\xi}},$$

where $\langle A(t) \rangle_{\mathsf{b}} := \int d\boldsymbol{r}^N A(\boldsymbol{r}^N) \rho_{\mathsf{b}}(\boldsymbol{r}^N | \boldsymbol{R}, \Theta; t)$,

and $\boldsymbol{\xi}(t)$ is Gaussian white* noise with $\langle \boldsymbol{\xi}(t) \otimes \boldsymbol{\xi}(t') \rangle = \boldsymbol{\lambda} \delta_+(t-t') + \boldsymbol{\lambda}^T \delta_-(t-t')$

where $\delta_-(t-t') + \delta_+(t-t) = 2\delta(t-t')$.

*Chun, Durang and Noh, *PRE* (2018)

$$\begin{bmatrix} \dot{\boldsymbol{P}} \\ \dot{L} \end{bmatrix} = \begin{bmatrix} \langle \boldsymbol{F} \rangle_{\mathsf{b}} \\ \langle \Gamma \rangle_{\mathsf{b}} \end{bmatrix} - \underbrace{\begin{bmatrix} \boldsymbol{\zeta}_{\boldsymbol{PP}} & \boldsymbol{\zeta}_{\boldsymbol{P}L} \\ \boldsymbol{\zeta}_{L\boldsymbol{P}} & \zeta_{LL} \end{bmatrix}}_{\boldsymbol{\zeta}} \begin{bmatrix} M^{-1}\boldsymbol{P} \\ I^{-1}L \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{\xi}_{\boldsymbol{P}}(t) \\ \xi_L(t) \end{bmatrix}}_{\boldsymbol{\xi}},$$

where $\langle A(t) \rangle_{\mathsf{b}} := \int d\boldsymbol{r}^N A(\boldsymbol{r}^N) \rho_{\mathsf{b}}(\boldsymbol{r}^N | \boldsymbol{R}, \Theta; t)$,

and $\boldsymbol{\xi}(t)$ is Gaussian white* noise with $\langle \boldsymbol{\xi}(t) \otimes \boldsymbol{\xi}(t') \rangle = \boldsymbol{\lambda} \delta_+(t-t') + \boldsymbol{\lambda}^T \delta_-(t-t')$

where $\delta_-(t-t') + \delta_+(t-t) = 2\delta(t-t')$.

$$\boldsymbol{\lambda} = \int_0^\infty \mathrm{d}\tau \begin{bmatrix} \langle \delta\boldsymbol{F}(\tau) \otimes \delta\boldsymbol{F}(0) \rangle_{\mathsf{b}} & \langle \delta\boldsymbol{F}(\tau) \delta\Gamma(0) \rangle_{\mathsf{b}} \\ \langle \delta\Gamma(\tau) \delta\boldsymbol{F}^T(0) \rangle_{\mathsf{b}} & \langle \delta\Gamma(\tau) \delta\Gamma(0) \rangle_{\mathsf{b}} \end{bmatrix},$$

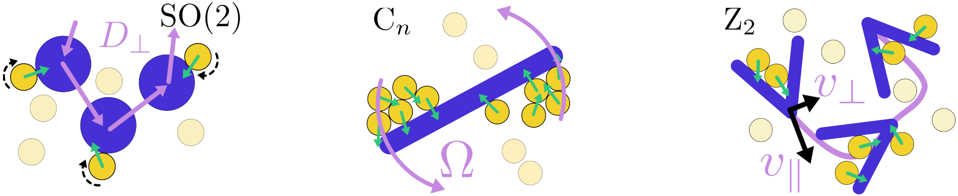
$$\boldsymbol{\zeta} = \int_0^\infty \mathrm{d}\tau \begin{bmatrix} \langle \delta\boldsymbol{F}(\tau) \otimes \boldsymbol{\nabla}_{\boldsymbol{R}} \log \rho_{\mathsf{b}}(0) \rangle_{\mathsf{b}} & \langle \delta\boldsymbol{F}(\tau) \partial_\Theta \log \rho_{\mathsf{b}}(0) \rangle_{\mathsf{b}} \\ \langle \delta\Gamma(\tau) \boldsymbol{\nabla}_{\boldsymbol{R}}^T \log \rho_{\mathsf{b}}(0) \rangle_{\mathsf{b}} & \langle \delta\Gamma(\tau) \partial_\Theta \log \rho_{\mathsf{b}}(0) \rangle_{\mathsf{b}} \end{bmatrix}.$$

*Chun, Durang and Noh, *PRE* (2018)

Object symmetries determine the dynamics

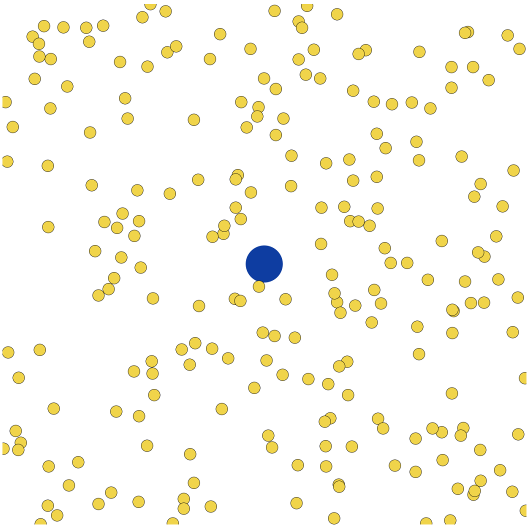
$$\begin{bmatrix} \dot{\boldsymbol{P}} \\ \dot{L} \end{bmatrix} = \begin{bmatrix} \langle \boldsymbol{F} \rangle_{\mathsf{b}} \\ \langle \Gamma \rangle_{\mathsf{b}} \end{bmatrix} - \underbrace{\begin{bmatrix} \boldsymbol{\zeta}_{\boldsymbol{PP}} & \boldsymbol{\zeta}_{\boldsymbol{P}L} \\ \boldsymbol{\zeta}_{L\boldsymbol{P}} & \zeta_{LL} \end{bmatrix}}_{\boldsymbol{\zeta}} \begin{bmatrix} M^{-1}\boldsymbol{P} \\ I^{-1}L \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{\xi}_P(t) \\ \xi_L(t) \end{bmatrix}}_{\boldsymbol{\xi}},$$

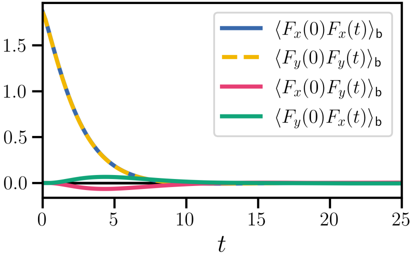
$$\boldsymbol{\zeta}^{\text{disk}} = \begin{bmatrix} \zeta_{\parallel} & \zeta_{\perp} & 0 \\ -\zeta_{\perp} & \zeta_{\parallel} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\zeta}^{\text{rod}} = \begin{bmatrix} \zeta_{xx} & \zeta_{xy} & 0 \\ \zeta_{yx} & \zeta_{yy} & 0 \\ 0 & 0 & \zeta_{\Theta\Theta} \end{bmatrix}, \quad \boldsymbol{\zeta}^{\text{wedge}} = \begin{bmatrix} \zeta_{xx} & \zeta_{xy} & \zeta_{x\Theta} \\ \zeta_{yx} & \zeta_{yy} & \zeta_{y\Theta} \\ \zeta_{\Theta x} & \zeta_{\Theta y} & \zeta_{\Theta\Theta} \end{bmatrix}$$



