UNIVERSITÄT ZU KÖLN

CRC 1238 Control and Dynamics of Quantum Materials

Universal non-equilibrium behavior of limit-cycle phases

Romain Daviet

Together with Carl P. Zelle, Armin Assadohali, Achim Rosch, Sebastian Diehl

- Universal phenomenology at critical exceptional points, Phys. Rev. X (2024)
- Nonequilibrium criticality at the onset of time-Crystalline order, Phys. Rev. Lett. (2024)
- Kardar-Parisi-Zhang scaling in time-crystalline matter, arXiv:2412.09677 (2024)

Career

PhD LPTMC
with N. Dupuis

Post-doc Uni Köln
S. Diehl

2018 — 2021 — today

- 1d quantum fluids
  - Disorder and interactions
  - Mott glass
  - Static chaos

- Non-equilibrium phases of matter
  - Driven-dissipative systems
  - Non-reciprocal active systems

Field theory methods
Renormalization group

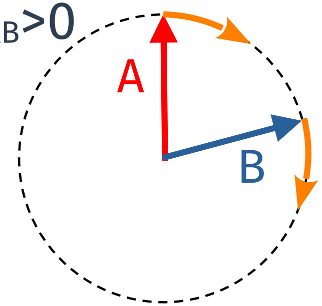
Introduction: Time-dependent (periodic) orders

- Rotating and oscillating orders emerge out of equilibrium:

Classical systems Active matter	Driven magnets	Open quantum condensates

Non reciprocal interactions:
- Ising [Avni et al 24]
- XY [Fruchart et al 21] :

$J_{BA}=-J_{AB}>0$

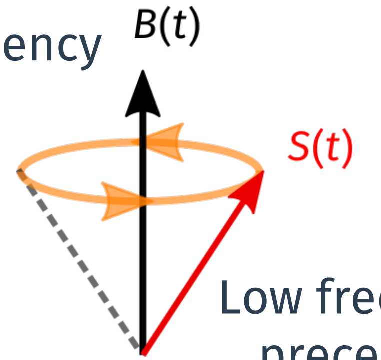


→ Rotation of spins

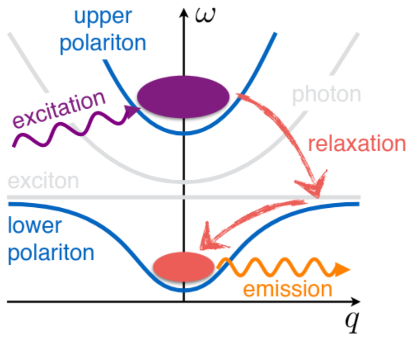
Our work, oscillating fields:
- Ferrimagnet + $B_0\cos(t)$

[Zelle, RD, Rosch, Diehl PRX 24]

High frequency drive



Low frequency precession



Exciton-polaritons
[Carusotto Ciuti 13]

