Temporal Bell inequalities in non-relativistic many-body physics

Andrea Tononi

ICFO – The Institute of Photonic Sciences

LPTMC – 1/10/2024

based on [Tononi, Lewenstein, arXiv:2409.17290]

PASQuanS2

Funded by the European Union

Quantum mechanics

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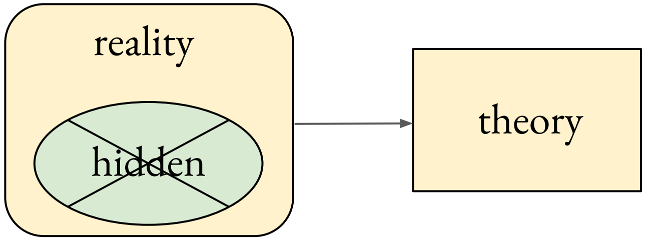
Einstein-Podolsky-Rosen (1935)

Can we assume quantum mechanics to be complete and get inconsistencies?

Completeness: "any element of reality has a counterpart in the theory"



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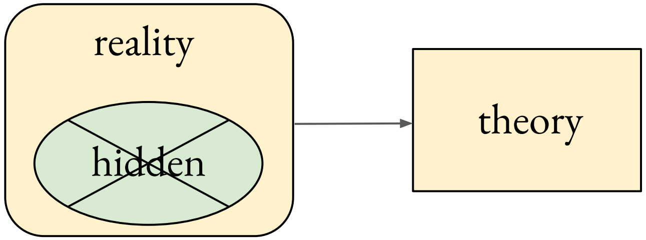

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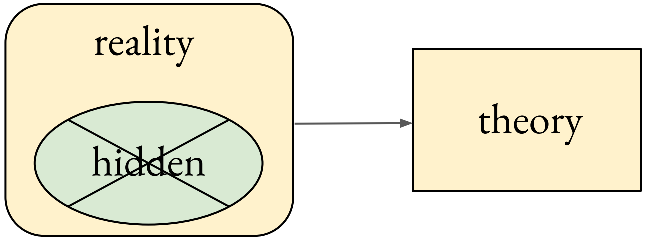

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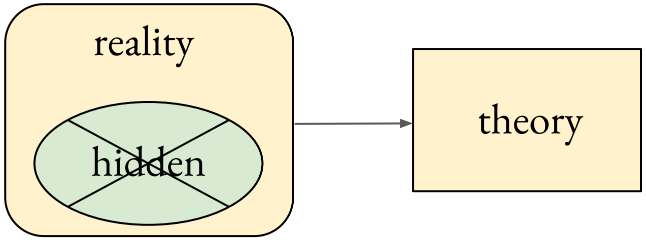

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Application: Discriminate what is intrinsically *quantum* from what would be reproducible by classical physics

How to prove completeness?

Suppose that hidden-variable theories exist and formulate inequalities for them

...Experiment: Bell inequalities are violated by quantum mechanics!

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Temporal Bell inequalities in non-relativistic many-body physics

Bell inequalities contain time only implicitly...

Can we explicitly include time in Bell inequalities,
and use them to probe the time evolution of a many-body system?

Contributions by:

Leggett-Garg (1985),
... Tononi-Lewenstein (today's talk)
[Tononi, Lewenstein, arXiv:2409.17290]

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  position: $\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \varphi_x(x_2) v_x(x_1) dx,$ or momentum: $\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \psi_p(x_2) u_p(x_1) dp,$

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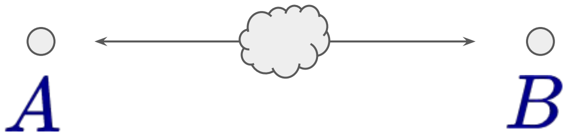
- But this is **impossible** because they are non-commuting operators! PARADOX!