Passive disk in a chiral active bath

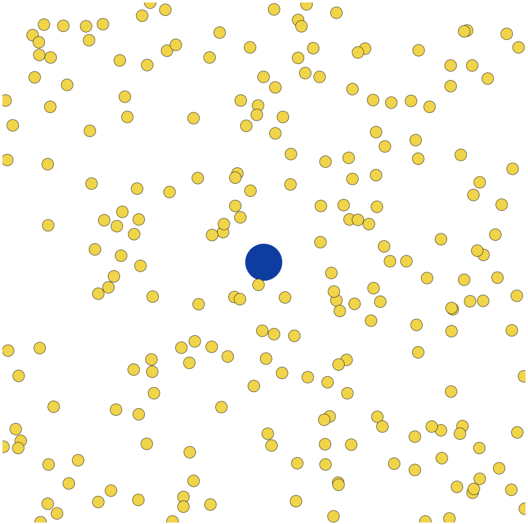
fixed disk

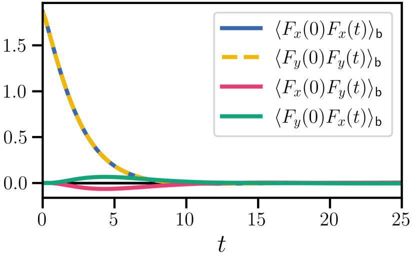


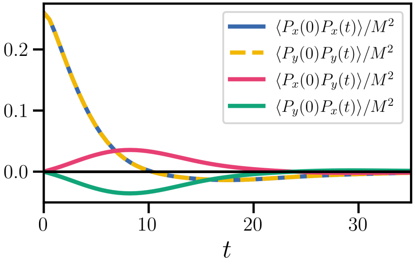


Passive disk in a chiral active bath

lightweight disk $(M = 1)$

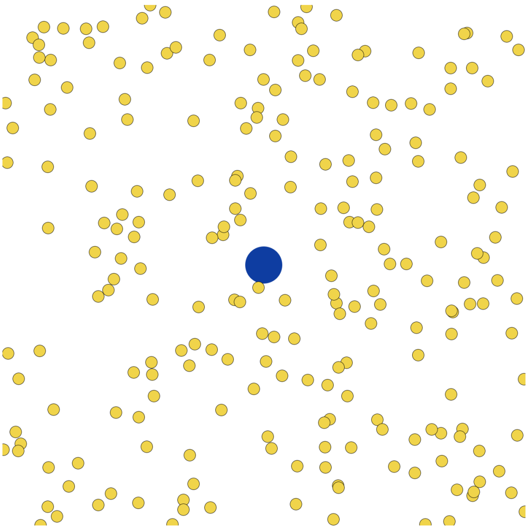


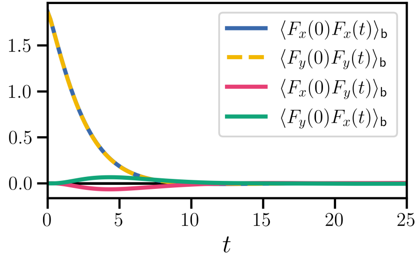


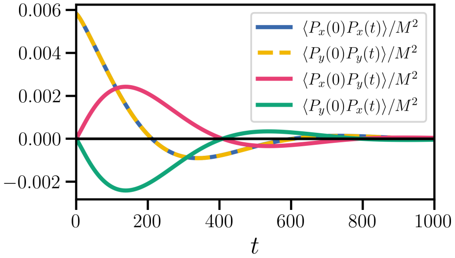


Passive disk in a chiral active bath

heavy disk ($M = 100$, 10x faster)

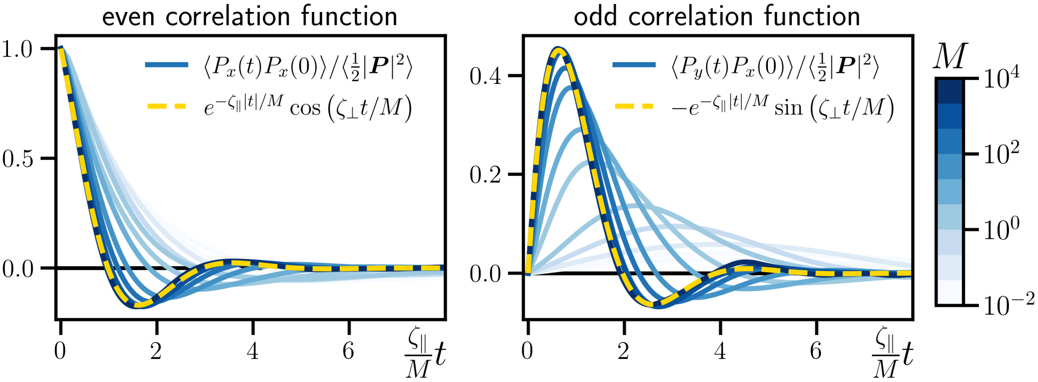




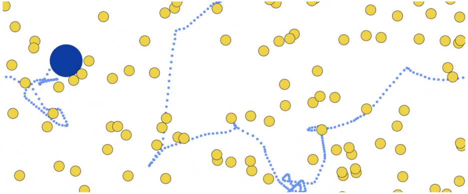


Passive disk in a chiral active bath

Momentum correlations match the Langevin prediction at $M \to \infty$



Diffusion

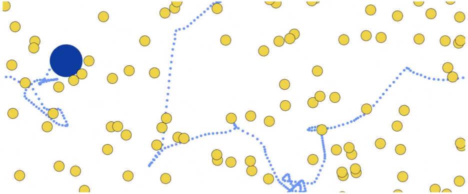


$$\mathbf{D} = \frac{1}{M^2}\int_0^\infty dt\ \langle \boldsymbol{P}(t)\boldsymbol{P}(0)\rangle$$
$$= \frac{1}{M^2}\int_0^\infty dt\ e^{-\zeta t/M}\ \underbrace{\langle \boldsymbol{P}(0)\boldsymbol{P}(0)\rangle}_{M{\color{blue}T_{\text{eff}}}\boldsymbol{\delta}}$$

Solon and Horowitz, *J. Phys. A* **55** (2022)

Einstein relations

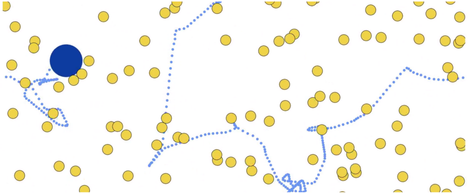
Diffusion



$$\mathbf{D} = \frac{1}{M^2}\int_0^\infty dt\ \langle \boldsymbol{P}(t)\boldsymbol{P}(0)\rangle$$
$$= \frac{1}{M^2}\int_0^\infty dt\ e^{-\zeta t/M}\ \underbrace{\langle \boldsymbol{P}(0)\boldsymbol{P}(0)\rangle}_{M T_{\text{eff}}\boldsymbol{\delta}}$$
$$= T_{\text{eff}}\boldsymbol{\zeta}^{-1} =: \begin{bmatrix} D_\parallel & D_\perp \\ -D_\perp & D_\parallel \end{bmatrix}.$$

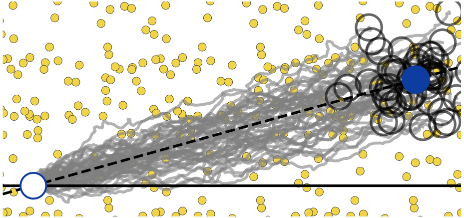
Solon and Horowitz, *J. Phys. A* **55** (2022)

Diffusion



$$\mathbf{D} = \frac{1}{M^2}\int_0^\infty dt\ \langle \boldsymbol{P}(t)\boldsymbol{P}(0)\rangle$$
$$= \frac{1}{M^2}\int_0^\infty dt\ e^{-\zeta t/M}\underbrace{\langle \boldsymbol{P}(0)\boldsymbol{P}(0)\rangle}_{MT_{\text{eff}}\boldsymbol{\delta}}$$
$$= T_{\text{eff}}\boldsymbol{\zeta}^{-1} =: \begin{bmatrix} D_\parallel & D_\perp \\ -D_\perp & D_\parallel \end{bmatrix}.$$

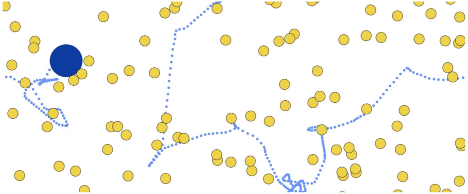
Mobility



$$\dot{\boldsymbol{P}} = -M^{-1}\boldsymbol{\zeta}\boldsymbol{P} + \boldsymbol{F}^{\text{ext}} + \boldsymbol{\xi}$$

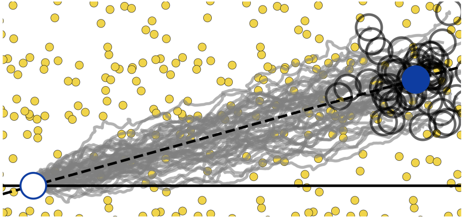
Solon and Horowitz, *J. Phys. A* **55** (2022)

Diffusion



$$\mathbf{D} = \frac{1}{M^2}\int_0^\infty dt\ \langle \boldsymbol{P}(t)\boldsymbol{P}(0)\rangle$$
$$= \frac{1}{M^2}\int_0^\infty dt\ e^{-\zeta t/M}\underbrace{\langle \boldsymbol{P}(0)\boldsymbol{P}(0)\rangle}_{MT_{\text{eff}}\boldsymbol{\delta}}$$
$$= T_{\text{eff}}\boldsymbol{\zeta}^{-1} =: \begin{bmatrix} D_\parallel & D_\perp \\ -D_\perp & D_\parallel \end{bmatrix}.$$

Mobility

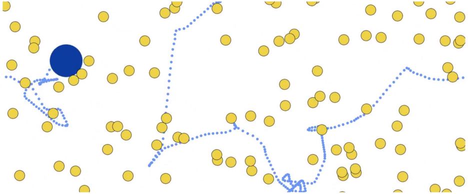


$$\dot{\boldsymbol{P}} = -M^{-1}\boldsymbol{\zeta}\boldsymbol{P} + \boldsymbol{F}^{\text{ext}} + \boldsymbol{\xi}$$
$$\implies \boldsymbol{\mu} = \boldsymbol{\zeta}^{-1}$$

Solon and Horowitz, *J. Phys. A* **55** (2022)

Einstein relations

Diffusion

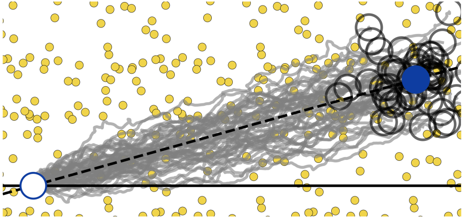


$$\mathbf{D} = \frac{1}{M^2}\int_0^\infty dt\ \langle \boldsymbol{P}(t)\boldsymbol{P}(0)\rangle$$

$$= \frac{1}{M^2}\int_0^\infty dt\ e^{-\zeta t/M}\underbrace{\langle \boldsymbol{P}(0)\boldsymbol{P}(0)\rangle}_{MT_{\text{eff}}\boldsymbol{\delta}}$$

$$= T_{\text{eff}}\boldsymbol{\zeta}^{-1} =: \begin{bmatrix} D_\parallel & D_\perp \\ -D_\perp & D_\parallel \end{bmatrix}.$$

