Precision Field theory for condensed matter systems

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Outline

Introduction

TOPIC: Perturbative and non-perturbative approaches to condensed matter systems.

EXPERTISE: Higher-orders renormalization-group techniques.

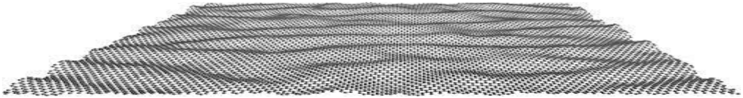
BACKGROUND:

- 2025: Post-Doctoral position - LAPTh - Annecy
- 2023-2024: Post-Doctoral position - INPAC - Shanghai Jiao Tong University
- 2020-2023: PhD thesis - LPTHE - SU - Supervisor: S. Teber

TODAY'S GOALS:

- Precision interaction effects
- Benchmark less controlled approaches
- New physics beyond leading order

TODAY'S SUBJECT: higher-order elastic and electronic interactions in membranes.

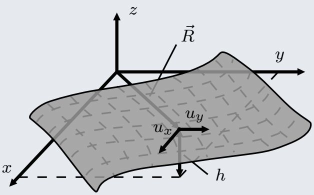


Outline

Model – Flat crystalline membranes

Low-temperature action: A scalar derivative field theory

[Neslon, Peliti '87]



In-plane (phonon) $\vec{u}(\vec{x})$ and out-of-plane (flexuron) $\vec{h}(\vec{x})$ fields

$$S = \int \mathrm{d}^d x \left[\frac{\kappa}{2} (\partial^2 h_\alpha)^2 + \mu T_{ab}^2 + \frac{\lambda}{2} T_{aa}^2 \right],$$

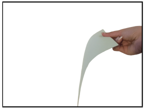
$$\text{Strain tensor} \equiv T_{ab} \approx \frac{1}{2} (\partial_a u_b + \partial_b u_a + \partial_a h_\alpha \partial_b h_\alpha)$$

$\kappa \equiv$ Bending rigidity, $\mu \equiv$ Shear modulus, $\lambda \equiv$ 2$^{\text{nd}}$ Lamé coef.

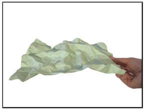
Renormalization: Anomalous elasticity exponent η

Flexuron: $\langle hh \rangle \sim p^{-(4-\eta)}$, Lamé coefs: $\lambda \sim \mu \sim p^{4-d-2\eta}$,

Phonon: $\langle uu \rangle \sim p^{-(6-d-2\eta)}$, Bending rigidity: $\kappa \sim p^{-\eta}$.



No deformations ($\eta = 0$)



With deformations ($\eta > 0$)

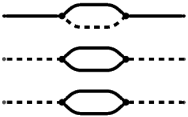
X-ray scattering experiments:

$\eta^{\text{[Schmidt et al. '93]}}_{\text{(blood-cells)}} = 0.70(20)$, $\eta^{\text{[Lopez-Polin et al. '15]}}_{\text{(graphene)}} \approx 0.82$,

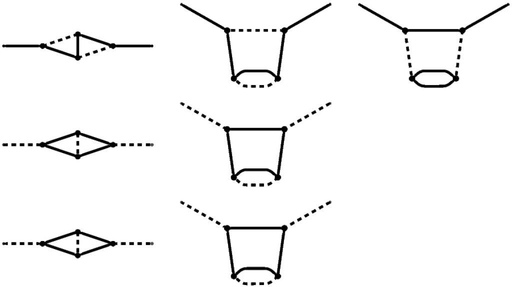
$\eta^{\text{[Gourier, Daillant '97]}}_{\text{(amphiphilic-films)}} = 0.70(20)$, $\eta^{\text{[Jackson et al. '23]}}_{\text{(fly-egg)}} \approx 0.8$.

1 & 2 loop approach

One loop: 3 diagrams, ~ 10 integrals, can be done by hand [Aronovitz, Lubensky '88]

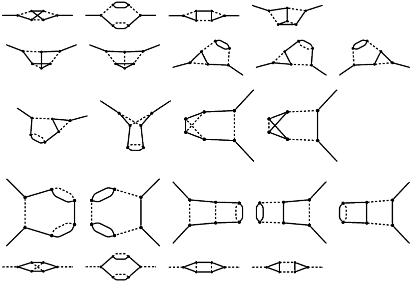


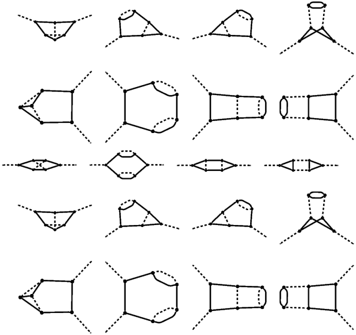
Two-loops: 7 diagrams, ~ 1000 integrals, need some automatization [Coquand, Mouhanna, Teber '20]