Bohm-Aharonov (1957)

(as quoted by Bell)

Pair of particles moving in opposite directions:



Spin singlet state: $$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right)$$

A measures the spin in some direction and gets +1

$\Rightarrow$ the spin of B in this direction is certainly -1

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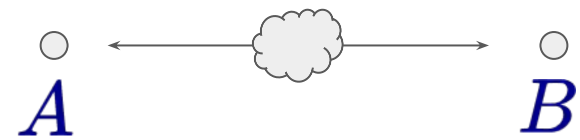

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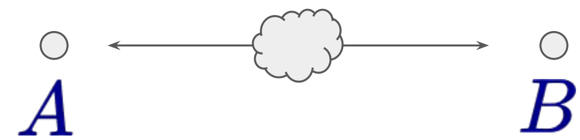

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Solution of the "paradox":

The quantum information of an entangled pair is not stored and retrievable locally. Measuring *one part* means measuring *the whole* system!

Bell (1964)

From *thought experiment*… to *experiment*

He encoded the elements of reality not captured by quantum mechanics into hidden variables λ

Assuming that the hidden-variables exist, the correlation function between two detectors' axes

$P(\vec{a},\vec{b}) = \int d\lambda\, \rho(\lambda) A(\vec{a},\lambda) B(\vec{b},\lambda)$ satisfies $|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 1 + P(\vec{b},\vec{c})$

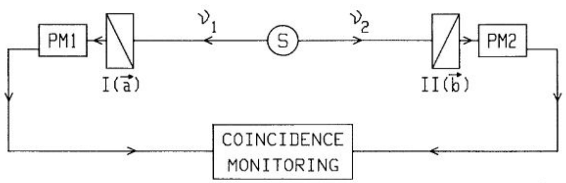

FIG. 1. Optical version of the Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*. The pair of photons ν_1 and ν_2 is analyzed by linear polarizers I and II (in orientations $\vec{a}$ and $\vec{b}$) and photomultipliers. The coincidence rate is monitored.

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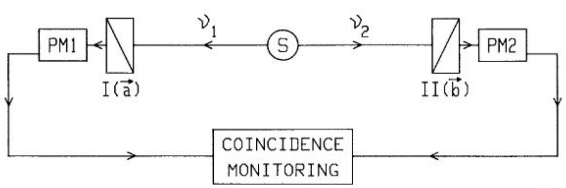
Aspect et al. (1982)



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Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

Alain Aspect, Jean Dalibard, and Gérard Roger

Phys. Rev. Lett. **49**, 1804 – Published 20 December 1982

Correlations of linear polarizations of pairs of photons have been measured with time-varying analyzers. The analyzer in each leg of the apparatus is an acousto-optical switch followed by two linear polarizers. The switches operate at incommensurate frequencies near 50 MHz. Each analyzer amounts to a polarizer which jumps between two orientations in a time short compared with the photon transit time. The results are in good agreement with quantum mechanical predictions but violate Bell's inequalities by 5 standard deviations.

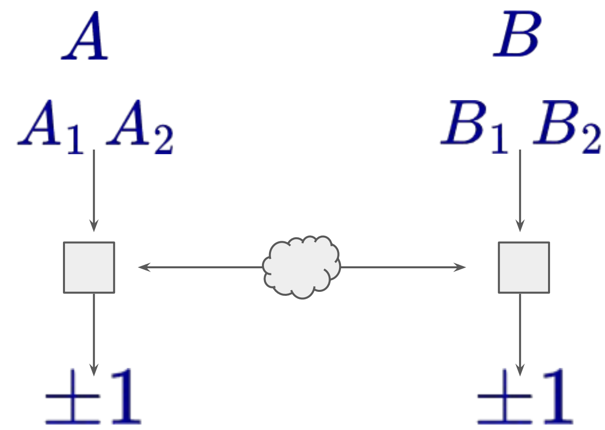
Other Bell-type inequalities were derived by Clauser-Horne-Shimony-Holt (CHSH, 1969), and Clauser-Horne (CH, 1974)