Mobility



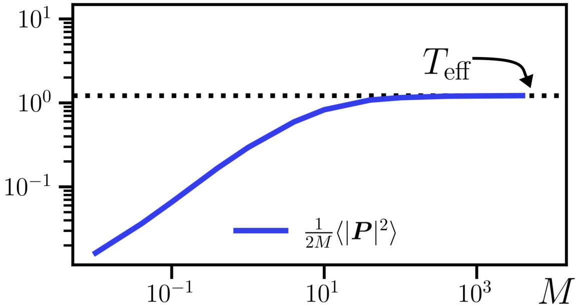
$$\dot{\boldsymbol{P}} = -M^{-1}\boldsymbol{\zeta}\boldsymbol{P} + \boldsymbol{F}^{\text{ext}} + \boldsymbol{\xi}$$

$$\implies \boldsymbol{\mu} = \boldsymbol{\zeta}^{-1}$$

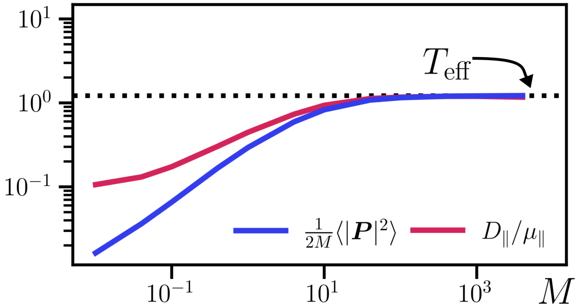
$$\boxed{\boldsymbol{\mu}T_{\text{eff}} = \mathbf{D}}$$

Solon and Horowitz, *J. Phys. A* **55** (2022)

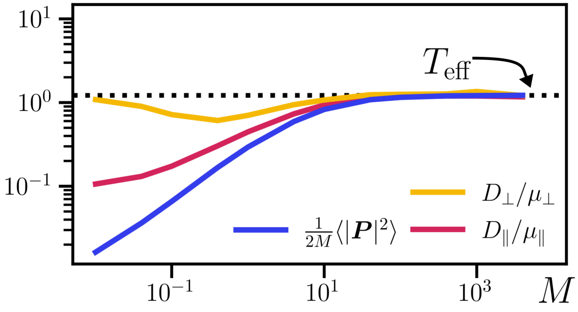
Even and odd **thermometers** for T_{eff}



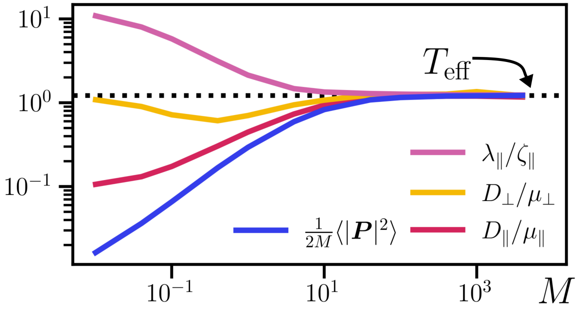
Even and odd **thermometers** for T_{eff}



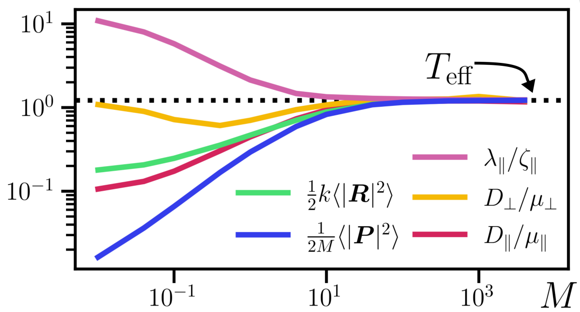
Even and odd **thermometers** for T_{eff}

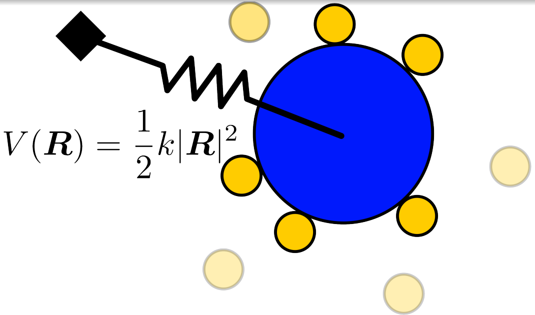


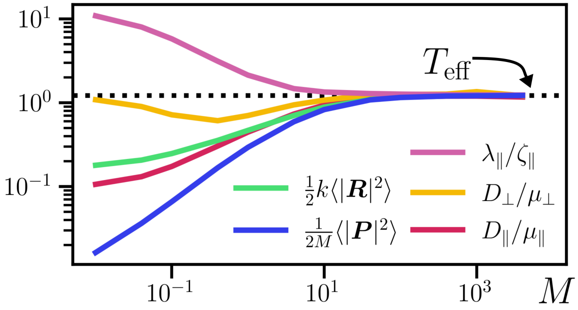
Even and odd **thermometers** for T_{eff}



Even and odd **thermometers** for T_{eff}

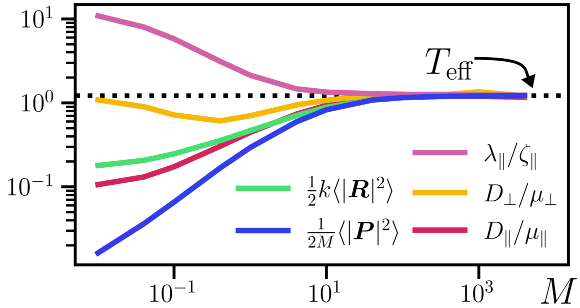






Adiabatic steady state is Boltzmann-distributed

$$\rho_\circ^{\mathsf{ss}} = \rho_{\boldsymbol{R}}^{\mathsf{ss}}(\boldsymbol{R})\rho_{\boldsymbol{P}}^{\mathsf{ss}}(\boldsymbol{P}) \propto \mathsf{e}^{-\left(V(\boldsymbol{R})+\frac{1}{2M}|\boldsymbol{P}|^2\right)/T_{\mathsf{eff}}}$$

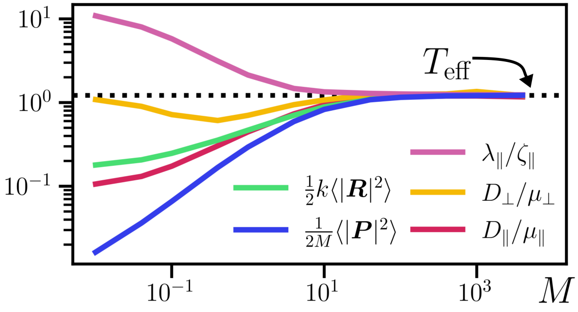


Adiabatic steady state is Boltzmann-distributed

$$\rho_\circ^{\text{ss}} = \rho_{\boldsymbol{R}}^{\text{ss}}(\boldsymbol{R})\rho_{\boldsymbol{P}}^{\text{ss}}(\boldsymbol{P}) \propto \mathrm{e}^{-\left(V(\boldsymbol{R})+\frac{1}{2M}|\boldsymbol{P}|^2\right)/T_{\text{eff}}}$$

$$\rho_{\boldsymbol{R}}(\boldsymbol{R},t) = \langle\delta(\boldsymbol{R}-\boldsymbol{R}(t))\rangle\,,$$

$$0 = \partial_t\rho_{\boldsymbol{R}}^{\text{ss}} = -\boldsymbol{\nabla}\cdot\underbrace{\left[(T_{\text{eff}}\boldsymbol{\mu}-\mathbf{D})\boldsymbol{\nabla}_{\boldsymbol{R}}\rho_{\boldsymbol{R}}^{\text{ss}}\right]}_{\boldsymbol{J}_{\boldsymbol{R}}^{\text{ss}}}$$



Adiabatic steady state is Boltzmann-distributed

$$\rho^{\text{ss}}_{\circ} = \rho^{\text{ss}}_{\boldsymbol{R}}(\boldsymbol{R})\rho^{\text{ss}}_{\boldsymbol{P}}(\boldsymbol{P}) \propto \mathrm{e}^{-\left(V(\boldsymbol{R})+\frac{1}{2M}|\boldsymbol{P}|^2\right)/T_{\text{eff}}}$$

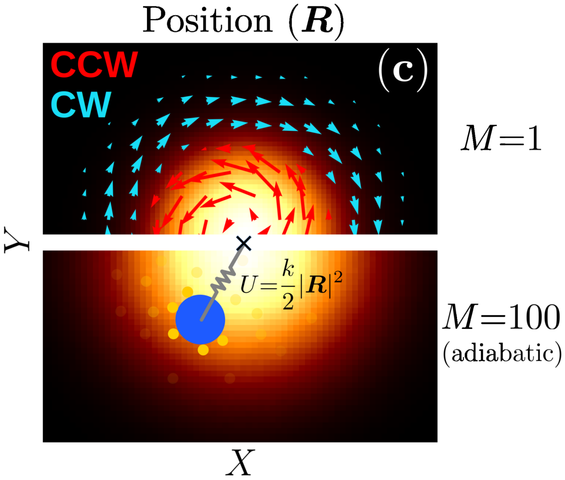
$$\rho_{\boldsymbol{R}}(\boldsymbol{R},t) = \langle\delta(\boldsymbol{R}-\boldsymbol{R}(t))\rangle\,,$$

$$0 = \partial_t\rho^{\text{ss}}_{\boldsymbol{R}} = -\boldsymbol{\nabla}\cdot\underbrace{\left[(T_{\text{eff}}\boldsymbol{\mu}-\mathbf{D})\boldsymbol{\nabla}_{\boldsymbol{R}}\rho^{\text{ss}}_{\boldsymbol{R}}\right]}_{\boldsymbol{J}^{\text{ss}}_{\boldsymbol{R}}}$$

