RESONANT SCATTERING IN LOW DIMENSIONS

Ludovic Pricoupenko, Phys. Rev. Lett. 100, 170404

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ABSTRACT

- MOTIVATION

  Reduced geometries and l-wave **resonant** 2-body scattering:

  ☞ Fermions in quasi-1D trap: Fermi-Tonks Gas ($l = 1$)

  M. Girardeau and E. M. Wright, Phys. Rev. Lett. **95**, 010406 (2005)

  $\exists$ mapping: strongly interacting fermionic gas $\equiv$ non interacting bosons

  ☞ Exotic superfluidity in 2D ($l > 0$)

  V. Gurarie, L. Radzihovsky, and A. V. Andreev, Phys. Rev. Lett. **94**, 230403 (2005).

- TOOL

  Short range potential $\Rightarrow$ use the zero range approach.

- RESULTS

  ☞ For 1D and 2D atomic wave guides, determination of the scattering amplitude in high partial waves as a function of the 3D scattering parameters and of the atomic wave-guide frequency.

  ☞ Characterization of the Confinement Induced Resonances in high partial waves.

- CONSEQUENCES

  ☞ The finite width of the resonances is an essential parameter for describing the BEC-BCS transition in 1D and 2D fermionic gases.

  ☞ For spin polarized fermions in quasi-1D: strong constraints for achieving a Fermi-Tonks gas.

CONTEXT

- **Dilute phase** of particles (density n).

- Traps with **high aspect ratio**.

- **Short range** 2-body potential of radius b ($nb^3 \ll 1$):

$$b \equiv \left(\frac{\mu C_6}{\hbar^2}\right)^{1/4} O(1) \qquad : \text{ of the order of the van der Waals range.}$$

- **Feshbach resonance** in the l-wave channel

☞ Resonance between the scattering state and a l-wave **molecular state** in the closed channel.

☞ 2-body scattering amplitude f_l described in 3D with 2 parameters:

$$\begin{cases} w_l & \text{:generalized scattering length.} \\ \alpha_l & \text{:generalized effective range} \rightarrow \text{linked to the width of the resonance.} \end{cases}$$

$$f_l = -\frac{1}{g(k)k^{-2l} + ik} \quad \text{and} \quad g(k) = \frac{1}{w_l} + \alpha_l k^2 \tag{1}$$

☞ Molecular state tuned with a magnetic field $B \longrightarrow w_l \propto \dfrac{1}{B - B_0}$

PARAMETERS AT RESONANCE IN 3D

- $\boxed{l = 0}$

☞ w_0: s-wave scattering length , $\alpha_0 = -r_e/2$, r_e: effective range.
No specific constraint on r_e in general.

☞ $w_0 \gg b$: existence of a shallow bound state of energy $-\frac{\hbar^2}{2\mu w_0^2}$.

☞ **Unitary limit** ($f_0 = \frac{1}{ik} \quad \forall kb \ll 1$) corresponds to the regime:

$$|w_0| \to \infty \quad \text{and} \quad \alpha_0 \equiv O(b)$$

- $\boxed{l > 0}$

☞ Zero energy resonance for $|w_l| \gg b^{2l+1}$:
existence of a shallow (quasi-)bound state of energy $-\frac{\hbar^2}{2\mu w_l \alpha_l}$.

☞ Interaction on a compact support of radius b:

$$\alpha_l > \frac{(2l-3)!!(2l-1)!!}{b^{2l-1}} \tag{2}$$

L.P., PRA **73**, 012701 (2006)

$\Longrightarrow$ **No possibility of unitary limit** ($f_l = \frac{1}{ik} \quad \forall kb \ll 1$).

$\Longrightarrow$ Question:

"Consequences of Eq.(2) on the low-D scattering properties ?"

PRINCIPLE OF THE ZERO RANGE APPROACH

1) Wave function solution of the **non-interacting Schrödinger equation** for non vanishing interparticle distances ($r \neq 0$).

2) Interaction replaced by a **contact condition as r** $\rightarrow 0$:
the wave function coincide with the real one outside the potential range.