Clauser-Horne (1974)

Two observers named Alice and Bob

each one choosing to measure one of two possible observables:

each observable with possible outcomes:



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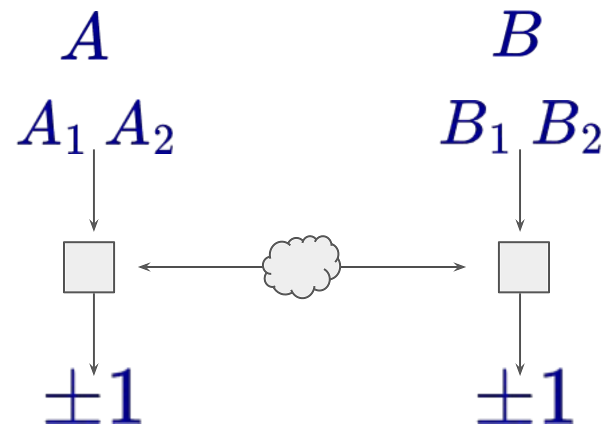

Clauser & Horne derived the quantity:

$$I'_{CH} = p_{11}(A_1, B_1) + p_{11}(A_1, B_2) + p_{11}(A_2, B_1) - p_{11}(A_2, B_2) - P_A(1|A_1) - P_B(1|B_1)$$

which can be quantified after many experimental repetitions.

For a hidden-variable theory: $-1 \leq I'_{CH} \leq 0$

But quantum mechanics violates it!

Bell-like inequalities explicitly featuring time…
Dynamics which can be described exclusively with quantum mechanics?

Leggett-Garg (1985)

Bell-like inequalities explicitly featuring time...
Dynamics which can be described exclusively with quantum mechanics?

Measure the observable O, with outcomes ± 1, at three time instances. The quantity:

$$K = \langle O(t_1)O(t_2)\rangle + \langle O(t_2)O(t_3)\rangle - \langle O(t_1)O(t_3)\rangle$$

satisfies (assuming macroscopic realism): $-3 \leq K \leq 1$

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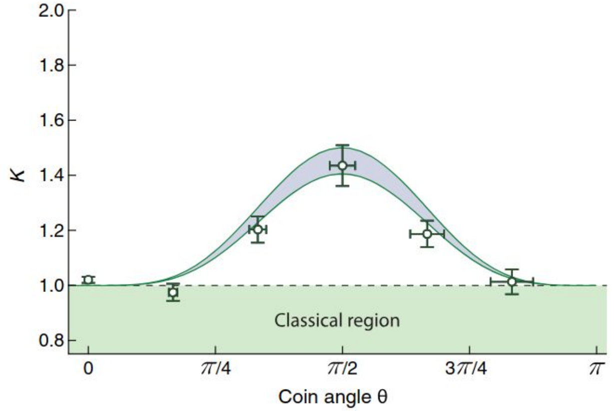
Also violated experimentally!

$\Rightarrow$ evidence against macroscopic realism

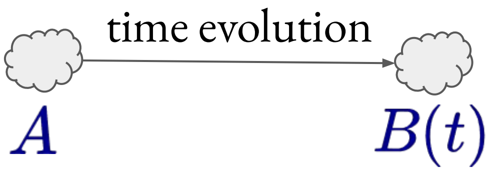


[Robens et al. PRX 5, 011003 (2015)]

Can we formulate temporal inequalities for non-relativistic many-body systems?

How to formulate temporal Bell inequalities?