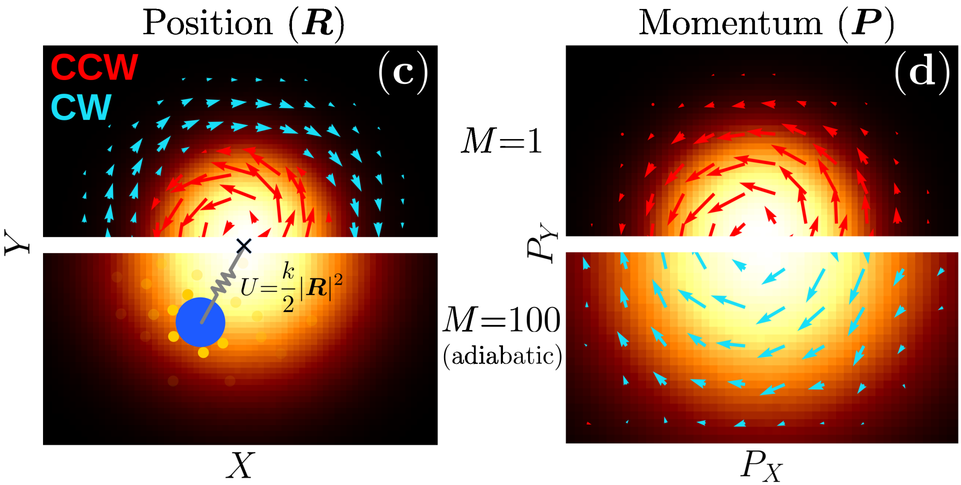
$$\rho_{\boldsymbol{P}}(\boldsymbol{P},t) = \langle\delta(\boldsymbol{P}-\boldsymbol{P}(t))\rangle\,,$$

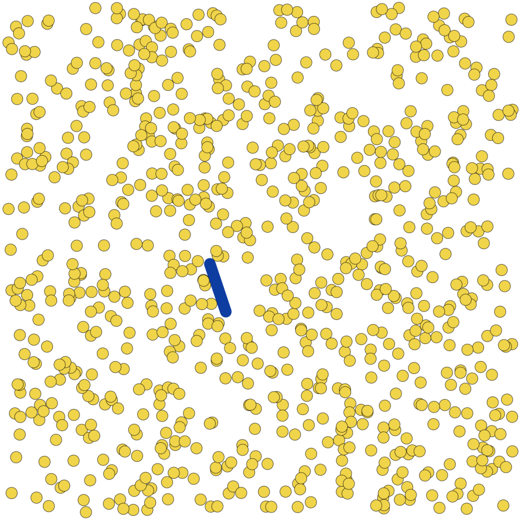
$$0 = \partial_t\rho^{\text{ss}}_{\boldsymbol{P}} = -\boldsymbol{\nabla}\cdot\underbrace{\left[(T_{\text{eff}}\boldsymbol{\zeta}-\boldsymbol{\lambda})\boldsymbol{\nabla}_{\boldsymbol{P}}\rho^{\text{ss}}_{\boldsymbol{P}}\right]}_{\boldsymbol{J}^{\text{ss}}_{\boldsymbol{P}}\neq 0\text{ even when adiabatic}}$$

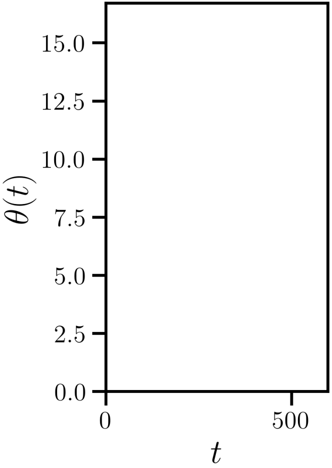
Odd circulation of the disk is a signature of nonequilibrium



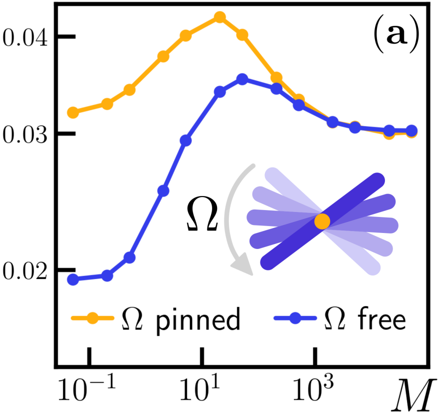
Odd circulation of the disk is a signature of nonequilibrium



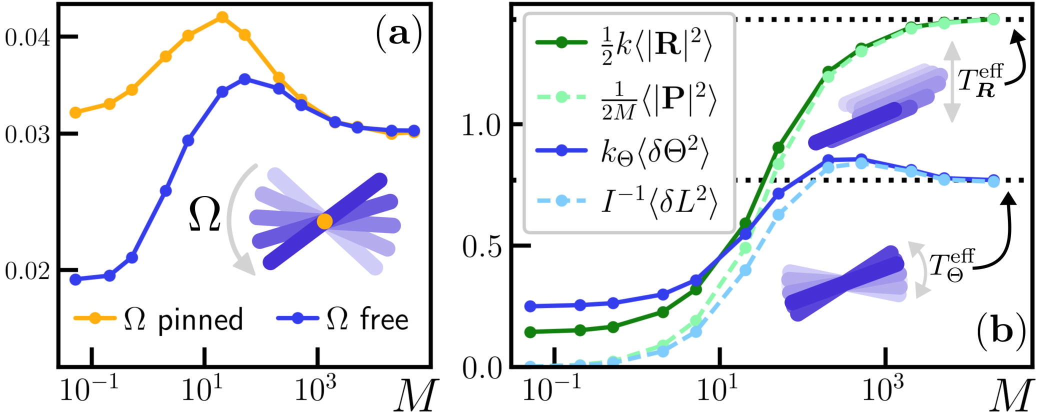



15.0
12.5
10.0
7.5
5.0
2.5
0.0
$\theta(t)$
0
500
t

Measures of rod **adiabaticity**

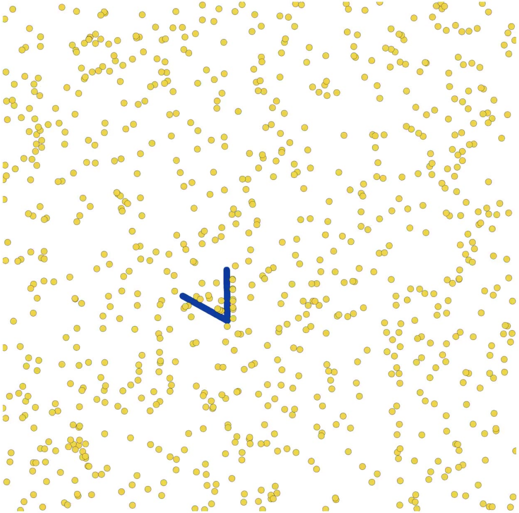


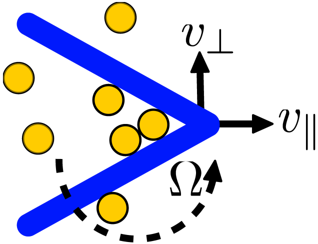
Measures of rod **adiabaticity**

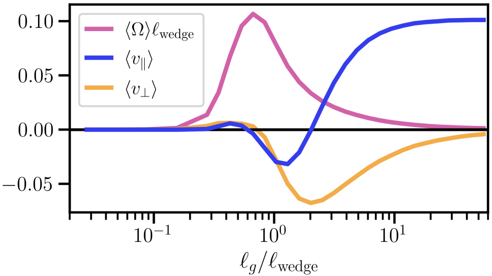


Wedge dynamics

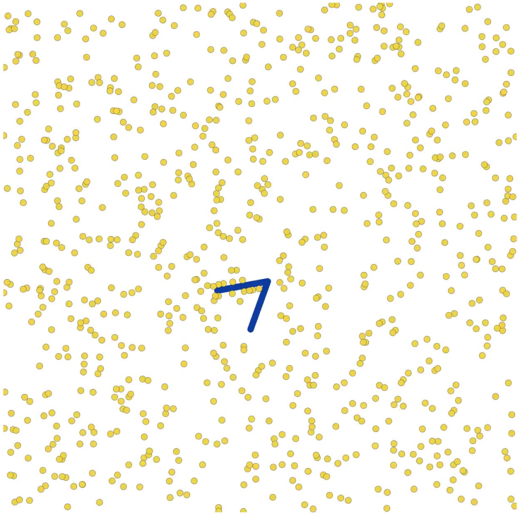
Achiral: $\ell_g/\ell_\text{wedge} \to \infty$

