ABSTRACT

- MOTIVATION

  Experiments on **ultra-cold polarized fermions** where two-body scattering is **resonant** in the *p*-wave channel

- TOOL

  Generalization of the zero range approach:

  ☞ Energy independent **boundary condition** for the wave function

  ☞ **Family of pseudopotentials** generated by a free parameter: $\lambda \longrightarrow \lambda$-potential

  ☞ Introduction of a **regularized scalar product** to restore Hermiticity

- APPLICATION

  Modelization of the many-body system with an effective model:

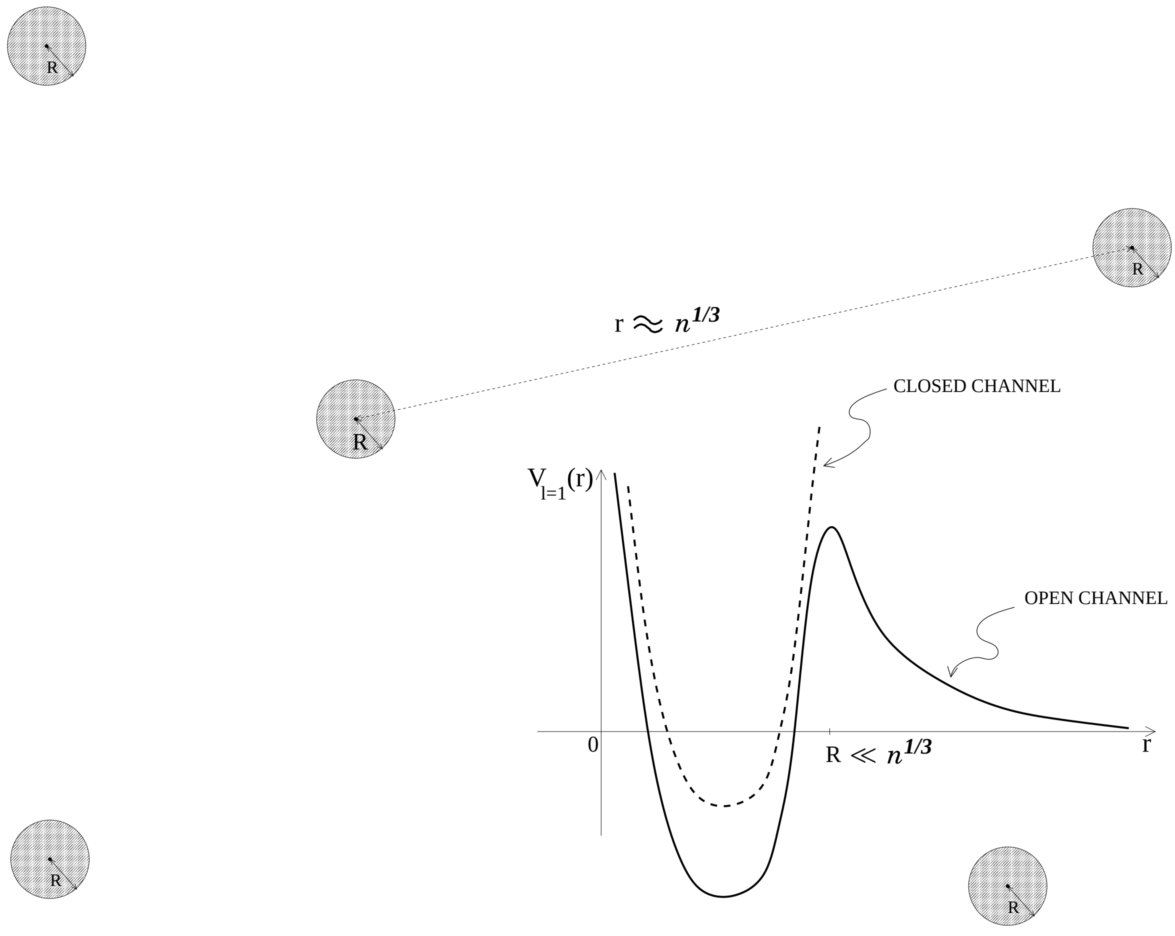
  ☞ Simple interpretation close to resonance of transfer between atomic and molecular states with a **"two-branches" picture**