3 loop approach

Three loops: 42 diagrams, ~ 200000 integrals, need full automatization [SM, Mouhanna, Teber '21]

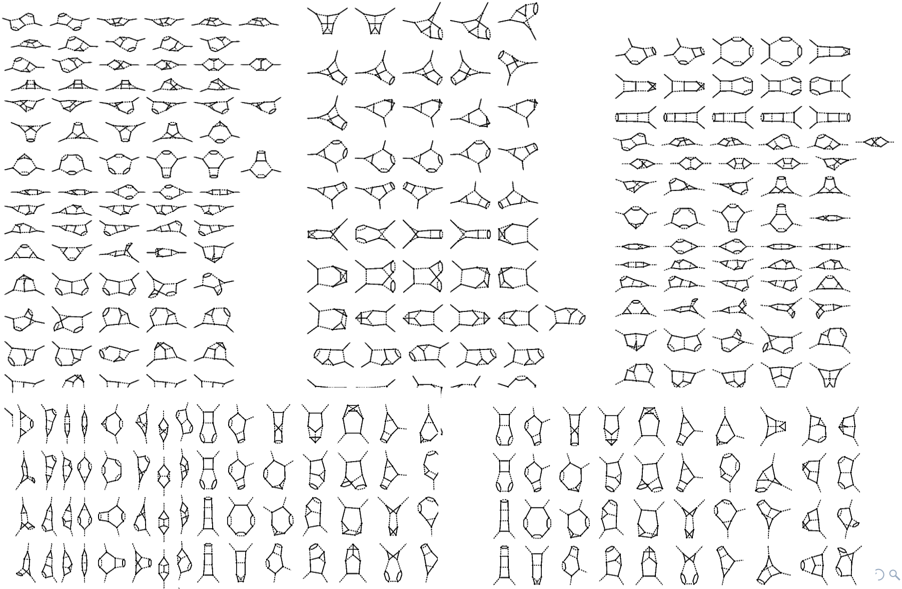




Highly automated computation using software from high-energy physics:
Qgraf [Nogueira '93], Fire [Smirnov '16], LiteRed [Lee '12], Form [Vermaseren '89] ...

4 loop approach

Four loops: 337 diagrams, ~ 60 million integrals, full automatization + supercomputer [SM '24]



Results

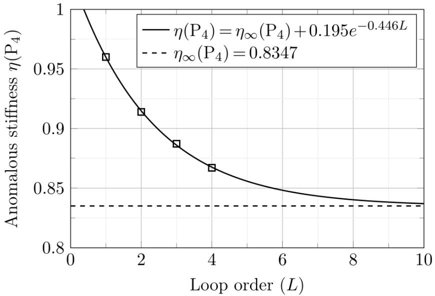
Analytical result at the stable fixed point in $d=4-2\varepsilon$: [SM '24]

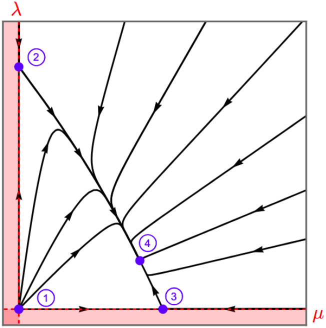
$$\eta = \underbrace{\frac{24\varepsilon}{25}}_{\text{1-loop}} - \underbrace{\frac{144\varepsilon^2}{3125}}_{\text{2-loop}} - \underbrace{\frac{4(1286928\zeta_3 - 568241)\varepsilon^3}{146484375}}_{\text{3-loop}} - \underbrace{\frac{4(139409079893 + \ldots)\varepsilon^4}{54931640625}}_{\text{4-loop}} + \mathrm{O}(\varepsilon^5)$$

Numerically:

$$\eta = 0.96\varepsilon - 0.046\varepsilon^2 - 0.027\varepsilon^3 - 0.020\varepsilon^4 + \ldots \underset{\varepsilon\to 1}{\approx} 0.867$$

Smaller and smaller corrections! (unexpected)





Benchmark less controlled methods

NPRG and SCSA provide non-perturbative results:

NPRG: $$\frac{(d-1)(d+8)(d-\eta+4)(4-d-2\eta)}{2\eta(d-\eta+8)} - d_c = 0$$ [Kownacki, Mouhanna '09]

SCSA: $$\frac{d(d-1)\Gamma(2-\eta)\Gamma(2-\eta/2)\Gamma(\eta/2)\Gamma(\eta+d)}{\Gamma(2-\eta-d/2)\Gamma((4-\eta+d)/2)\Gamma((\eta+d)/2)\Gamma(\eta+d/2)} - d_c = 0$$ [Le Doussal, Radzihovsky '92]

And non-perturbative answers $\eta_{\text{NPRG}} = 0.849$ and $\eta_{\text{SCSA}} = 0.821$. Can we trust them?

We can benchmark numerically in $d = 4-2\varepsilon$:

$$\eta_{\text{4-loop}} = 0.96\varepsilon - 0.046\varepsilon^2 - 0.027\varepsilon^3 - 0.020\varepsilon^4 + O(\varepsilon^5),$$ [SM '24]

$$\eta_{\text{NPRG}} = 0.96\varepsilon - 0.037\varepsilon^2 - 0.027\varepsilon^3 - 0.018\varepsilon^4 + O(\varepsilon^5),$$ [Kownacki, Mouhanna '09]

$$\eta_{\text{SCSA}} = 0.96\varepsilon - 0.048\varepsilon^2 - 0.028\varepsilon^3 - 0.018\varepsilon^4 + O(\varepsilon^5),$$ [Le Doussal, Radzihovsky '92]

They are numerically extremely successful for this model!

Literature – A 30 years old story

Simulations ↓ / Theory ↑

η	Method	Year/ref
≈ 0.66	Monte Carlo (membrane)	1990 Abraham, Nelson
0.667	Large D (LO)	1988 Guitter, *et al.*
≈ 0.7	Monte Carlo (vesicles)	1991 Komura, Baumgärtner
0.72 ± 0.04	Monte Carlo (membrane)	1989 Leibler, Maggs
0.75 ± 0.05	Monte Carlo (membrane)	1990 Guitter *et al.*
$0.750(5)$	Monte Carlo (membrane)	1996 Bowick *et al.*
0.789	SCSA (large-d_c NLO, semi-numerical)	2009 Gazit
$0.795(10)$	Monte Carlo (graphene)	2013 Tröster
$0.81(3)$	Monte Carlo (membrane)	1993 Zhang *et al.*
≈ 0.82	Molecular dynamics simulations	1996 Zhang *et al.*
≈ 0.82	SCSA (LO, semi-numerical)	2010 Zakharchenko *et al.*
0.821	SCSA (LO, analytical)	1992 Le Doussal, Radzihovsky
0.835	1 to 4-loop (extrapolation)	2024 SM
0.849	NPRG (analytical)	2009 Kownacki, Mouhanna
≈ 0.85	NPRG (semi-numerical)	2009 Braghin, Hasselmann
≈ 0.85	Monte Carlo (graphene)	2009 Los *et al.*
0.867	4-loop	2021 Pikelner
0.887	3-loop	2021 SM *et al.*
0.9 ± 0.04	Molecular dynamics simulations	1993 Petsche, Grest
0.914	2-loop	2020 Coquand *et al.*
0.960	1-loop	1988 Aronovitz, Lubensky
1	Mean field	1987 Nelson, Peliti

Recalling $\eta_{(\text{exp.})} \approx 0.8$ [Schmidt *et al.* '93; Gourier *et al.* '97; Lopez-Polin *et al.* '15; Jackson *et al.* '23]]

Outline

Model: Quenched disordered membranes

The quenched disordered action

[Neslon, Radzihovsky '91]

Introduce local random curvature and local random stress. Take quenched Gaussian distribution with zero mean. Use replica trick $\implies$ fields promoted with replica indices $A=1,\ldots,n\to 0$.

$$S=\int \mathrm{d}^d x\Big[\frac{\kappa}{2}(\partial^2 h_\alpha^A)^2+\frac{\lambda}{2}(T_{aa}^A)^2+\mu(T_{ab}^A)^2+\Delta_\kappa \partial^2 h_\alpha^A \partial^2 h_\alpha^B+\frac{\Delta_\lambda}{2}T_{aa}^A T_{bb}^A+\Delta_\mu T_{ab}^A T_{ab}^B\Big].$$

Content: 2 fields, 6 couplings, 3 indices type ... challenging.