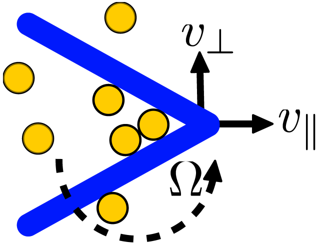


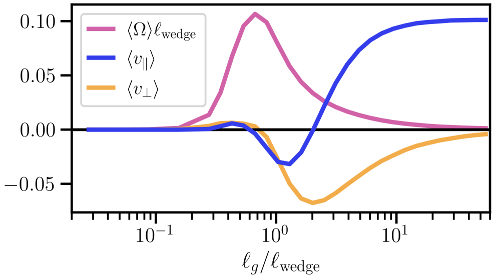




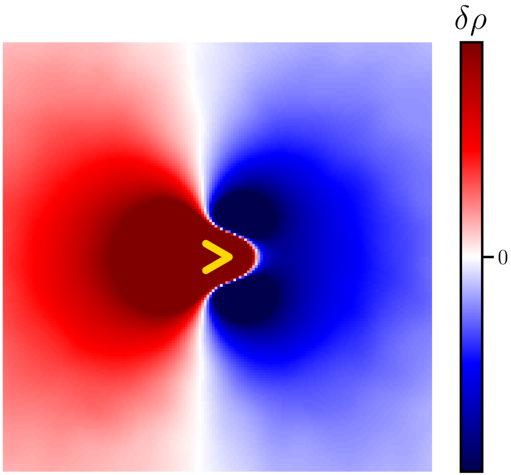
Chiral: $\ell_g/\ell_{\text{wedge}} = 1$







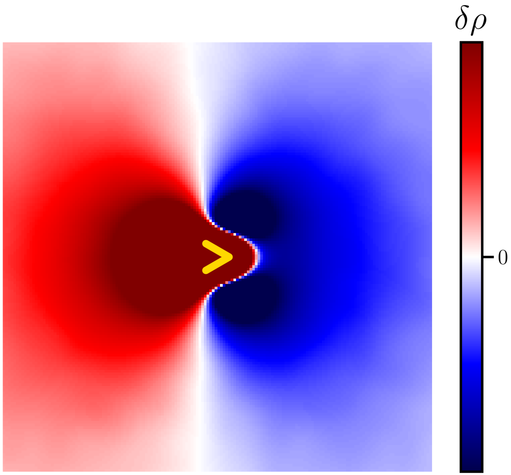
How does the object affect the **bath** itself?



Baek et al., *PRL* (2018)
Granek et al., *J. Stat. Mech.* (2022)

$$\langle \boldsymbol{F} \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \delta\rho(\boldsymbol{r}) \boldsymbol{\nabla} V$$

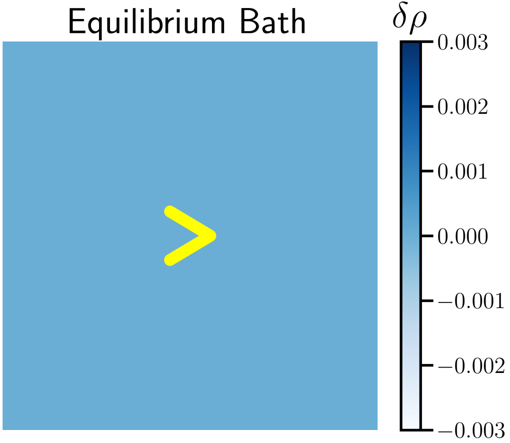
$$\langle \Gamma \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \delta\rho(\boldsymbol{r}) \boldsymbol{r} \times \boldsymbol{\nabla} V$$



Baek et al., *PRL* (2018)
Granek et al., *J. Stat. Mech.* (2022)

$$\langle \boldsymbol{F} \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \delta\rho(\boldsymbol{r}) \boldsymbol{\nabla} V$$

$$\langle \Gamma \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \delta\rho(\boldsymbol{r}) \boldsymbol{r} \times \boldsymbol{\nabla} V$$

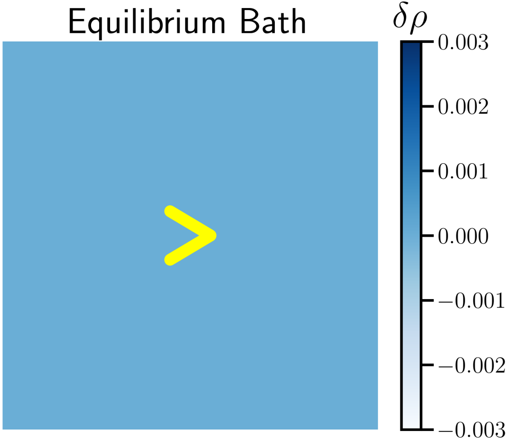


Baek et al., *PRL* (2018)
Granek et al., *J. Stat. Mech.* (2022)

How does the object affect the **bath** itself?

$$\langle \boldsymbol{F} \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \rho_0 e^{-V(\boldsymbol{r})/T} \boldsymbol{\nabla} V = 0$$

$$\langle \Gamma \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \rho_0 e^{-V(\boldsymbol{r})/T} \boldsymbol{r} \times \boldsymbol{\nabla} V = 0$$

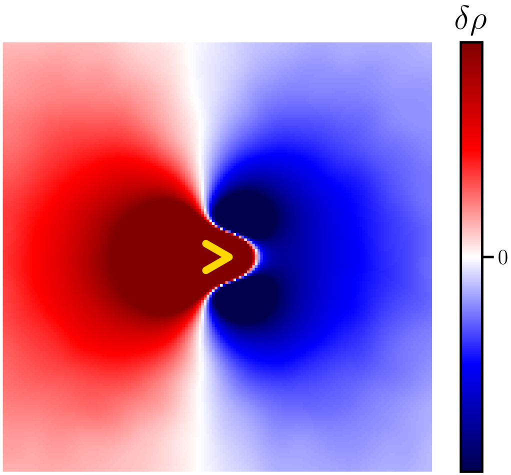


Baek et al., *PRL* (2018)
Granek et al., *J. Stat. Mech.* (2022)

How does the object affect the **bath** itself?

$$\langle \boldsymbol{F} \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \delta\rho(\boldsymbol{r}) \boldsymbol{\nabla} V$$

$$\langle \Gamma \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \delta\rho(\boldsymbol{r}) \boldsymbol{r} \times \boldsymbol{\nabla} V$$



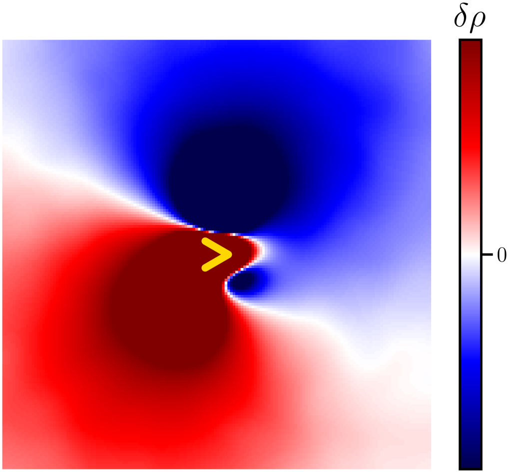
Baek et al., *PRL* (2018)
Granek et al., *J. Stat. Mech.* (2022)

How does the object affect the **bath** itself?

$$\langle \boldsymbol{F} \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \delta\rho(\boldsymbol{r}) \boldsymbol{\nabla} V$$

$$\langle \Gamma \rangle_{\mathsf{b}} = \int d\boldsymbol{r}\ \delta\rho(\boldsymbol{r}) \boldsymbol{r} \times \boldsymbol{\nabla} V$$

What is the effect of chirality?



Baek et al., *PRL* (2018)
Granek et al., *J. Stat. Mech.* (2022)

A **multipole expansion** connects ratchet forces to the far field

In steady state $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$. Defining $\boldsymbol{J} = \boldsymbol{\delta J} - \mathbf{D}\boldsymbol{\nabla}\rho$ then yields

$$D_{\|}\nabla^2\rho = -\boldsymbol{\nabla} \cdot \boldsymbol{\delta J}\,,$$

A **multipole expansion** connects ratchet forces to the far field

In steady state $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$. Defining $\boldsymbol{J} = \boldsymbol{\delta J} - \mathbf{D}\boldsymbol{\nabla}\rho$ then yields

$$D_{\parallel} \nabla^2 \rho = -\boldsymbol{\nabla} \cdot \boldsymbol{\delta J} \, ,$$

which admits the multipole solution

$$\rho(\boldsymbol{r}) = \rho_0 + \frac{\mu}{2\pi D_{\parallel}} \left\{ \frac{\boldsymbol{p} \cdot \boldsymbol{r}}{r^2} + \frac{\boldsymbol{r} \cdot \mathbf{q} \cdot \boldsymbol{r}}{2r^4} \right\} + \mathcal{O}(r^{-3}) \, ,$$

A **multipole expansion** connects ratchet forces to the far field

In steady state $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$. Defining $\boldsymbol{J} = \boldsymbol{\delta J} - \mathbf{D}\boldsymbol{\nabla}\rho$ then yields

$$D_{||}\nabla^2\rho = -\boldsymbol{\nabla} \cdot \boldsymbol{\delta J}\,,$$

which admits the multipole solution

$$\rho(\boldsymbol{r}) = \rho_0 + \frac{\mu}{2\pi D_{||}}\left\{\frac{\boldsymbol{p}\cdot\boldsymbol{r}}{r^2} + \frac{\boldsymbol{r}\cdot\mathbf{q}\cdot\boldsymbol{r}}{2r^4}\right\} + \mathcal{O}(r^{-3})\,,$$

with the dipole and quadrupole moments

$$\boldsymbol{p} = -\int d\boldsymbol{r}\ \rho(\boldsymbol{r})\boldsymbol{\nabla}V(\boldsymbol{r}) =$$

$$\mathbf{q} = \text{Traceless}\left(-2\int d\boldsymbol{r}\ \left[\rho(\boldsymbol{r})\boldsymbol{r}\otimes\boldsymbol{\nabla}V + \frac{1}{v_0}\mathbf{D}\boldsymbol{m}(\boldsymbol{r})\otimes\boldsymbol{\nabla}V\right]\right)$$