Introduction: Time-dependent (periodic) orders

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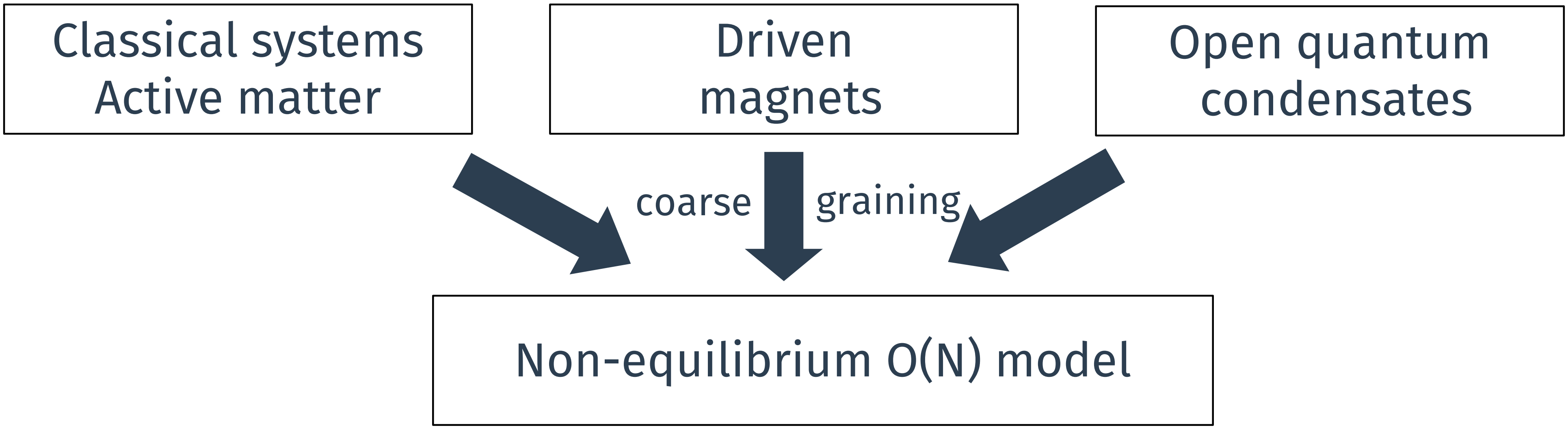


- Sponteanous breaking of continuous time translations → Time crystal
- New non-equilibrium universal phenomenology associated to its breaking ?

Langevin dynamics

- O(N) order parameter in d spatial dimensions: $\vec{\phi}(x,t) \in \mathbb{R}^N$

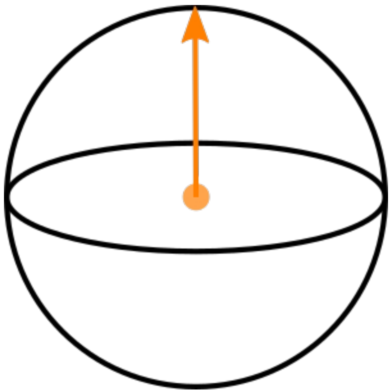
Langevin dynamics without conservation law

$$\partial_t^2 \vec{\phi} + \gamma \partial_t \vec{\phi} + (\underbrace{r + \lambda\rho}_{U'(\rho)} - Z_1 \nabla^2)\vec{\phi} + \vec{\xi} = 0.$$

$$\rho = \vec{\phi} \cdot \vec{\phi} \qquad \langle \xi_i(x,t)\xi_j(x',t')\rangle = D\delta_{ij}\delta(x-x')\delta(t-t')$$

- For r ~ $T\text{-}T_c < 0$, O(N) symmetry broken: $\langle \vec{\phi} \rangle = \sqrt{-r/\lambda}\,\hat{e}_1$
- Non-equilibrium terms are RG irrelevant

→ Effective thermal equilibrium



New transition mechanism: negative damping

- Antidamping γ<0 ↔ pumps supersede losses [Zelle, RD, Rosch, Diehl PRX 24]

Langevin dynamics without conservation law

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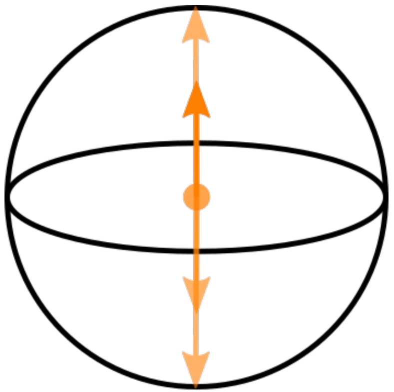
For N=1, Van der Pol equation, known to have a limit cycle:

$$\langle\vec{\phi}\rangle(t) = (\phi_{\mathrm{VdP}}(t), 0, \dots, 0)^T$$



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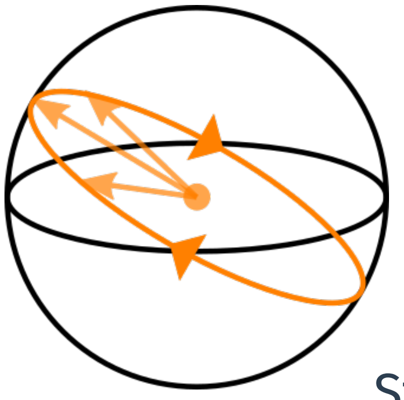
For N>1, rotating order:

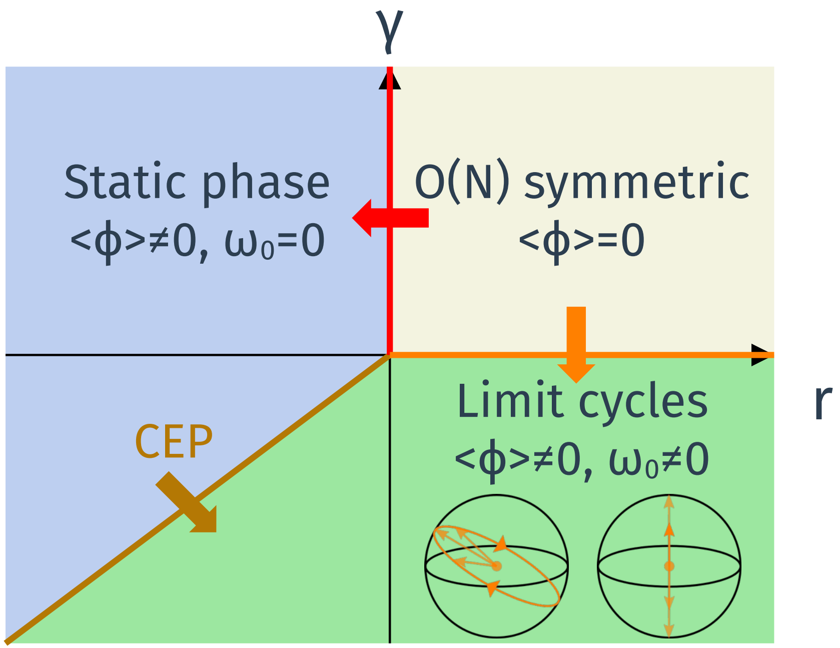
$$\langle \vec{\phi} \rangle(t) = \sqrt{\frac{-\gamma}{2u}} R(\omega_0 t)\hat{e}_1 \quad \text{with } R \in SO(N).$$

Spontaneously chosen circle.