Renormalization

With disorder, correlation functions become tensorial in the replica indices:

$$\langle h^A h^B\rangle \sim \delta^{AB} p^{-(4-\eta)}+J^{AB}p^{-(4-\eta-\phi)},$$
$$\langle u^A u^B\rangle \sim \delta^{AB} p^{-(6-d-2\eta)}+J^{AB}p^{-(6-d-2\eta-2\phi)}.$$

There is now an extra disorder exponent ϕ:

$\phi>0 \equiv$ Temperature dominates (Clean phase),

$\phi<0 \equiv$ Disorder dominates (Glassy phase),

$\phi=0 \equiv$ Temperature and disorder coexist (Marginal phase).

Compute η in the glassy and marginal phase!

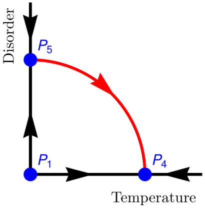
Results: Exponents (1 & 2)

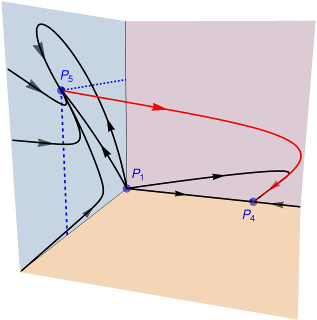
At one loop: [Neslon Radzihovsky '91]

- A single disordered fixed point ($\phi = ?$): $\eta_5 = \frac{6\varepsilon}{7} + O(\varepsilon^2)$

and ϕ is not resolved properly...

The RG-flow is weird:





Disorder seems irrelevant...

At two loops: Some progress, but the authors could not resolve the fixed points properly. [Coquand Mouhanna '21].

Results: Exponents (3)

At three loop, the fixed points are finaly resolved [SM, Mouhanna '22]

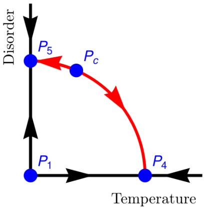
- Disorder dominated fixed point ($\phi < 0$?):

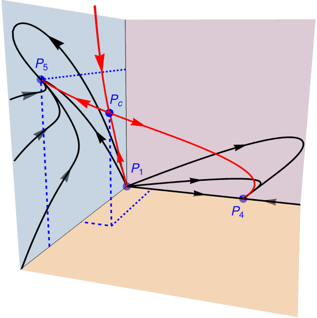
$$\eta_5 = \frac{6\varepsilon}{7} - \frac{3629\varepsilon^2}{24010} - \frac{(698184144\zeta_3 - 759884263)\varepsilon^3}{823543000} + O(\varepsilon^4) \underset{\varepsilon\to 1}{\approx} 0.610$$

- New critical fixed point ($\phi = 0$), finite disorder finite temperature glassy phase:

$$\eta_c = \frac{6\varepsilon}{7} - \frac{507\varepsilon^2}{3430} - \frac{(9504432\zeta_3 - 10463737)\varepsilon^3}{10084200} + O(\varepsilon^4) \underset{\varepsilon\to 1}{\approx} 0.614$$

New physics beyond LO. Disorder is now relevant:





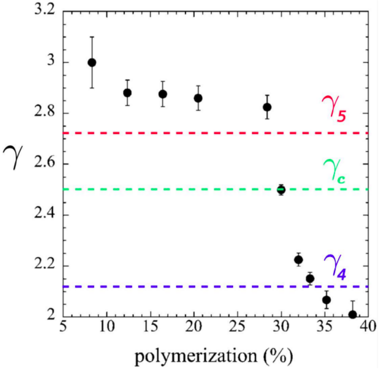
But P_c is still marginally unstable (in contradiction with NPRG), and ϕ is still not resolved properly at P_5...
calls for 4-loop to settle everything.

Literature comparison

$\varepsilon = 1$	This work			Other approaches		
	1-loop	2-loop	3-loop	NPRG	SCSA	large-d_c
η_5	0.857^a	0.706^b	0.610^c	0.449^d	0.449^e	0.228^f
η_c	0.857^a	0.709^b	0.614^c	0.492^d		
η_4	0.960^g	0.914^h	0.887^i	0.849^j	0.821^k	1.68^l

The NPRG is the only other approach to distinguish P$_5$ and P_c!
SCSA (and large-d_c) can't distinguish P$_{c/5}$ at LO.