A **multipole expansion** connects ratchet forces to the far field

In steady state $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$. Defining $\boldsymbol{J} = \boldsymbol{\delta J} - \mathbf{D}\boldsymbol{\nabla}\rho$ then yields

$$D_{\|}\nabla^2\rho = -\boldsymbol{\nabla} \cdot \boldsymbol{\delta J} \,,$$

which admits the multipole solution

$$\rho(\boldsymbol{r}) = \rho_0 + \frac{\mu}{2\pi D_{\|}}\left\{\frac{\boldsymbol{p}\cdot\boldsymbol{r}}{r^2} + \frac{\boldsymbol{r}\cdot\mathbf{q}\cdot\boldsymbol{r}}{2r^4}\right\} + \mathcal{O}(r^{-3}) \,,$$

with the dipole and quadrupole moments

$$\boldsymbol{p} = -\int d\boldsymbol{r}\ \rho(\boldsymbol{r})\boldsymbol{\nabla}V(\boldsymbol{r}) = -\langle\boldsymbol{F}\rangle_{\mathrm{b}} =$$

$$\mathbf{q} = \text{Traceless}\left(-2\int d\boldsymbol{r}\ \left[\rho(\boldsymbol{r})\boldsymbol{r}\otimes\boldsymbol{\nabla}V + \frac{1}{v_0}\mathbf{D}\boldsymbol{m}(\boldsymbol{r})\otimes\boldsymbol{\nabla}V\right]\right)$$

A **multipole expansion** connects ratchet forces to the far field

In steady state $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$. Defining $\boldsymbol{J} = \boldsymbol{\delta J} - \mathbf{D}\boldsymbol{\nabla}\rho$ then yields

$$D_{\|}\nabla^2\rho = -\boldsymbol{\nabla} \cdot \boldsymbol{\delta J}\,,$$

which admits the multipole solution

$$\rho(\boldsymbol{r}) = \rho_0 + \frac{\mu}{2\pi D_{\|}}\left\{\frac{\boldsymbol{p}\cdot\boldsymbol{r}}{r^2} + \frac{\boldsymbol{r}\cdot\mathbf{q}\cdot\boldsymbol{r}}{2r^4}\right\} + \mathcal{O}(r^{-3})\,,$$

with the dipole and quadrupole moments

$$\boldsymbol{p} = -\int d\boldsymbol{r}\;\rho(\boldsymbol{r})\boldsymbol{\nabla}V(\boldsymbol{r}) = -\langle\boldsymbol{F}\rangle_{\mathsf{b}} = \gamma\int d\boldsymbol{r}\;\boldsymbol{J}(\boldsymbol{r})$$

$$\mathbf{q} = \text{Traceless}\left(-2\int d\boldsymbol{r}\;\left[\rho(\boldsymbol{r})\boldsymbol{r}\otimes\boldsymbol{\nabla}V + \frac{1}{v_0}\mathbf{D}\boldsymbol{m}(\boldsymbol{r})\otimes\boldsymbol{\nabla}V\right]\right)$$

A **multipole expansion** connects ratchet forces to the far field

In steady state $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$. Defining $\boldsymbol{J} = \boldsymbol{\delta J} - \mathbf{D}\boldsymbol{\nabla}\rho$ then yields

$$D_{\parallel} \nabla^2 \rho = -\boldsymbol{\nabla} \cdot \boldsymbol{\delta J} \, ,$$

which admits the multipole solution

$$\rho(\boldsymbol{r}) = \rho_0 + \frac{\mu}{2\pi D_{\parallel}} \left\{ \frac{\boldsymbol{p} \cdot \boldsymbol{r}}{r^2} + \frac{\boldsymbol{r} \cdot \mathbf{q} \cdot \boldsymbol{r}}{2r^4} \right\} + \mathcal{O}(r^{-3}) \, ,$$

with the dipole and quadrupole moments

$$\boldsymbol{p} = -\int d\boldsymbol{r} \; \rho(\boldsymbol{r}) \boldsymbol{\nabla} V(\boldsymbol{r}) = -\langle \boldsymbol{F} \rangle_{\mathsf{b}} = \gamma \int d\boldsymbol{r} \; \boldsymbol{J}(\boldsymbol{r})$$

$$\mathbf{q} = \text{Traceless}\left(-2 \int d\boldsymbol{r} \; \left[\rho(\boldsymbol{r}) \boldsymbol{r} \otimes \boldsymbol{\nabla} V + \frac{1}{v_0} \mathbf{D} \boldsymbol{m}(\boldsymbol{r}) \otimes \boldsymbol{\nabla} V \right] \right)$$

$$\text{Asymm}(\mathbf{q}) = -\langle \Gamma \rangle_{\mathsf{b}} + (\cdots) = \gamma \int d\boldsymbol{r} \; \boldsymbol{r} \times \boldsymbol{J}(\boldsymbol{r})$$

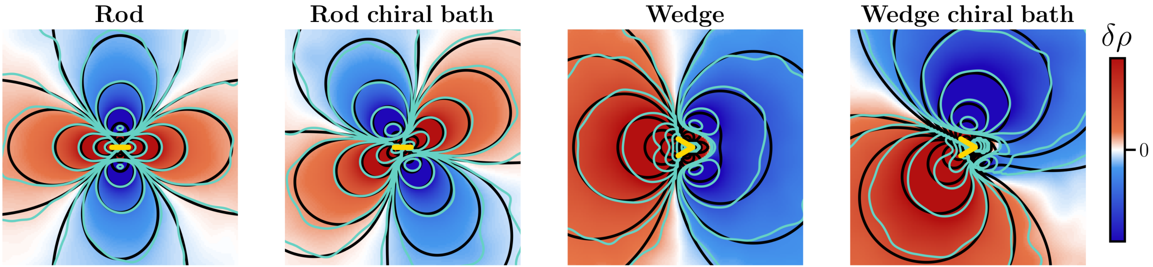
A **multipole expansion** connects ratchet forces to the far field

In steady state $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$. Defining $\boldsymbol{J} = \boldsymbol{\delta J} - \mathbf{D}\boldsymbol{\nabla}\rho$ then yields

$$D_{\parallel} \nabla^2 \rho = -\boldsymbol{\nabla} \cdot \boldsymbol{\delta J} \,,$$

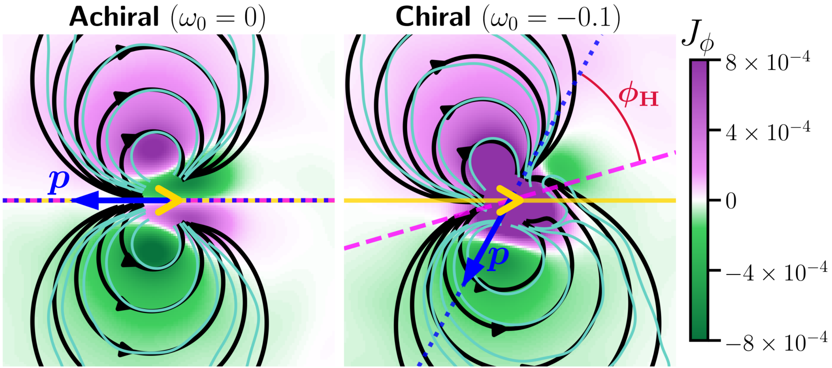
which admits the multipole solution

$$\rho(\boldsymbol{r}) = \rho_0 + \frac{\mu}{2\pi D_{\parallel}} \left\{ \frac{\boldsymbol{p} \cdot \boldsymbol{r}}{r^2} + \frac{\boldsymbol{r} \cdot \mathbf{q} \cdot \boldsymbol{r}}{2r^4} \right\} + \mathcal{O}(r^{-3}) \,,$$



Near field: chirality drives circulation

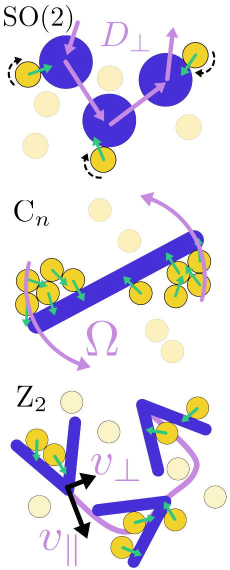
Far field: $\boldsymbol{\nabla} \times \boldsymbol{J} = 0$



$$\phi_{\mathsf{H}} = \arctan(D^{\mathrm{b}}_{\perp}/D^{\mathrm{b}}_{\parallel}) \,, \quad J_\phi = \boldsymbol{r} \times \boldsymbol{J}/|\boldsymbol{r}|$$

Conclusions

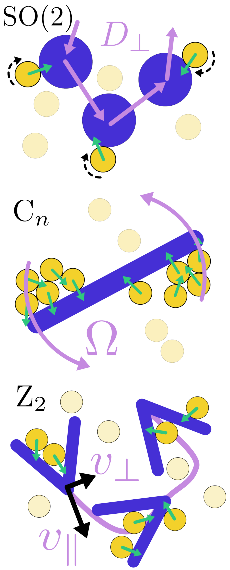
In a chiral active bath, **how do a passive tracer's shape, mass, and size control odd transport and ratchet effects?**