  ☞ In the dilute regime at resonance: equation of state linear with density

CONTEXT

- **Dilute phase** of particles (density n)
- **Short range** two-body potential of characteristic radius R ($nR^3 \ll 1$)
- **Feshbach resonance** in the p-wave channel
  $\longrightarrow$ two-body scattering amplitude described using two parameters:

$$f(\vec{k},\vec{k}\,') = \frac{3\mathcal{V}_s\,\vec{k}.\vec{k}\,'}{1+\alpha\mathcal{V}_s k^2 + i\mathcal{V}_s k^3} \quad \begin{cases} \mathcal{V}_s & \text{Scattering volume} \\ \alpha & \text{“Effective range”} \end{cases} \tag{1}$$



☞ Resonance between the scattering state and a **molecular state** in the closed channel

☞ Molecular state tuned with a magnetic field $B \longrightarrow \mathcal{V}_S \propto \dfrac{1}{B-B_0}$

Zero energy resonance : $|\mathcal{V}_S| \rightarrow \infty$

ZERO RANGE APPROACH

- Limit $R \to 0$ while the low energy behavior is fixed to the real one
  Separates the low energy scale from the high energy scale $\dfrac{\hbar^2}{mR^2}$:
  1) Wave function solution of the **free Schrödinger equation** $(r \neq 0)$
  2) Interaction term replaced by a **boundary condition** as $r \to 0$. For p-wave interacting particles defined by the low energy scattering behavior in Eq.(1), the wave function Ψ satisfies:

$$\lim_{r\to 0}\left[\left(\mathcal{V}_s\partial_r^3 + 2\alpha\mathcal{V}_s\partial_r^2 + 2\right) r^2 \int_{S_r} d^2\Omega\ \vec{e}_r\Psi\right] = \vec{0} \qquad (\vec{e}_r = \vec{r}/r) \quad (2)$$

  ☞ Surface integration over the sphere $\mathcal{S}_r$ of radius r centered on the singularity at $r = 0 \longrightarrow$ **acts in the p-wave channel only**

  ☞ Analog to the **Bethe-Peierls** approach in s-wave channel

- PSEUDOPOTENTIAL : a way to implement the zero range scheme

☞ Cancels the “delta” term coming from the action of the Laplacian on the wave function in the Schrödinger equation: $\Delta\left(\dfrac{\vec{p}.\vec{r}}{r^3}\right) = 4\pi\vec{p}.(\vec{\nabla}\delta)(\vec{r})$

☞ Imposes the correct boundary condition Eq.(2) on the wave-function

☞ Can be used in a first order Born approximation

$$\langle\vec{r}\,|V_\lambda|\Psi\rangle = -g_\lambda(\vec{\nabla}\delta)(\vec{r}\,)\,.\,\vec{\mathcal{R}}_\lambda[\Psi\,] \qquad (3)$$

Coupling constant $\quad g_\lambda = \dfrac{12\pi\hbar^2\mathcal{V}_s}{m(1-\lambda\mathcal{V}_s)}$

Regularizing operator $\vec{\mathcal{R}}_\lambda[\Psi] = \lim_{r\to 0}\left[(\dfrac{\partial_r^3}{2} + \alpha\partial_r^2 + \lambda)r^2\displaystyle\int_{S_r}\frac{d^2\Omega}{4\pi}\vec{e}_r\Psi(\vec{r}\,)\right]$

λ: **free parameter** $\longrightarrow$ exact results don't depend on it

TWO-BODY EIGENSTATES IN FREE SPACE

- $E = \dfrac{\hbar^2 k^2}{m} > 0$: Scattering states

$$\Psi_{\vec{k}}(\vec{r}) = \exp(i\vec{k}.\vec{r}) + \frac{3i\mathcal{V}_s \vec{k}.\vec{e}_r}{1 + \alpha \mathcal{V}_s k^2 + i\mathcal{V}_s k^3}\, \partial_r \left(\frac{\exp(ikr)}{r} \right)$$

☞ Coincide with the real scattering states for $r > R$

☞ **Can** be obtained in the first order Born approximation with the choice $\lambda = -k^2(\alpha + ik)$.