Alice measures at 0, Bob at T=t:



...if the observers are causally-connected, the measurement probabilities are immediately describable in terms of hidden-variable theories

[Tononi, Lewenstein, arXiv:2409.17290]

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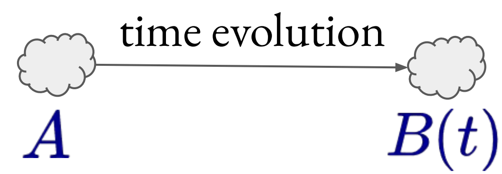


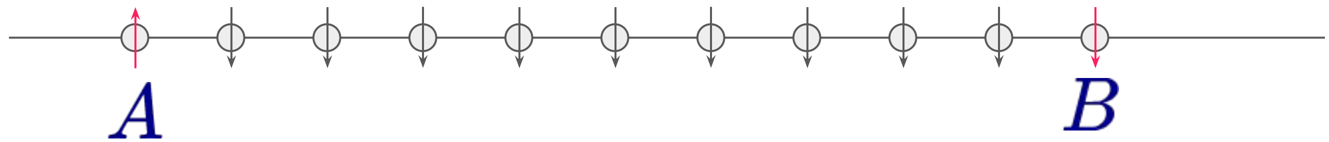
...if the observers are causally-connected, the measurement probabilities are immediately describable in terms of hidden-variable theories

To prevent A immediately "signaling" her measurement to B we need some kind of "medium":

- space-time separation (causality)
- <u>a medium with finite velocity of propagation of quantum information</u>

[Tononi, Lewenstein, arXiv:2409.17290]

Many body medium: a spin chain connecting Alice and Bob



The propagation of quantum information in a spin chain is limited by a (Lieb-Robinson) bound:

The Finite Group Velocity of Quantum Spin Systems

Elliott H. Lieb*

Dept. of Mathematics, Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

Derek W. Robinson**

Dept. of Physics, Univ. Aix-Marseille II, Marseille-Luminy, France

Received May 15, 1972

Abstract. It is shown that if Φ is a finite range interaction of a quantum spin system, τ_t^{Φ} the associated group of time translations, τ_x the group of space translations, and A, B local observables, then

$$\lim_{\substack{|t|\to\infty \\ |x|>v|t|}} \|[\tau_t^{\Phi}\tau_x(A), B]\|\, e^{\mu(v)t} = 0$$

whenever v is sufficiently large ($v > V_{\Phi}$) where $\mu(v) > 0$. The physical content of the statement is that information can propagate in the system only with a finite group velocity.

Measurement protocol

A spin chain connecting an entangled Bell pair $|\phi\rangle = (|\uparrow\rangle_1 \otimes |\uparrow\rangle_N + |\downarrow\rangle_1 \otimes |\downarrow\rangle_N)/\sqrt{2}$

$T = 0^-$

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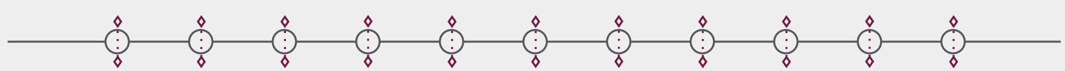
The system evolves in time, then Bob chooses between B_1 B_2

$T = t$



$B(t)$

Temporal CH inequality: $0 \leq I_{CH}(t) \leq 1$

obtained from the CH inequality by adding the time dependence to Bob's operators (Heisenberg picture):

$$I_{CH}(t) = p_{11}(A_1, B_2(t)) + p_{-1-1}(A_1, B_1(t)) + p_{11}(A_2, B_1(t)) - p_{11}(A_2, B_2(t))$$

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The probabilities of interest can be calculated analytically:

$$p_{a_i b_j}(A_i, B_j(t)) = \langle \psi(0^-) | \Pi_{a_i}^{A_i} \Pi_{b_j}^{B_j(t)} \Pi_{a_i}^{A_i} | \psi(0^-) \rangle$$

$T = 0^-$

$|\phi\rangle = (|\uparrow\rangle_1 \otimes |\uparrow\rangle_N + |\downarrow\rangle_1 \otimes |\downarrow\rangle_N)/\sqrt{2}$