Conclusions

In a chiral active bath, **how do a passive tracer's shape, mass, and size control odd transport and ratchet effects?**

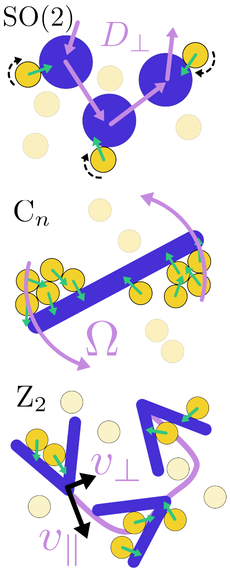
- **Shape**: Object symmetry + bath chirality determines the dynamics



Conclusions

In a chiral active bath, **how do a passive tracer's shape, mass, and size control odd transport and ratchet effects?**

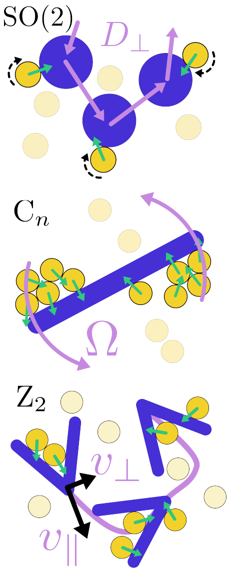
- **Shape**: Object symmetry + bath chirality determines the dynamics
- **Mass**: Effective equilibrium at $M \to \infty$ retains odd signatures of the nonequilibrium bath



Conclusions

In a chiral active bath, **how do a passive tracer's shape, mass, and size control odd transport and ratchet effects?**

- **Shape**: Object symmetry + bath chirality determines the dynamics
- **Mass**: Effective equilibrium at $M \to \infty$ retains odd signatures of the nonequilibrium bath
- **Size**: Odd dynamics are maximized for $\ell_g \sim \ell_{\text{tracer}}$

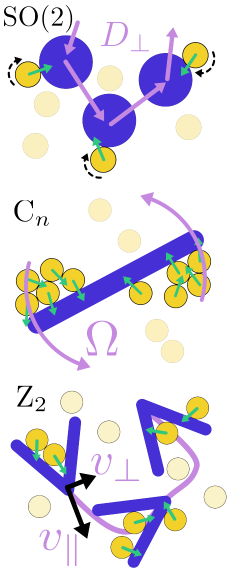


Conclusions

In a chiral active bath, **how do a passive tracer's shape, mass, and size control odd transport and ratchet effects?**

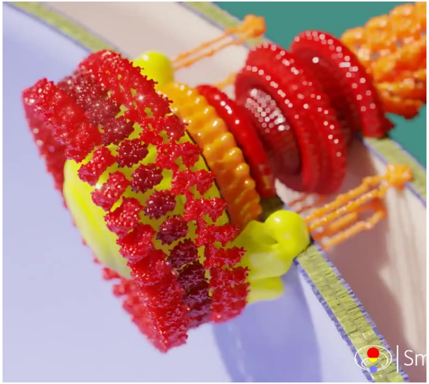
- **Shape**: Object symmetry + bath chirality determines the dynamics
- **Mass**: Effective equilibrium at $M \to \infty$ retains odd signatures of the nonequilibrium bath
- **Size**: Odd dynamics are maximized for $\ell_g \sim \ell_{\text{tracer}}$

Future directions: tracer interactions and self-assembly, chiral nematic bath, application to bacterial ratchets



Backup slides

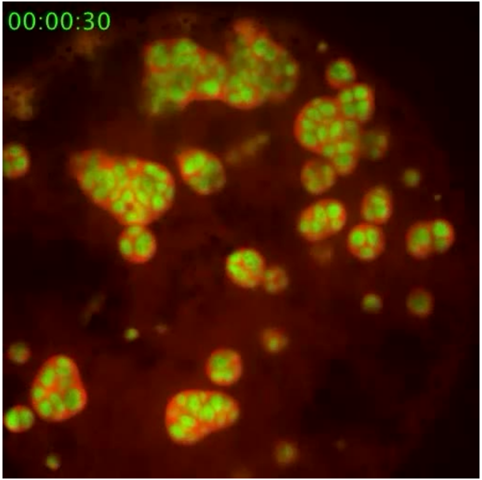
Molecular motors are chiral



Bacterial Flagellum, *Smart Biology*

(https://www.youtube.com/watch?v=dYt5135_0bs)

An active bath can drive assembly



Feric et al. *Cell* 2016

The adiabatic limit: $M \to \infty$

Define the small parameter

$$\epsilon = \sqrt{\text{“}m\text{”}/M} = \sqrt{\gamma/D_r M}\,.$$

Object: rigid Newtonian dynamics

$$\dot{\boldsymbol{R}} = \frac{1}{M}\boldsymbol{P}, \quad \dot{\Theta} = \frac{1}{I}L,$$
$$\dot{\boldsymbol{P}} = \boldsymbol{F} \qquad \dot{L} = \Gamma$$

van Kampen and Oppenheim, *Physica A* **138** (1986)

The adiabatic limit: $M \to \infty$

Define the small parameter

$$\epsilon = \sqrt{\text{“}m\text{”}/M} = \sqrt{\gamma / D_r M}\,.$$

Rescale: $\boldsymbol{P} = \epsilon^{-1} \boldsymbol{P}^*\,, \;\; L = \epsilon^{-1} L^*.$

For simplicity, set “m” $= 1$.

Object: rigid Newtonian dynamics

$$\dot{\boldsymbol{R}} = \frac{1}{M}\boldsymbol{P}, \quad \dot{\Theta} = \frac{1}{I}L,$$
$$\dot{\boldsymbol{P}} = \boldsymbol{F} \qquad \dot{L} = \Gamma$$

van Kampen and Oppenheim, *Physica A* **138** (1986)

The adiabatic limit: $M \to \infty$

Define the small parameter

$$\epsilon = \sqrt{\text{“}m\text{”}/M} = \sqrt{\gamma / D_r M}\,.$$

Rescale: $\boldsymbol{P} = \epsilon^{-1}\boldsymbol{P}^*$, $L = \epsilon^{-1}L^*$.
For simplicity, set “m” $= 1$.

Object: rigid Newtonian dynamics

$$\dot{\boldsymbol{R}} = \frac{1}{M}\boldsymbol{P}, \quad \dot{\Theta} = \frac{1}{I}L,$$
$$\dot{\boldsymbol{P}} = \boldsymbol{F} \qquad \dot{L} = \Gamma$$

$$\Downarrow$$

$$\dot{\boldsymbol{R}} = \epsilon\boldsymbol{P}^*, \quad \dot{\Theta} = \epsilon L^*,$$
$$\dot{\boldsymbol{P}}^* = \epsilon\boldsymbol{F} \qquad \dot{L}^* = \epsilon\Gamma$$

van Kampen and Oppenheim, *Physica A* **138** (1986)

The adiabatic limit: $M \to \infty$

Define the small parameter

$$\epsilon = \sqrt{\text{“}m\text{”}/M} = \sqrt{\gamma / D_r M}\,.$$

Rescale: $\boldsymbol{P} = \epsilon^{-1}\boldsymbol{P}^*\,,\quad L = \epsilon^{-1}L^*.$
For simplicity, set “m” $= 1$.

$\rho(\boldsymbol{r}^N, \theta^N, \boldsymbol{R}, \boldsymbol{P}^*, \Theta, L^*, t)$ evolves as

$$\partial_t \rho = \left(\epsilon \mathcal{L}_{\text{object}} + \mathcal{L}_{\text{bath}}\right)\rho$$

Object: rigid Newtonian dynamics

$$\dot{\boldsymbol{R}} = \frac{1}{M}\boldsymbol{P}, \quad \dot{\Theta} = \frac{1}{I}L,$$
$$\dot{\boldsymbol{P}} = \boldsymbol{F} \qquad \dot{L} = \Gamma$$

$$\Downarrow$$

$$\dot{\boldsymbol{R}} = \epsilon\boldsymbol{P}^*, \quad \dot{\Theta} = \epsilon L^*,$$
$$\dot{\boldsymbol{P}}^* = \epsilon\boldsymbol{F} \qquad \dot{L}^* = \epsilon\Gamma$$

van Kampen and Oppenheim, *Physica A* **138** (1986)

The adiabatic limit: $M \to \infty$

Define the small parameter

$$\epsilon = \sqrt{\text{“}m\text{”}/M} = \sqrt{\gamma/D_r M}\,.$$

Rescale: $\boldsymbol{P} = \epsilon^{-1}\boldsymbol{P}^*$, $L = \epsilon^{-1}L^*$.
For simplicity, set “m” $= 1$.

