ONE PARTICLE IN A BOX:

THE SIMPLEST MODEL FOR A FERMI GAS IN THE UNITARY LIMIT

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ABSTRACT

We consider a single quantum particle in a spherical box interacting with a fixed scatterer at the center, to construct a model of a degenerate atomic Fermi gas close to a Feshbach resonance. One of the key predictions of the model is the existence of two branches for the macroscopic state of the gas, as a function of the magnetic field controlling the value of the scattering length. This model is able to draw a qualitative picture of all the different features recently observed in a degenerate atomic Fermi gas close to the resonance, even in the unitary limit.

CONTEXT

Two spin-component degenerate Fermi gases in the unitary limit produced using a Feshbach resonance:
scattering length a of two atoms with different spin components $\gg$ mean interparticle separation

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SCOPE OF THE WORK

To give the simplest possible physical picture of a two spin-component Fermi gas for arbitrary values of the scattering length a at temperature $\simeq 0$.

$\longrightarrow$ Spatially homogeneous gas: $\dfrac{N}{2}$ spins $+\dfrac{1}{2}$ and $\dfrac{N}{2}$ spins $\dfrac{-1}{2}$

$\Longrightarrow$ qualitative understanding of experimental observations.

$\Longrightarrow$ useful guidelines for experiments to come.

MODEL

A given spin $+\frac{1}{2}$ particles interacts with the $\frac{N}{2}$ spin $\frac{-1}{2}$ particles:

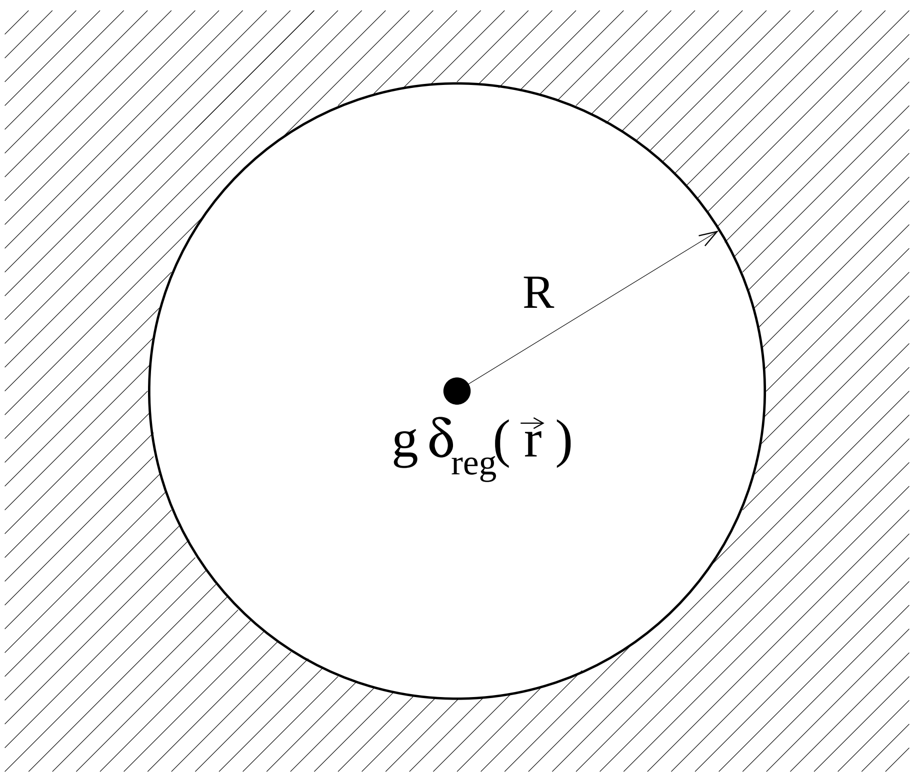
(i) Interaction of a fictitious particle of mass equal to the reduced mass $\frac{m}{2}$, with a fixed scatterer at the center of a spherical box of radius R:

$$V(r) = g\delta_{reg}(\vec{r}) = g\delta(\vec{r})\partial_r(r.) \quad \text{with} \quad g = \frac{4\pi\hbar^2 a}{m}$$

$\Longleftrightarrow$ represents the interaction of two opposite spin particles in the center of mass frame and in the singlet spin state.