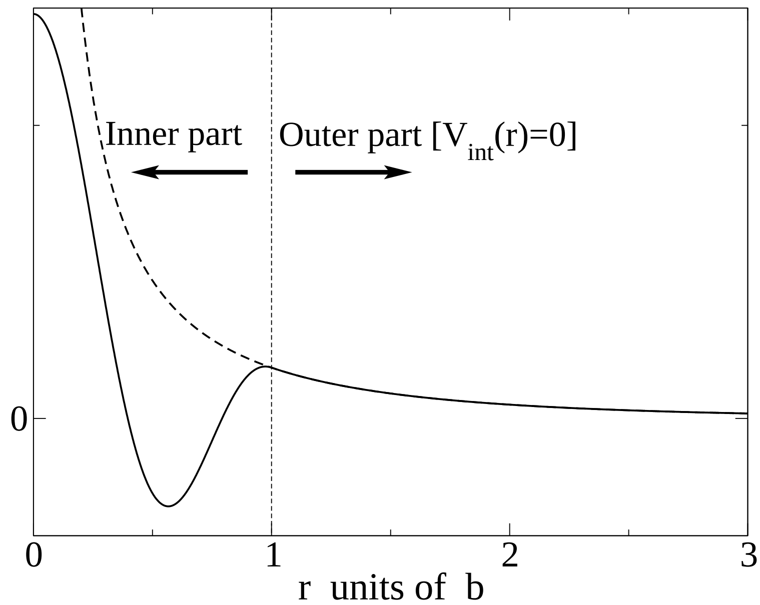


Figure 1: Dotted line: radial wave function of a s-wave bound state in a square well of radius b. Solid line: radial wave function obtained in a zero-range approach. By construction, the outer parts ($r > b$) of the two wave functions coincide.

3) Contact condition constructed using Eq.(1) only:
asymptotic 3D scattering states are eigenstates of the model.

ZERO RANGE APPROACH INCLUDING $l \geq 0$ CHANNELS

- Formulation in the $\mathbf{k}$-representation $\Longrightarrow$ simplification of the calculations.

- 2 particles of reduced mass μ, energy $E = \frac{\hbar^2 q^2}{\mu}$, wave function $|\Psi\rangle$:

$$|\Psi\rangle = |\Psi_0\rangle + \frac{2\pi\hbar^2}{\mu} \sum_{l\geq 0} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{k^l \langle \mathbf{k}|\delta_\epsilon\rangle \, (\mathcal{R}_l \cdot \mathcal{S}_{l,\mathbf{k}})}{\mathcal{H}_0 - E - i0^+} |\mathbf{k}\rangle, \qquad (3)$$

☞ $\mathcal{H}_0$: free Hamiltonian which includes the external potential.

☞ $|\Psi_0\rangle$ belongs to the kernel of $\mathcal{H}_0 - E$: regular solution.

☞ $\lim_{\epsilon\to 0} \langle \mathbf{r}|\delta_\epsilon\rangle = \delta(\mathbf{r})$. We choose here: $\langle \mathbf{k}|\delta_\epsilon\rangle = \exp(-k^2\epsilon^2/4)$.

☞ $(\mathcal{R}_l \cdot \mathcal{S}_{l,\mathbf{k}})$: contraction of two Symmetric Trace Free tensors of rank l.

☞ $\mathcal{S}_{l,\mathbf{k}}$: tensors appearing in the standard multipolar expansion

$$\mathcal{S}_{l,\mathbf{k}}^{[\alpha\beta\ldots]} = \frac{(-1)^l}{(2l-1)!!} k^{l+1} \left(\partial_{k_\alpha} \partial_{k_\beta} \ldots \right) k^{-1}$$

$$e.g.: \ \mathcal{S}_2^{\alpha,\beta} = \frac{k^\alpha k^\beta}{k^2} - \frac{1}{3}\delta^{\alpha,\beta}$$

☞ $\mathcal{R}_l$ fixes the balance between the regular and irregular solutions of (3).

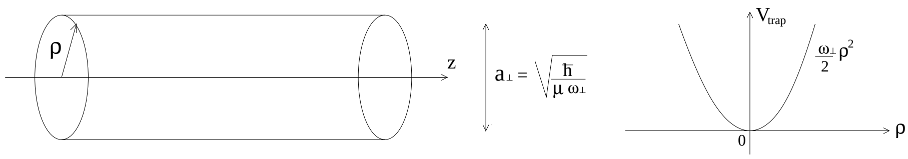
- Contact condition:

$$\operatorname*{Reg}_{\epsilon\to 0} \int \frac{d^3\mathbf{k}}{(2\pi)^3} k^l \langle \mathbf{k}|\Psi\rangle \mathcal{S}_{l,\mathbf{k}} = -\frac{l!\, g_l(q)\, \mathcal{R}_l}{(2l+1)!\,!}$$

☞ $\operatorname*{Reg}_{\epsilon\to 0}$: means the regular part of the integral obtained as $\epsilon \to 0$.

SPIN POLARIZED FERMIONS IN 1D

$$\mathcal{H}_0 = -\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}} + \frac{1}{2}\mu\omega_\perp^2\rho^2 - \hbar\omega_\perp, \tag{4}$$



- Interaction in the p-wave channel (Pauli)
- monomode regime $E < 2\hbar\omega_\perp$:

$$\langle \mathbf{r}|\Psi\rangle \underset{|z|\gg a_\perp}{=} \exp\left(-\rho^2/2a_\perp^2\right) \times \left[\exp(iqz) + \mathrm{sign}(z) f^{\mathrm{odd}} \exp(iq|z|)\right]. \tag{5}$$

$$f_p^{\mathrm{odd}} \underset{q\to 0}{=} \frac{-iq}{\frac{1}{l_p} + iq + q^2\xi_p}, \tag{6}$$

$l_p = 6a_\perp \left[\frac{a_\perp^3}{w_1} - 12\,\zeta(-\frac{1}{2})\right]^{-1}$: odd-wave scattering length