Stable u < u′

Mean-field phase diagram



→ Equilibrium O(N) transition

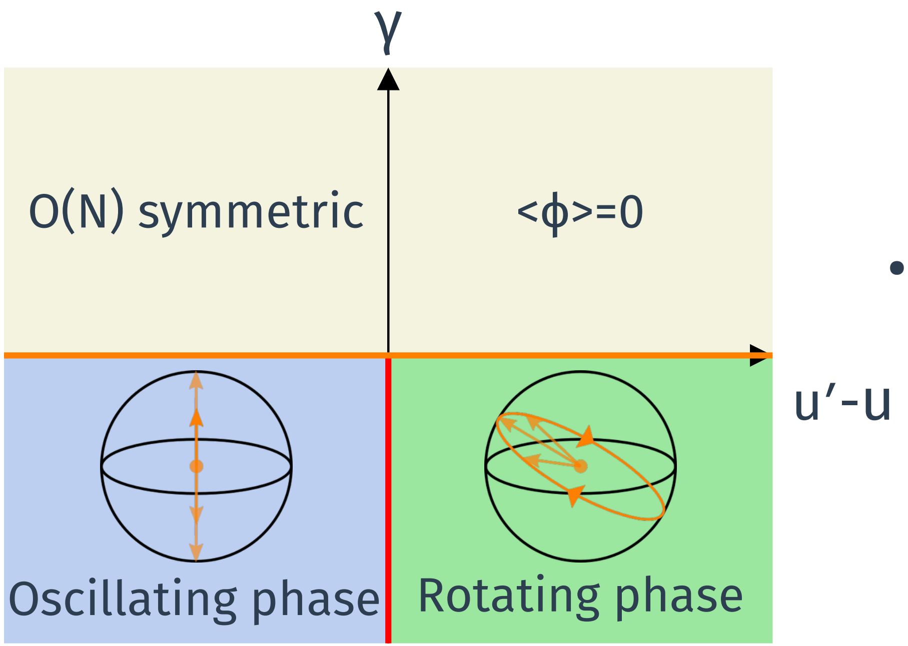
→ Finite frequency transition

→ Critical exceptional point (CEP):

- 2nd order mean-field.
- 1st order induced by fluctuations for d<4 [Zelle, RD, Rosch, Diehl PRX 24]

- Controled resummation/solution of Schwinger-Dyson equations. [Brazovskii 73]

Finite frequency transition



[RD, Zelle, Rosch, Diehl PRL 24]

- **Divergent correlations $q \to 0$, $\omega \to \pm\sqrt{r}$**

$$\langle \phi_i \phi_i \rangle(q,t) \sim \cos(\sqrt{r}t)\frac{e^{-(\frac{Z_1}{2}q^2+\gamma)|t|}}{\frac{Z_1}{2}q^2+\gamma}$$

$$\vec{\phi}(x,t) = \vec{\phi}_1(x,t)\cos(\sqrt{r}t) + \vec{\phi}_2(x,t)\sin(\sqrt{r}t)$$

- Effective theory for $\phi_{1/2}$ by averaging over the fast time scale $\omega_0 \sim\sqrt{r}$.
- Emergent SO(2) symmetry from reparameterization $\omega_0 t \to \omega_0 t + \theta$

Effective O(N)xSO(2) theory: d=4-ϵ perturbation theory

$$(\partial_t - Z\nabla^2 + \gamma)\vec{\psi} + \frac{g_d + ig_c}{2}(\vec{\psi}\cdot\vec{\psi}^*)\vec{\psi} + (\kappa_d + i\kappa_c)(\vec{\psi}\cdot\vec{\psi})\vec{\psi}^* + \vec{\xi} = 0, \quad \vec{\psi} = \vec{\phi}_1 + i\vec{\phi}_2 \in \mathbb{C}^N$$

- Equilibrium case ($g_c = \kappa_c = 0$) → O(N)xO(2): Frustrated magnets ; Spinors BEC.
  [Kawamura 98 ; Delamotte et al 05 ; Kawaguchi, Ueda 12 ; Debelhoir, Dupuis 16]

N=1 case

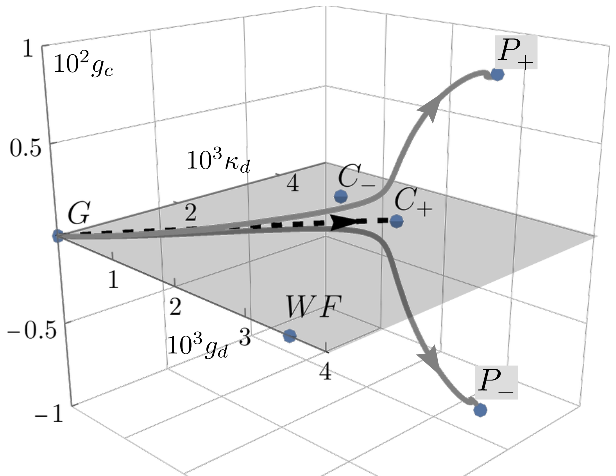
- Classical oscillators [Risler et al 05]
- Dissipative BEC [Sieberer et al 14]

→ Effective thermal equilibrium O(2) universality class

N>1 case [RD, Zelle, Rosch, Diehl PRL 24]