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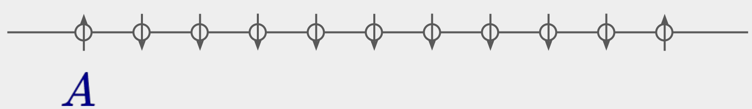
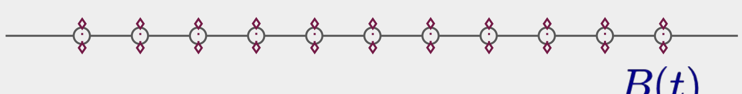


$$\Pi_{a_i}^{A_i} = (1 + a_i A_i)/2$$

$T = t$



$$\Pi_{b_j}^{B_j(t)} = (1 + b_j B_j(t))/2$$

We consider the following operators:

$$A_1 = \sigma^z \quad A_2 = \sigma^x \quad B_1 = (\sigma^z + \sigma^x)/\sqrt{2} \quad B_2 = (\sigma^z - \sigma^x)/\sqrt{2}$$

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$t = 0$, we know that $I_{CH}(0) = (1 + \sqrt{2})/2 \approx 1.207 > 1$

$t > 0$, we need to specify the system Hamiltonian

XX Hamiltonian in transverse field

$$H = -\frac{J}{2}\sum_{i=1}^{N-1}\left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y\right) - \frac{\mu}{2}\sum_{i=1}^{N}\left(\sigma_i^z + 1_i\right),$$

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admits an exact mapping to fermions

$$H = -J\sum_{i=1}^{N-1}\left(f_i^\dagger f_{i+1} + f_{i+1}^\dagger f_i\right) - \mu\sum_{i=1}^{N} f_i^\dagger f_i$$

$$= \sum_{\substack{m=1,\\ k=k_m}}^{N} \epsilon_k c_k^\dagger c_k, \qquad \epsilon_k = -2J\lambda_k - \mu \qquad \lambda_k = \cos k$$

$$k_m = \pi m/(N+1)$$

With this Hamiltonian, everything is analytical:

$$f_j^\dagger(t) = \sum_{i=1}^{N} G_{ij}(t) f_i^\dagger \qquad G_{ij}(t) = \sum_{\substack{m=1,\\ k=k_m}}^{N} u_{ik}u_{jk}e^{i\epsilon_k t}$$

$$u_{jk} = (-1)^{j-1}U_{j-1}(\lambda_k)/\left[\sum_{l=1}^{N} U_{l-1}^2(\lambda_k)\right]^{1/2} \qquad U_{j-1}(\lambda_k) = \sin(jk)/\sin(k)$$