(ii) Boundary conditions: the wavefunction $\phi(\vec{r})$ of the fictitious particle vanishes on the surface of the box.

$\Longleftrightarrow$ mimics the interaction effect of the $\frac{N}{2} - 1$ other spin $-\frac{1}{2}$ particles and the Fermi statistical effect of the remaining $\frac{N}{2} - 1$ spin $+\frac{1}{2}$ particles.



Relating the radius of the box with the mean density : $g = 0$

Total energy : $E = \frac{3}{5} N \epsilon_F = \frac{1}{2} N \epsilon$,

- $\epsilon_F = \dfrac{\hbar^2 k_F^2}{2m}$ the Fermi energy
- $\epsilon =$ energy of the fictitious particle

$$g = 0 \Longrightarrow \epsilon = \frac{\hbar^2}{m} \left(\frac{\pi}{R}\right)^2 \Longrightarrow \boxed{k_F R = \left(\frac{5}{3}\right)^{1/2} \pi}$$

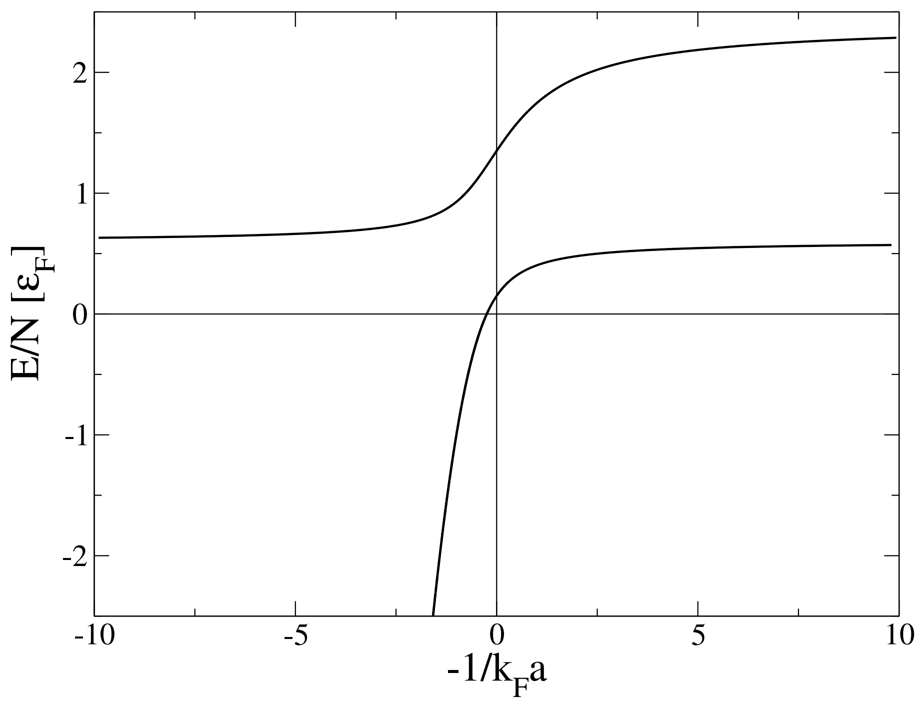
with $k_F = \left(3\pi^2 \rho\right)^{1/3}$

Solving a one body problem:

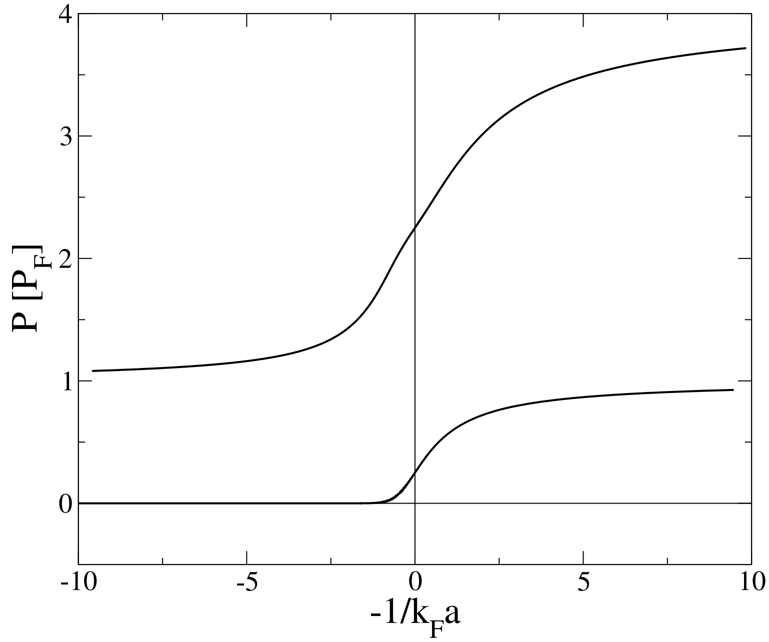
$$-\frac{\hbar^2}{m} \Delta\phi = \epsilon\phi \quad ; \quad \lim_{r \to 0} \frac{\partial_r (r\phi)}{r\phi} = -\frac{1}{a}$$

($r =$ distance to the origin)

- **positive energies** $\epsilon = \dfrac{\hbar^2 k^2}{m}$: $\phi(r) \propto \dfrac{\sin[k(r-R)]}{r}$; $\tan kR = ka$
- **negative energies** $\epsilon = \dfrac{-\hbar^2 \kappa^2}{m}$: $\phi(r) \propto \dfrac{\sinh[k(r-R)]}{r}$; $\tanh \kappa R = \kappa a$



Total energy of the gas in units of the Fermi energy, as function of $\dfrac{-1}{k_F a}$. Only the first two branches of the model are represented. The right part corresponds to magnetic fields above the Feshbach resonance, and the left part to magnetic fields below the resonance.



Pressure P of the gas in units of the Fermi pressure $P_F = \frac{\hbar^2 k_F^5}{15\pi^2 m}$. Only the first two branches are represented. The pressure on the ground branch is lower than the ideal Fermi gas pressure. The decrease of the pressure across the Feshbach resonance from the $a < 0$ part to the $a > 0$ part is observable in a trap through a shrinking of the size of the cloud.

GROUND BRANCHE : 2 regimes

i) $k_F a \to 0^-$: **weakly attractive Fermi gas**
$\longrightarrow$ BCS phase in a more complete treatment.

ii) $k_F a \to 0^+$: **dilute gas of dimers**. 1 Dimer = bound state of the two-body problem in free space
$\longrightarrow$ Bose-Einstein condensate of dimers in a more complete treatment.

UPPER BRANCH $k_F a \to 0^+$: **weakly repulsive Fermi gas**

UNITARY LIMIT $k_F |a| \to +\infty$

- Mean energy per particle $\propto$ Fermi energy $<$ ideal gas value.
- Effective attraction due to atomic interactions.
- $E > 0$, $P > 0$, $\chi > 0 \Longrightarrow$ no collapse.

UPPER BRANCH $\longrightarrow$ GROUND BRANCH : 2 mechanisms

i) **Three body collisions** between atoms: 3 atoms $\longrightarrow$ 1 dimer + 1 atom
$\longrightarrow$ **molecules with a high** center of mass kinetic **energy** $\simeq \dfrac{\hbar^2}{ma^2}$

ii) **Follow adiabatically the ground branch**: slowly crosses the resonance from right to left:
$\longrightarrow$ formation of **Bose-Einstein condensates of molecules**

CONCLUSION

Essential features of a two spin-component Fermi gas for s-wave interactions with an arbitrary scattering length:

- Existence of a **metastable atomic phase** (upper branch)
- Existence of a **molecular phase** (lower branch) for $a \to 0^+$
- **Continous connection** between the molecular regime and the weakly attractive regime when a is varied from 0^+ to 0^- across the Feshbach resonance.