Attractive non-equilibrium fixed point $P_\pm$

→ New universality class O(N)xSO(2)



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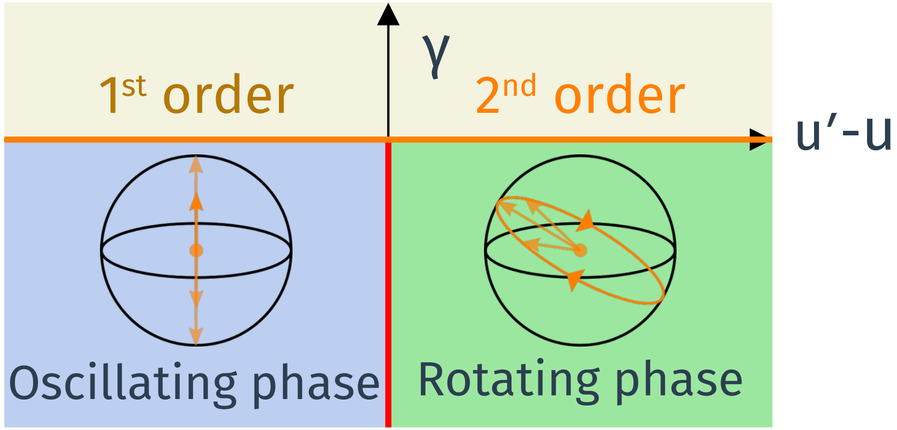
  O(2) universality class

O(N=2)xSO(2) condensates:

- Polarized Exciton-polaritons
- Yttrium-Iron-Garnet magnon

N>1 case [RD, Zelle, Rosch, Diehl PRL 24]

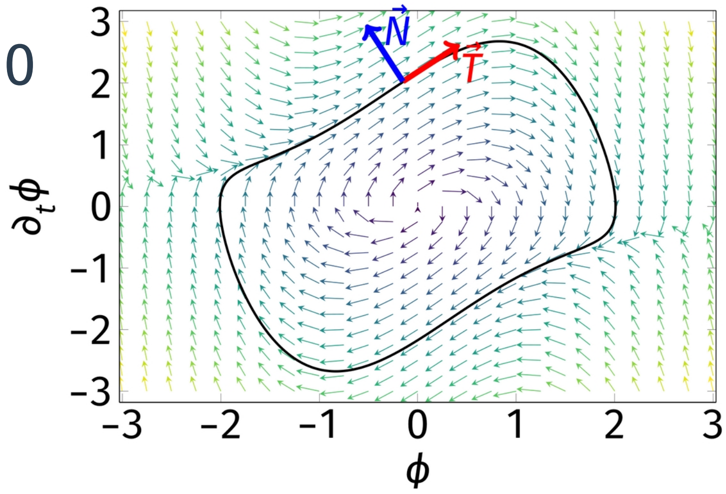
Attractive non-equilibrium fixed point $P_\pm$
→ New universality class O(N)xSO(2)



Limit-cycle phase and Kardar-Parisi-Zhang equation (N=1)

Broken continuous time-translation → slow (Goldstone) mode

$$\partial_t^2\phi + (\gamma + u\rho - Z_2\nabla^2)\partial_t\phi + (r - Z_1\nabla^2)\phi + \xi = 0$$



$$\vec{\Phi}_0(t) = (\phi_{\text{VdP}}(t), \phi'_{\text{VdP}}(t))^T$$

$$\vec{\Phi}(x,t) = \vec{\Phi}_0\big(t + \theta(x,t)\big) + a(x,t)\vec{N}(t)$$

$\theta \in [0, 2\pi[$ is an equivalent solution

From symmetry and explicit calculations, at timescale $t_G >> \omega_0^{-1}$:

KPZ equation [Kardar Parisi Zhang 86] + compactness

$$\partial_t\theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

[RD, Zelle, Assadohali, Diehl, arXiv:2412.09677]