[a] [Morse, Lubensky, 92'], [b] [Coquand, Mouhanna, 21'], [c] [Metayer, Mouh 18'], [e] [Radzihovsky, Le Doussal, 92'], [f] [Saykin, Kachorovskii, Burmistro Mouhanna, Teber, 20'], [i] [Metayer, Mouhanna, Teber, 21'], [j] [Kownacki, Doussal, Radzihovsky, 92'], [l] [Saykin, Gornyi, Kachorovskii, Burmistrov, 2



New fixed point P$_c$ observed in partial polymerization experiments [Chaieb *et al.* '06] ($\gamma = 3 - \eta$), first seen theoretically in NPRG [Coquand, Essafi, Kownacki, Mouhanna, 18'].

Conclusion

Takeway:

- Apparently convergent ε-series.
- Good benchmark for NPRG, SCSA, large-d_c ...
- $\exists$ a new non-trivial, finite-temperature, finite-disorder phase transition towards a glassy phase in polymerized membranes.

Ongoing projects:

- ϕ not completely resolved even at 3-loop, 4-loop needed for a final answer.
- SCSA beyond LO might distinguish η_5 and η_c

Model perspectives:

- Crumpled-to-flat phase transition model
- Surface growth models (KPZ)
- Smectic-glass transition in liquid crystals

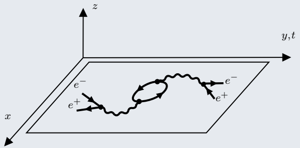
Techniques perspectives:

- Auxiliary field technique, big shortcut?
- NPRG
- Automatic NLO SCSA package

Outline

Model for electronic interaction in graphene

Model – QED in three dimensions



Massless (S)QED$_3$ in large-N_f:

$$S = \int \mathrm{d}^3x \left[\mathrm{i}\bar{\psi}\not{D}\psi - \frac{1}{4}F_{\mu\nu}^2 \right] + \mathrm{GF}_{(\xi)} + \mathrm{SUSY}_{(\mathcal{N}=1)},$$

N_f electrons flavors (ψ) coupled (e) to photons (A^μ)

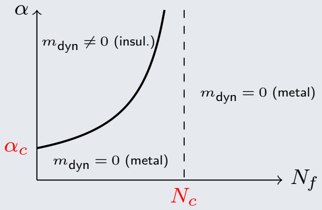
Naively superrenormalizable $[e]=1$, but non-trivial IR fixed point in the large-N_f limit $\Rightarrow$ renormalizable with dimless coupling $1/N_f$ [Appelquist, Pisarski '81]

Dynamical electron mass generation

Renormalization of the (s)electron mass:

$$m_\psi \sim p^{1+\gamma_{m_\psi}}, \quad m_\phi \sim p^{1+\gamma_{m_\phi}}$$

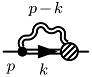
Search for N_c. Non-perturbative effect!



Renormalization and dynamical mass generation

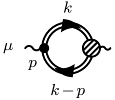
Non-perturbative effect, need to solve the SD equations self-consistently:

$$-\mathrm{i}\Sigma(p) =$$



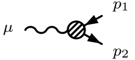
$$= \int [\mathrm{d}^{d_e} k] \frac{\Gamma^\mu(k,p)\Gamma_0^\nu D^{(0)}_{\mu\nu}(p-k)}{(\not{k}-\Sigma(k))(1-\Pi(p-k))},$$

$$\mathrm{i}\Pi^{\mu\nu}(p) =$$



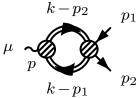
$$= -N_f \int [\mathrm{d}^{d_e} k] \mathrm{Tr} \frac{\Gamma^\mu(k,k-p)\Gamma_0^\nu}{(\not{k}-\Sigma(k))(\not{k}-\not{p}-\Sigma(k-p))},$$

$$\Gamma^\mu(p_1,p_2) =$$



$$= -\mathrm{i}e\gamma^\mu - \mathrm{i}e\Lambda^\mu(p_1,p_2),$$

$$\Lambda^\mu(p_1,p_2) =$$



$$= -N_f \int [\mathrm{d}^{d_e} k] \frac{\Gamma^\mu(k-p_1,k-p_2)K(p_1,k-p_2,k-p_1,p_2)}{(\not{k}-\not{p}1-\Sigma(k-p1))(\not{k}-\not{p}2-\Sigma(k-p2))},$$

Massaging the equations around Σ, we conjecture the simple all-order gap equation: [SM, Teber '21]