$\xi_p = \frac{\alpha_1 a_\perp^2}{6}$: 1D p-wave effective range

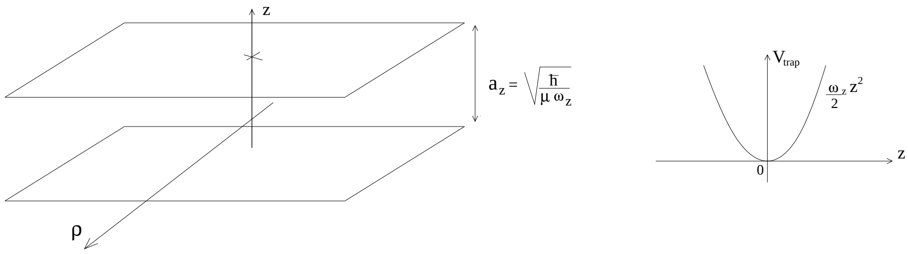
☞ Gas of atomic density n, conditions for the Fermi-Tonks regime:

$$|l_p| = \infty \qquad \text{and} \qquad n\xi_p \ll 1$$

Not satisfied in general because Eq.(2) gives $\alpha_1 > \frac{1}{b} \Longrightarrow \xi_p > \frac{a_\perp^2}{6b}$
e.g. ^{40}K, $\omega_\perp = 2\pi \times 70$ kHz and $\omega_z = 2\pi \times 10$ Hz at $T = 0$ K
$\Longrightarrow$ FTG for $N \ll 14$ atoms: few-body systems.

SCATTERING IN QUASI-2D

$$\mathcal{H}_0 = -\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}} + \frac{1}{2}\mu\omega_z^2 z^2 - \frac{\hbar\omega_z}{2}. \tag{7}$$



$$\Psi(\boldsymbol{\rho}) \underset{\rho \gg a_z}{=} \exp(-z^2/2a_z^2) \times \left[e^{i\mathbf{q}\cdot\boldsymbol{\rho}} - \frac{i}{4} \sum_{m=-\infty}^{m=\infty} i^m f^{[m]} H_m^{(1)}(q\rho) e^{im\angle(\boldsymbol{\rho},\mathbf{q})} \right]$$

$$f^{[m]} \underset{q\to 0}{\propto} \frac{q^{2m}}{(w_m^{-1} - w_m^{\star -1})} \quad w_1^\star \simeq 5.39 \times a_z^3 \quad \text{and} \quad w_2^\star \simeq 1.3 \times a_z^5$$

- resonance shift $\nearrow$ as $a_z \searrow$
- (quasi-) bound state energy: $E_b \sim -\hbar^2(w_m^{-1} - w_m^{\star -1})/(2\mu\alpha_m)$
- Scattering cross section $\sigma = |f_m|^2/4q$ resonant for $E \sim E_b$ $\Longrightarrow$ only in presence of a quasi-bound state.
- resonance width of the m-wave resonance:

$$\Delta E/E_b \propto (E_b/\hbar\omega_z)^{m-1}/(\alpha_m a_z^{2m-1}) \ll 1$$

- Exact expressions for the low energy behavior of $f^{[m]}$ induced by a 3D resonant m-wave interaction

  e.g., for $m = 1$: $f^{[1]} = \dfrac{6\sqrt{\pi}q^2}{a_z}\left[g_1(q) + \dfrac{6}{a_z^3\sqrt{\pi}} J_1\left(\dfrac{E}{2\hbar\omega_z} + i0^+\right)\right]^{-1}$

  for $m = 2$: $f^{[2]} = \dfrac{15\sqrt{\pi}q^4}{2a_z}\left[g_2(q) + \dfrac{60}{a_z^5\sqrt{\pi}} J_2\left(\dfrac{E}{2\hbar\omega_z} + i0^+\right)\right]^{-1}$

J_m defined by $J_m(\tau) = \text{P.f.}\int_0^\infty du\, u^{-(m+1)} \exp(\tau u)[1 - \exp(-u)]^{-1/2}$.

PERSPECTIVES

- BEC-BCS crossover in the resonant quasi-1D spin-polarized fermionic gas:
  $\Longrightarrow$ many-body problem using the Λ-potential which reproduces Eq.(6):

  $$\langle z|V_\Lambda|\phi\rangle = -\frac{\hbar^2 l_p}{2\mu(1-\Lambda l_p)}\delta'(z)\lim_{\epsilon\to 0^+}(\Lambda+\partial_\epsilon-\xi_p\partial_\epsilon^2)\left[\psi_{1D}(\epsilon)-\psi_{1D}(-\epsilon)\right]$$

  $\forall\Lambda\in\mathbb{R}$: Λ is a free parameter.

  $\Longrightarrow$ Equation of State and collective modes at $l_p=\infty$?

- BEC-BCS crossover in quasi-2D fermionic gases by varying $(w_m^{-1}-w_m^{\star -1})$ from positive to negative values