KEEP CALM

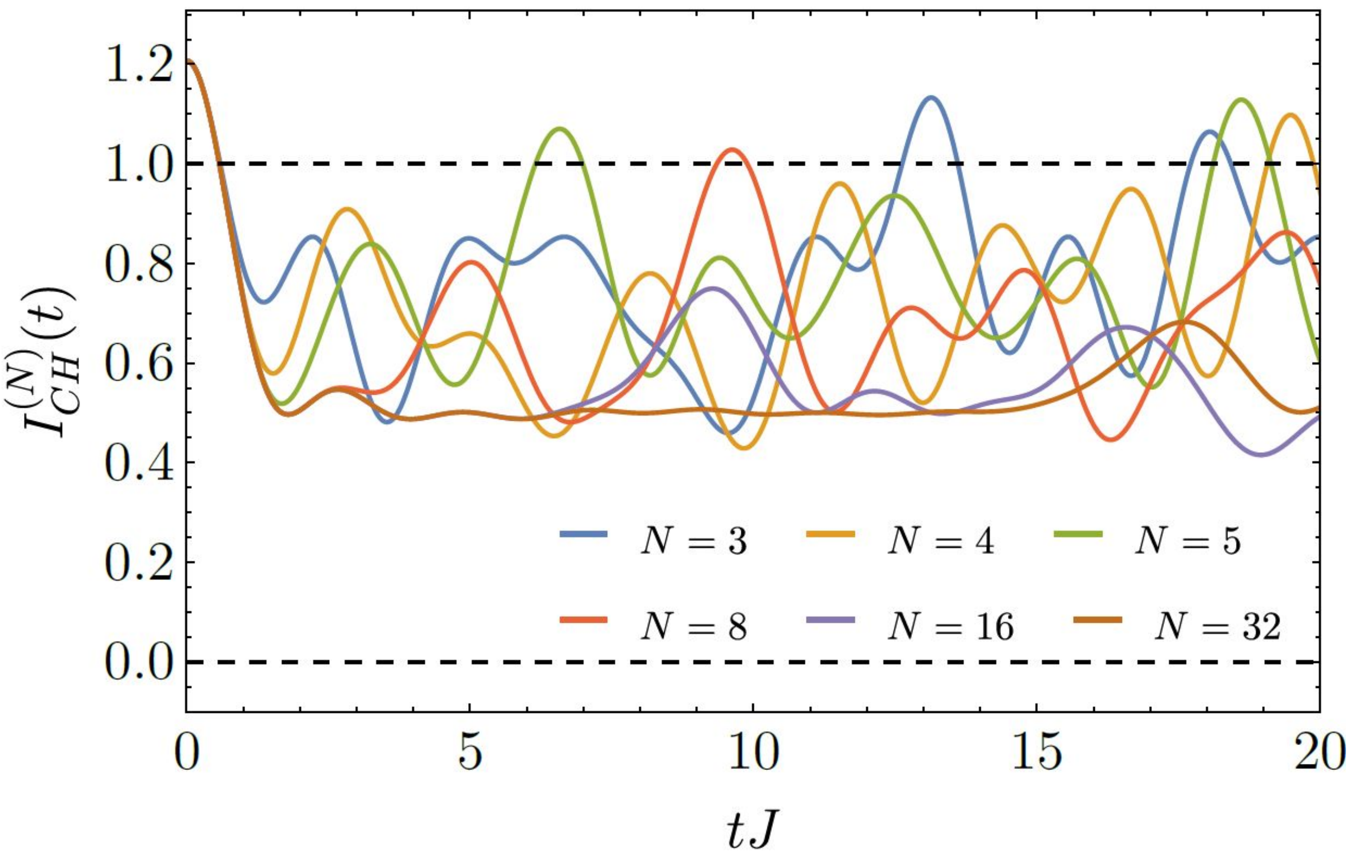
Analytical solution

Putting all together, after many calculations, we find:

$$I_{CH}^{(N)}(t) = \frac{1}{2} + \frac{\sqrt{2}}{4}\{|G_{NN}(t)|^2 + |G_{1N}(t)|^2 + \mathrm{Re}[G_{NN}(t)]\},$$

an analytical function of N tJ μ/J

Dynamics of the temporal CH inequality $0 \leq I_{CH}(t) \leq 1$



[Tononi, Lewenstein, arXiv:2409.17290]