Compact KPZ (N=1)

Compact KPZ equation

$$\partial_t \theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

- KPZ implies growth
  → Time-translation breaking

- Driven condensate [Fontaine et al 22] ; Active stripes [Chen, Toner 2013] ; Phase synchronization [Lauter et al 17] ; Generalized hydrodynamics [De Nardis et al 2023]

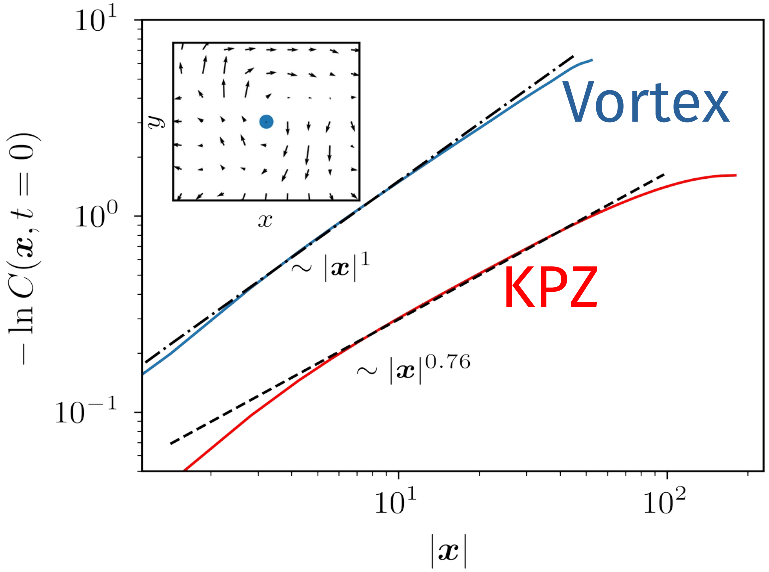
KPZ regime:

$$C(x,t) = \langle \phi(x,t)\phi(0,0)\rangle \sim \cos(\omega_0 t) e^{-\langle(\theta(x,t)-\theta(0,0))^2\rangle_c}$$

$$\Longrightarrow \quad C(x,0) \sim e^{-|x|^{2\chi}}, \quad C(0,t) \sim e^{-|t|^{2\beta}}$$

Vortex regime, high noise:

$$C(x,0) \sim e^{-|x|}, \quad C(0,t) \sim e^{-|t|}$$



2d: $2\chi \sim 0.78$, Numerics: $2\chi \sim 0.76$

Effective model for N>1

Generalizations of KPZ

Time Translation → $\partial_t \theta - Z_\theta \nabla^2 \theta + \lambda_\theta (\nabla\theta)^2 + \lambda_\alpha (\nabla\vec{\alpha})^2 + \xi_\theta = 0,$

O(N) symmetry → $\partial_t \vec{\alpha} - Z_\theta \nabla^2 \vec{\alpha} + 2\lambda_\times \nabla\theta \cdot \nabla\vec{\alpha} + \vec{\xi}_\alpha = 0,$

Oscillating phase : known model [Ertas Kardas 92], $\vec{\alpha} \in \mathbb{R}^{N-1}$

- Non KPZ behavior for N ≥ 4 → New RG fixed point in 1d.

Rotating phase : $\vec{\alpha} \in \mathbb{C}^M$ → New model (2M=2N-4)

- New non KPZ behaviors for all N ≥ 3 → Possible in spin systems.

[Zelle, RD, Diehl, in preparation]

Conclusion

- **Summary**
  - Universal phenomena associated with spontaneous breaking of time translations symmetry.
  - New non-equilibrium universality classes.
  - Ordered phase leads to generalization of KPZ.
- **Perspectives**
  - Effect of conservation laws? → Non reciprocal Cahn-Illiard [Saha et al 20; You et al 20]
  - True quantum effects in dissipative limit cycles/condensates?
  - Driven fermions?