- $E = \epsilon_B < 0$: **Shallow state** in the **resonant regime** $\mathcal{V}_s \alpha^3 \gg 1$

☞ Energy $\epsilon_B = -\dfrac{\hbar^2 \kappa_B^2}{m}$ with $\kappa_B^{-2} \simeq \alpha \mathcal{V}_s$

☞ Radial wave function $\mathcal{R}_B(r) = \mathcal{N}_B\, \partial_r \left(\dfrac{\exp(-\kappa_B r)}{r} \right)$

☞ **Outer part of the molecular state**: emerges from the coupling between the closed and open channels.

☞ Populated by pairs of particles in the **BEC region** of the BCS-BEC crossover.

REGULARIZED SCALAR PRODUCT

- NON HERMITIAN APPROACH ?

☞ $\vec{k} \neq \vec{k}\,' \longrightarrow |\langle \Psi_{\vec{k}\,'} | \Psi_{\vec{k}} \rangle| = \infty$

☞ **Laplacian not Hermitian** for wave functions satisfying Eq.(2).

- A NEW METRICS

$\longrightarrow$ Modify the usual scalar product to restore Hermiticity

$$(\Psi|\Phi)_0 = \lim_{r_0 \to 0} \left\{ \int_{r>r_0} d^3\vec{r}\,\Psi^*(\vec{r})\Phi(\vec{r}) + (\alpha r_0^4 - r_0^3) \int_{r=r_0} d^2\Omega\,\Psi^*(\vec{r})\Phi(\vec{r}) \right\}$$

$\equiv$ weighted scalar product with $g(r) = 1 + \delta(r)\left[(\alpha r^2 - r)\,.\right]$

- NORMALIZATION

For $R \neq 0$: alternative expression of the standard scalar product

$$\langle \Psi|\Psi\rangle = \int_{r>R} d^3\vec{r}\,|\Psi|^2 - \frac{\hbar^2 R^2}{m} \int_{r=R} d^2\Omega \left(\Psi^* \partial_r \partial_E \Psi - \partial_r \Psi^* \partial_E \Psi\right) \qquad (4)$$

KEY RESULT: $R \to 0 \quad (\Psi|\Psi)_0 \equiv$ r.h.s (Eq. 4)

☞ Regularized scalar product $\equiv$ method based on the analyticy of $f(\vec{k}, \vec{k}\,')$

☞ Renormalization of the scalar product in the **configuration space**

☞ Normalization of the shallow state: $\mathcal{N}_B^{-2} = \alpha - 3\kappa_B/2$

Probability that the state is in the open channel $< 1 \Longrightarrow \alpha \gtrsim \dfrac{1}{R}$

EFFECTIVE MODEL FOR THE MANY-BODY SYSTEM

- Homogeneous system of N spin-polarized identical fermions

  ☞ **Exact formulation** for the low energy behavior:
  interaction between particles modeled by the potential Eq.(3)

  ☞ **A first step**: effective model to extract the physics involved at the neighborhood of the resonant regime

- MODEL (mean-field approach **independent on** λ)

  ☞ **Fictitious particle** of mass equal to the reduced mass $\frac{m}{2}$ interacting with a fixed scatterer at the center of a box of radius L

  ☞ Wave function of the fictitious particle $\neq 0$ in the p-wave channel only
  $\longrightarrow$ represents the **pair function** of two fermions:
  1) Eigenstate of the pseudopotential Eq.(3)
  2) Vanishes on the surface of the box
  $\longrightarrow$ mimics the effects of correlations between pairs