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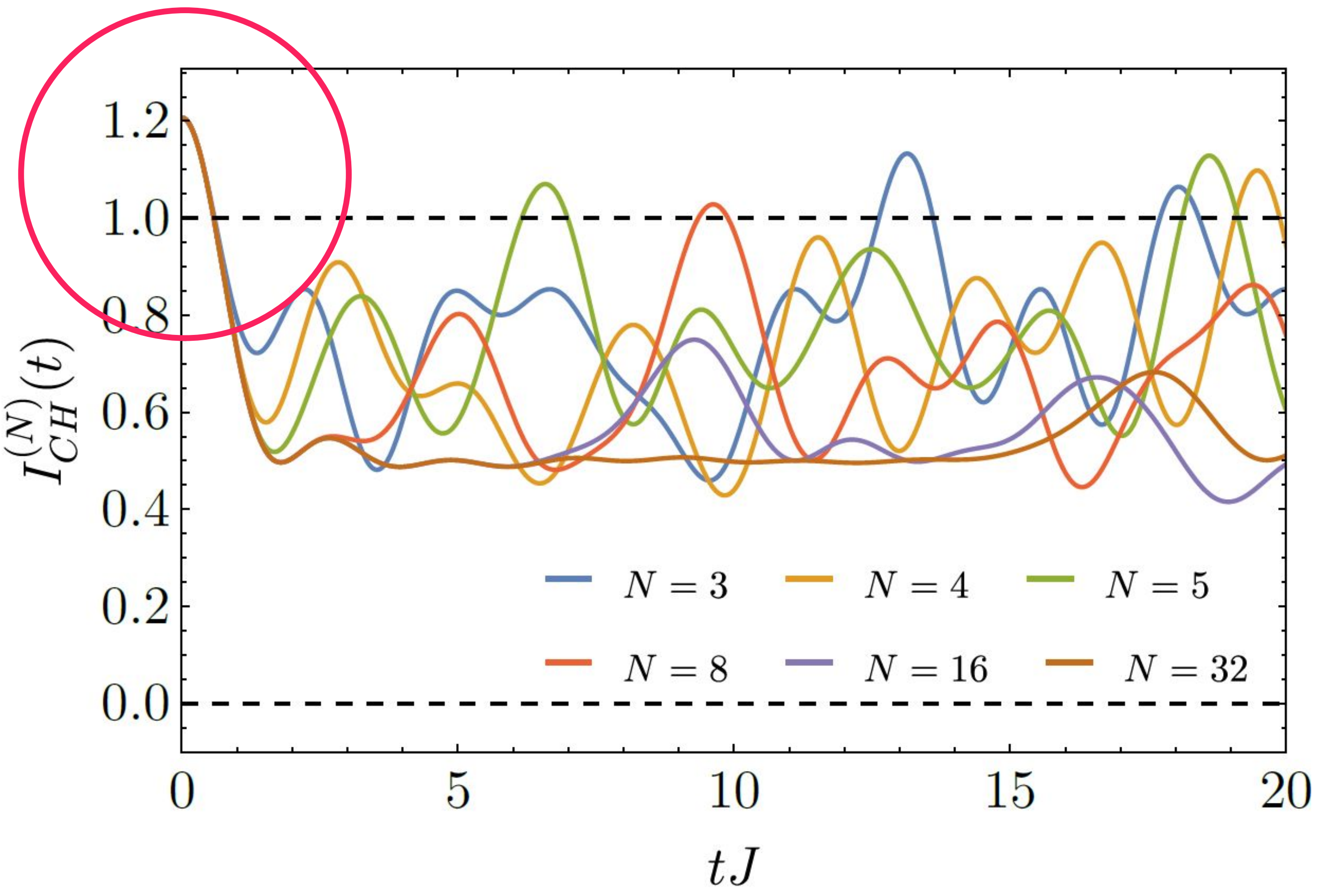
Violation of the temporal CH inequality at small time:



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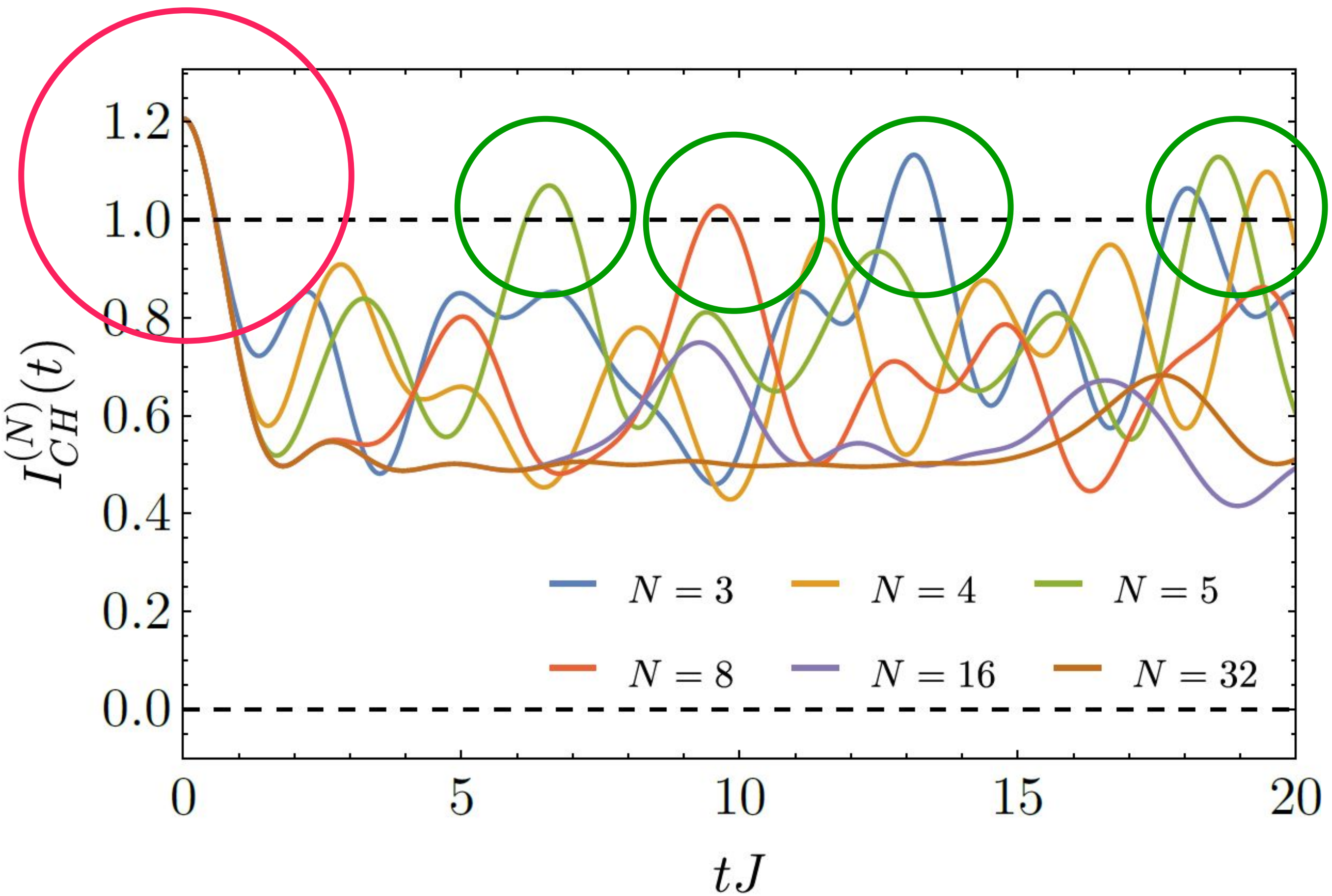

Breaking revivals at larger times

(they are less frequent as one approaches the thermodynamic limit of infinite chain)

[Tononi, Lewenstein, arXiv:2409.17290]

Conclusions

From proving the completeness of quantum mechanics to practical applications

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Problem: how to prevent the immediate spreading of quantum information?

Solution: connect Alice and Bob spins with a spin chain, where quantum information spreads with a finite velocity (Lieb-Robinson bound)

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Solution: connect Alice and Bob spins with a spin chain, where quantum information spreads with a finite velocity (Lieb-Robinson bound)

Results:

1) The dynamics of the Clauser-Horne temporal inequality is analytical

2) The quantum correlations survive for a finite time interval between Alice and Bob measurements!

3) Speed of light → model-dependent Lieb-Robinson bound, itself of physical interest

Conclusions

From proving the completeness of quantum mechanics to practical applications

Can we explicitly include time in Bell inequalities,
and use them to probe the time evolution of a many-body system?

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Thank you for your attention!