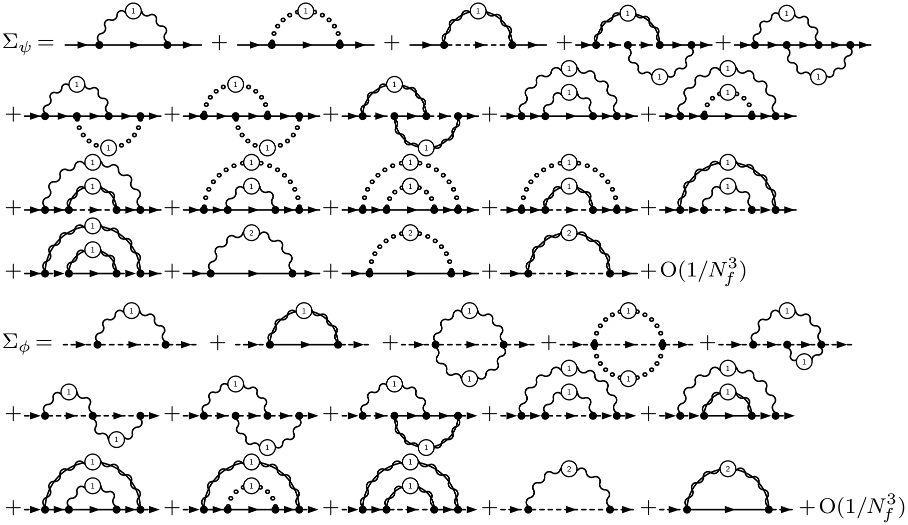
$$(1-b)b = (1-\gamma_{m_\psi})\gamma_{m_\psi}, \quad \text{with} \quad m_{\text{dyn}} = \Sigma(p\to 0) \sim p^{-b}$$

Which depends only on $\gamma_{m_\psi} \implies$ precision needed.

We need to go beyond LO in large-N_f expansion...

Large-N_f approach

NLO (s)electron self-energies, 34 diagrams, ~ 500 branchut integrals [SM, Teber '22]



Results – (S)QED$_3$

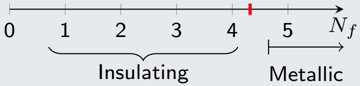
In QED$_3$: $\gamma_{m_\psi} = \frac{32}{3\pi^2 N_f} - \frac{64(28-3\pi^2)}{9\pi^4 N_f^2} + \mathrm{O}\left(1/N_f^3\right)$ [Gracey '93-'94]

In SQED$_3$: $\gamma_{m_\psi} = \gamma_{m_\phi} = \frac{4}{\pi^2 N_f} - \frac{4(14-\pi^2)}{\pi^4 N_f^2} + \mathrm{O}\left(1/N_f^3\right)$ [SM, Teber '22]

Using the gap equation we derive N_c at leading order:

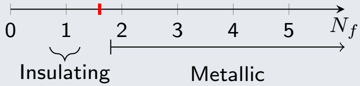
QED$_3$:

$$N_c = \frac{128}{3\pi^2} = 4.32$$



SQED$_3$:

$$N_c = \frac{16}{\pi^2} = 1.62$$



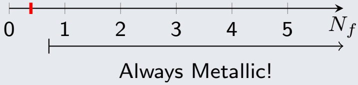
And beyond leading order? N_c reduces drastically $\Longrightarrow$ New physics.

QED$_3$:

$$N_c = \frac{2(4+3\pi^2)}{3\pi^2} = 2.27$$

SQED$_3$:

$$N_c = \frac{\pi^2 - 6}{\pi^2} = 0.39$$



Literature – N_c in QED$_3$, a 40 years old debate!

In the literature, seemingly all values for N_c has been found, from 0 to ∞ ...