  ☞ Non interacting case: link between the radius of the box and the density

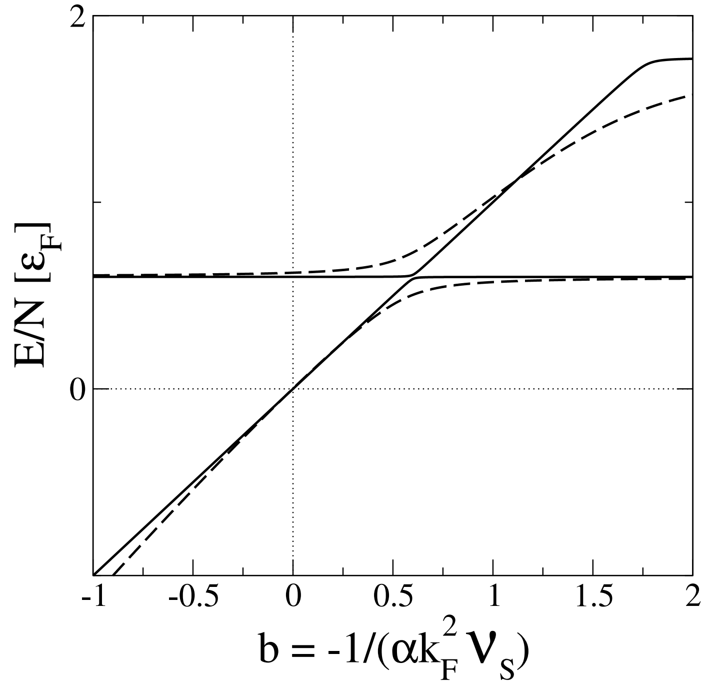
  Total energy : $E = \frac{3}{5} N \epsilon_F = \frac{1}{2} N \epsilon$
  - $k_F^3 = 6\pi^2 n$
  - $\epsilon_F = \dfrac{\hbar^2 k_F^2}{2m}$ Fermi energy
  - $\epsilon =$ energy of the fictitious particle

  Ground state in the box $\Longrightarrow k_F L \simeq 5.8 \Longrightarrow L \propto n^{-1/3}$

- EQUATION OF STATE

$$\frac{E}{N} = \frac{\epsilon}{2} = \frac{\hbar^2 k^2}{2m} \text{ , } k \text{ solution of } \quad \frac{kL\cos(kL) - \sin(kL)}{kL\sin(kL) + \cos(kL)} = -\frac{\mathcal{V}_s k^3}{1 + \alpha \mathcal{V}_s k^2}$$

- Two dimensionless parameters : $\frac{\alpha}{k_F} (\gg 1)$ small variation across resonance
  $b = -(\alpha k_F^2 \mathcal{V}_s)^{-1} \propto B - B_0$



Two branches picture for $\frac{\alpha}{k_F} = 10$ *(dashed) and* 10^3 *(continuous)*

☞ Left part/upper branch
$\longrightarrow$ metastable **weakly repulsive atomic phase**.

☞ Right part/ground branch $\longrightarrow$ weakly attractive atomic phase $\longrightarrow$ **BCS phase** at low temperature.

☞ Left part/ground branch
$\longrightarrow$ **molecular phase** composed of dimers of energy $\epsilon/\epsilon_F \simeq 2b$.

☞ **At resonance** $|\mathcal{V}_S| = \infty \longrightarrow E/N \propto \hbar^2 n/m\alpha \ll \epsilon_F$
$\longrightarrow$ result $\neq$ unitary regime in s-wave scattering ($n^{2/3}$ law)

☞ **Small level crossing** between the two branches $\leftrightarrow$ small heating in non adiabatic transfers from atomic branch to the ground branch.

CONCLUSIONS

- Box model supports a stable phase at resonance
- Life time of a dimer close to resonance ?
  $\longrightarrow$ solve a three-body problem in p-wave scattering
- Regularized scalar product simple to apply in inhomogeneous situations
- Formulation of the resonant regime with energy independent boundary conditions $\longrightarrow$ The formalism can be applied directly when the two-body scattering energy vary:
  - ☞ Inhomogeneous situations (trap)
  - ☞ Few-body problem
  - ☞ Time-dependent problems
- Generalization to arbitrary resonant partial wave channel using the same ideas: $\longrightarrow$ L.P. cond-mat/0508120