$\rho(\boldsymbol{r}^N, \theta^N, \boldsymbol{R}, \boldsymbol{P}^*, \Theta, L^*, t)$ evolves as

$$\partial_t \rho = \left(\epsilon \mathcal{L}_{\text{object}} + \mathcal{L}_{\text{bath}}\right)\rho$$

Object: rigid Newtonian dynamics

$$\dot{\boldsymbol{R}} = \frac{1}{M}\boldsymbol{P}, \quad \dot{\Theta} = \frac{1}{I}L,$$
$$\dot{\boldsymbol{P}} = \boldsymbol{F} \qquad \dot{L} = \Gamma$$

$$\Downarrow$$

$$\dot{\boldsymbol{R}} = \epsilon\boldsymbol{P}^*, \quad \dot{\Theta} = \epsilon L^*,$$
$$\dot{\boldsymbol{P}}^* = \epsilon\boldsymbol{F} \qquad \dot{L}^* = \epsilon\Gamma$$

Adiabatic approximation for $\epsilon \to 0$:

$$\rho(\boldsymbol{r}^N, \theta^N, \boldsymbol{R}, \boldsymbol{P}^*, \Theta, L^*, t) = \rho_{\text{tracer}}(\boldsymbol{R}, \boldsymbol{P}^*, \Theta, L^*, t)\rho_{\text{bath}}(\boldsymbol{r}^N, \theta^N | \boldsymbol{R}, \Theta)$$

van Kampen and Oppenheim, *Physica A* **138** (1986)

Fast variable treatment

Define the projection operator $\mathcal{P}$ where

$$\mathcal{P}\rho(\boldsymbol{R}, \boldsymbol{P}, L, \Theta, \mathbf{r}^N, \theta^N, t) = \rho_{\mathsf{bath}}(\mathbf{r}^N, \theta^N | \boldsymbol{R}, \Theta)\rho_{\mathsf{o}}(\boldsymbol{R}, \boldsymbol{P}, \Theta, L, t).$$

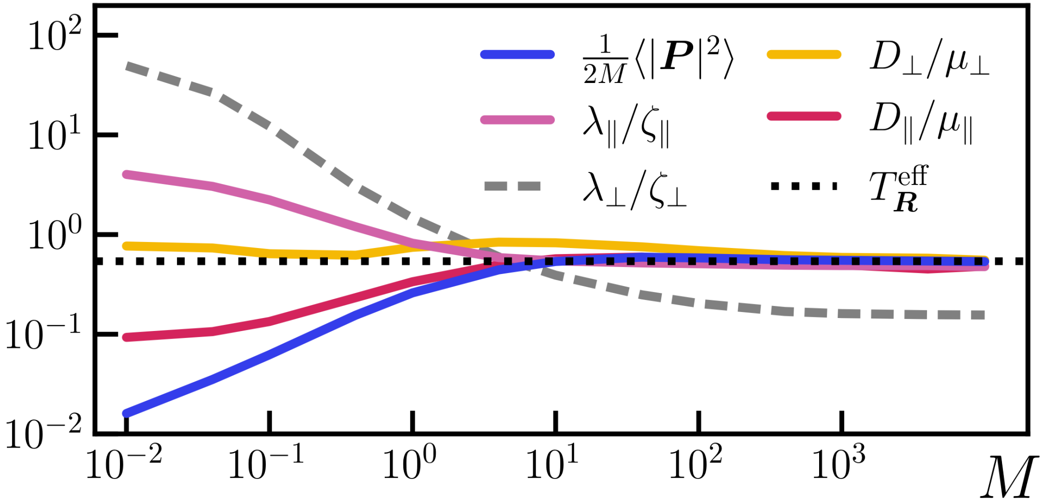
noting that $\mathcal{P}\mathcal{L}_{\mathsf{B}} = \mathcal{L}_{\mathsf{B}}\mathcal{P} = 0$, because $\mathcal{L}_{\mathsf{B}}\rho_{\mathsf{bath}} = 0$.
Then define the orthogonal projector $\mathcal{Q} \equiv 1 - \mathcal{P}$ to decouple the evolution as

$$\begin{aligned}
\partial_t \mathcal{P}\rho &= \epsilon \mathcal{P}\mathcal{L}_{\mathsf{T}}\mathcal{P}\rho + \epsilon \mathcal{P}\mathcal{L}_{\mathsf{T}}\mathcal{Q}\rho \\
\partial_t \mathcal{Q}\rho &= \epsilon \mathcal{Q}\mathcal{L}_{\mathsf{T}}\mathcal{P}\rho + \epsilon \mathcal{Q}\mathcal{L}_{\mathsf{T}}\mathcal{Q}\rho + \mathcal{Q}\mathcal{L}_{\mathsf{B}}\mathcal{Q}\rho\,.
\end{aligned}$$

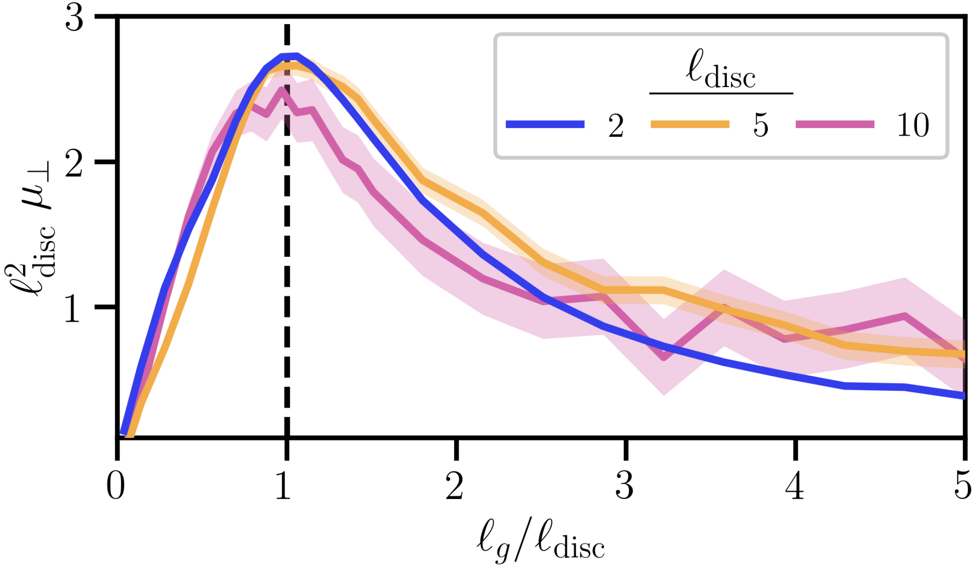
Perturbative solution: $\mathcal{Q}\rho = q^{(0)} + \epsilon q^{(1)} + \mathcal{O}(\epsilon^2) \quad \Rightarrow \quad \mathcal{Q}\mathcal{L}_{\mathsf{B}} q_1 + \mathcal{Q}\mathcal{L}_{\mathsf{T}}\mathcal{P}\rho = 0.$

$$\partial_t \mathcal{P}\rho = \rho_{\mathsf{bath}}\partial_t\rho_{\mathsf{o}} = \mathcal{P}\mathcal{L}_{\mathsf{T}}\mathcal{P}\rho + \mathcal{P}\mathcal{L}_{\mathsf{T}}\mathcal{Q}\left(\int_0^\infty ds e^{\frac{1}{\epsilon}\mathcal{L}_{\mathsf{B}} s}\right)\mathcal{Q}\mathcal{L}_{\mathsf{T}}\mathcal{P}\rho + \mathcal{O}(\epsilon^2)\,.$$

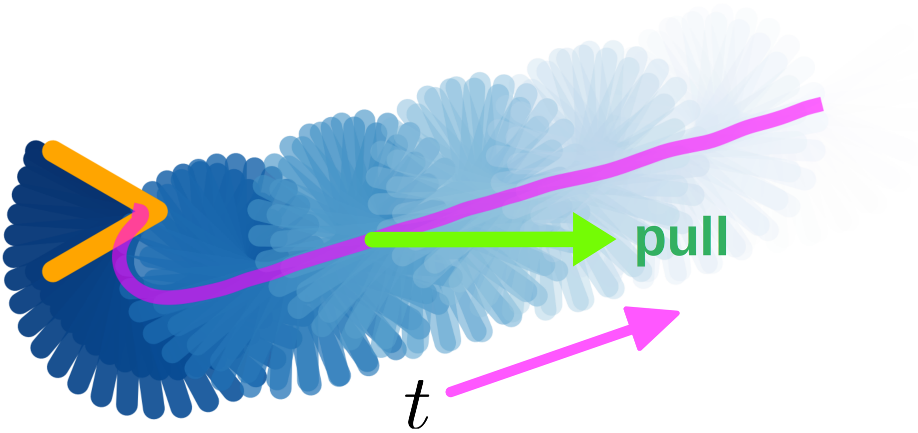
There is no FDT connecting $\lambda_\perp$ and $\zeta_\perp$.



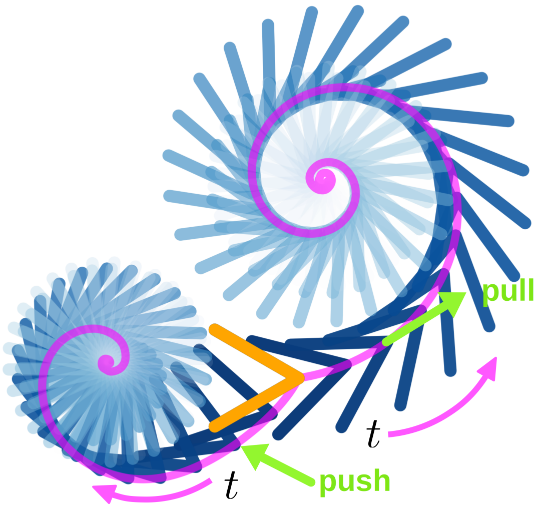
Disk mobility



Wedge pulling



Wedge pulling



Rotational symmetry