N_c in QED$_3$	Method	Year
∞	SD (LO)	1984 Pisarski
∞	SD (non-perturbative, Landau gauge)	1990, 1992 Pennington *et al.*
∞	RG study	1991 Pisarski
∞	lattice simulations	1993, 1996 Azcoiti *et al.*
< 4.4	F-theorem	2015 Giombi *et al.*
$(4/3)(32/\pi^2) = 4.32$	SD (LO, resummation)	1989 Nash
4.422	RG study (1-loop) ($N_c^{\text{conf}} \approx 6.24$)	2016 Janssen
4	functional RG ($4.1 < N_c^{\text{conf}} < 10.0$)	2014 Braun *et al.*
$3 < N_c < 4$	RG study	2001 Kubota, Terao
3.5 ± 0.5	lattice simulations	1988, 1989 Dagotto *et al.*
3.31	SD (NLO, Landau gauge)	1993 Kotikov
3.29	SD (NLO, Landau gauge)	2016 Kotikov *et al.*
$32/\pi^2 \approx 3.24$	SD (LO, Landau gauge)	1988 Appelquist *et al.*
$3.0084 - 3.0844$	SD (NLO, resummation)	2016 Kotikov, Teber
2.89	RG study (1-loop)	2016 Herbut
2.85	SD (NLO, resummation, $\forall\xi$)	2016 Gusynin *et al.* Kotikov *et al.*
$1 + \sqrt{2} = 2.41$	F-theorem	2016 Giombi *et al.*
2.27	Effective gap eq. (NLO, double resummation, $\forall\xi$)	2022 SM, Teber
$< 9/4 = 2.25$	RG study (1-loop)	2015 Di Pietro *et al.*
$< 3/2$	Free energy constraint	1999 Appelquist *et al.*
$1 < N_c < 4$	lattice simulations	2004 Hands *et al.* 2008 Strouthos *et al.*
0	SD (non-perturbative, Landau gauge)	1990 Atkinson *et al.*
0	lattice simulations	2015, 2016 Karthik, Narayanan

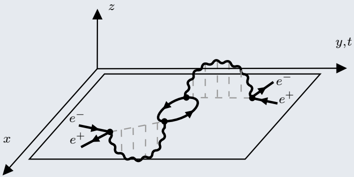
Recent results converges towards $N_c \in [2,3]$...

Outline

Model – Electronic interactions in graphene

(S)QED in mixed dimensions

[Gorbar, Gusynin, Miransky '01]



Massless (S)QED$_{4,3}$:

$$S = \mathrm{i}\int \mathrm{d}^3x \bar{\psi}\not{D}\psi - \frac{1}{4}\int \mathrm{d}^4x F_{\mu\nu}^2 + \mathsf{GF}_{(\xi)} + \mathsf{SUSY}_{(\mathcal{N}=1)},$$

N_f electrons flavors (ψ) in 3-dim coupled (e) to photons (A^μ) in 4-dim

Renormalizable with non-running dimensionful coupling $\alpha = e^2/4\pi$. [Gorbar, Gusynin, Miransky '02]

Optical conductivity

The photon (perpendicular) propagator renormalize as

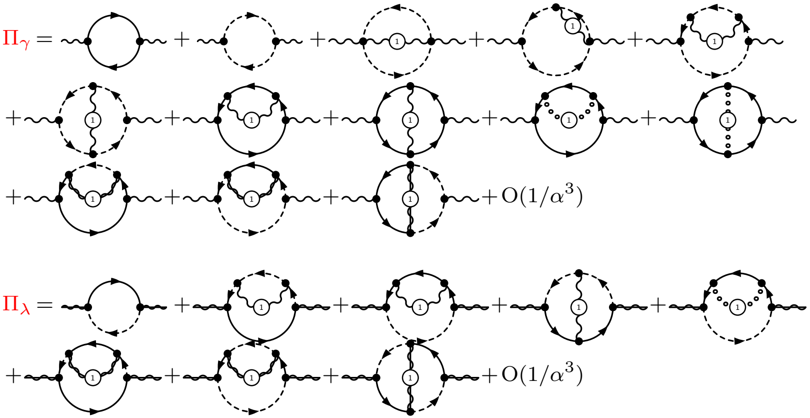
$$\langle A^\mu A^\nu \rangle_\perp = \frac{\mathrm{i}}{2p}\frac{P_\perp^{\mu\nu}}{1-\Pi_\gamma}, \quad \text{with} \quad \Pi_\gamma = -\frac{\pi N_f \alpha}{4}\left[1 + C_\gamma \alpha + \mathrm{O}(\alpha^2)\right].$$

The photon polarization Π_γ is finite & gauge-independant ⇒ physical! Need C_γ for precision.
Optical conductivity (Kubo formula):

$$\sigma(\omega) = -p \times \Pi_\gamma = \sigma_0(1 + C_\gamma \alpha + \mathrm{O}(\alpha^2))$$

Multi-loop approach

Two-loop photon and photino polarizations: 21 diagrams, ~ 500 branchut integrals [SM, Teber '21]



Result: $C_\gamma = \dfrac{92 - 9\pi^2}{18\pi} \approx 0.06$ and $C_\gamma^{\mathsf{SUSY}} = \dfrac{12 - \pi^2}{2\pi} \approx 0.34$ both very small corrections!

Result

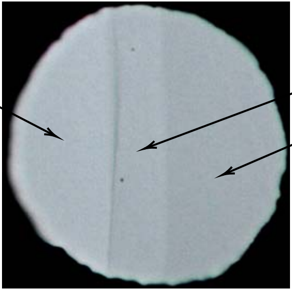
The optical conductivity is directly related to the universal optical absorbance (A): [SM, Teber '23]

$$A_{\text{graphene}} = \pi\alpha(1+\alpha C_\gamma + \dots) = (2.293 \pm 0.002)\%$$

$$A_{\text{super-graphene}} = 2\pi\alpha(1+\alpha C_\gamma^{\text{SUSY}} + \dots) = (4.59 \pm 0.15)\%$$

Interestingly, optical measurements provides [Nair *et al.* '08]:

$A^{(\text{exp})}_{\text{Air}} \approx 0\%$



$A^{(\text{exp})}_{\text{Graphene}} \approx 2.3\%$

$A^{(\text{exp})}_{\text{Bi-layer graphene}} \approx 4.6\%$

Also what super-graphene would look like!

- SUSY enhance the optical absorbance.
- Bi-layer and super-graphene have similar optical absorbance.

Conclusion

Takeway:

- In fermionic QEDs, dynamical matter mass generation is possible for small N_f
- SUSY strongly suppress dynamical matter mass generation
- SUSY enhances the optical absorbance of planar materials

Ongoing projects:

- Higher orders for QCD cusp anomalous dimension matrix

Models perspectives:

- Dynamical mass generation in QCD
- Josephson junction, sine-Gordon model
- ...

Technical perspectives:

- Bootstrap methods
- Automatic NLO SD solving package

Outline

Conclusion

Final takeaway:

- Higher precision for critical exponents is successful in many models
- Strong benchmark for less controlled methods
- Higher orders for non-perturbative methods is needed
- Perturbative input to access non-perturbative features
- New physics beyond leading order!

Thank you for your attention :)

Selection of seminars and posters:

- **Anomalous elasticity in polymerized membranes, analytical 4-loop result**
  2024: Seminar, "journée des utilisateurs du supercalculateur MeSU" – SU
- **Electronic interaction effects in low-dimensional abelian field theories**
  2024: Seminar, ShanghaiTech
- **Field theoretic approach to flat quenched disordered polymerized membranes**
  2023: Seminar, International conference "48th Middle European Cooperation in Statistical Physics" (MECO48) Slovakia
- **Membranes elastic degrees of freedom, a multi-loop approach**
  2022: Poster, international conference "47th Middle European Cooperation in Statistical Physics" (MECO47)
- **3-loop order approach to flat polymerized membranes**
  2022: Seminar, "Journée de physique statistique" - ENS - France
- **Membranes elastic degrees of freedom, a multi-loop approach**
  2021: Poster, international conference "Advanced Computing and Analysis Techniques in Physics Research" (ACAT21)
- **2-loop anomalous dimensions in reduced QED and dynamical mass generation**
  2021: Poster, international conference "Relativistic Fermions in Flatland"

Publications

- **Field-theoretic approach to flat polymerized membranes**
  [SM & S. Teber, 2025,arXiv:2412.18490]
- **Four-loop elasticity renormalization of low-temperature flat polymerized membranes**
  [SM, EPL, 2024, 10.1209/0295-5075/ad949a]
- **Critical Properties of Three-Dimensional Many-Flavor QEDs**
  [SM & S. Teber, Symmetry, 2023, 10.3390/sym15091806]
- **Electron mass anomalous dimension at $O(1/N_f^2)$ in 3D $\mathcal{N}$=1 supersymmetric QED**
  [SM & S. Teber, PLB, 2022, 838(2023)137729]
- **Flat polymerized membranes at three-loop order**
  [SM, D. Mouhanna & S. Teber, J. Phys. Conf. Ser., 2022, 2438(2023)1:012141]
- **The flat phase of quenched disordered membranes at three-loop order**
  [SM & D. Mouhanna, PRE, 2022, 106(6):064114]
- **Three-loop order approach to flat polymerized membranes**
  [SM, D. Mouhanna & S. Teber, PRE Letter, 2022, 105(1):L012603]
- **Two-loop mass anomalous dimension in RQED and dynamical fermion mass generation**
  [SM & S. Teber, JHEP, 2021 2021(9):107]
- **3D $\mathcal{N}$=1 supersymmetric QED at large-N_f and applications to super-graphene**
  [A. James, SM & S. Teber, 2021 arXiv:2102.02722]