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Chapter 4
Time Variable and Time Scales in Natural Systems and Their Modeling

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Abstract In this chapter, I discuss the status of time when modeling natural systems. In such an operational perspective, time is defined and measured by a clock, i.e. the comparison with a given physical phenomenon. A central notion is that of time scales. I argue that the representation of time (e.g. discrete vs. continuous) is essentially a modeling choice, depending on the time scale of the observation or description compared to the characteristic times of the system. In integrated approaches developed for investigating systems presenting several levels of organization, it may be fruitful to consider several independent time variables describing evolution at different nested time scales ("multiple scale method"). This would solve some puzzling but recurrent statements, such as an evolution composed of a succession of equilibrium states, and leads to a practical understanding of diverse systems, from combustion and kinetic theory of gases in physics to enzymatic catalysis in biochemistry to adaptive dynamics in ecology. At a more fundamental level, the confrontation of descriptions performed at different scales and within different frameworks, from molecular dynamics to statistical mechanics to thermodynamics, is essential to delineate the limits of each of them, and overall reach a more informative understanding of reality. In particular, such a multi-scale confrontation offers a simple answer to the long-lasting opposition between microscopic reversibility and macroscopic irreversibility of isolated systems (the celebrated Second Principle of the thermodynamics). In biology, time scales are also essential in discussing the robustness of living systems and their evolutionary strategies.

Foreword

As a classical physicist (i.e. not involved in quantum mechanics, cosmology, and general relativity) I do not feel in a position to address the question of the nature of time. I will rather focus on the representation of time when modeling natural

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systems and the notion of time scales. In this perspective, time is a relative and flexible notion. I will challenge philosophical discussions with a pragmatic view, as a physicist dealing with *phenomena*, meaning the observable and measurable appearance and not the *nature* of objects and processes.

4.1 Clocks and Time Measurement

The statement attributed to Poincaré (Poincaré 1898; Barbour 2008): "Time is what a clock measures" underlines that the measure of time, and even its everyday definition, rely on the joint observation of two phenomena: that of interest, and a given well-calibrated one, called a clock. In other words, clocks provide a time unit based not on an absolute reference but on a controlled (usually physical) process. Moreover, it indicates that for all practical purposes, the notion of time can be identified with the measure of time. Thus time has been originally defined with respect to the (apparent) motion of sun, thought to be an absolute reference. In parallel to sundials (not always functional nor very accurate), time has been defined using as a reference a process of externally bounded duration: how water flows in a water clock (clepsydra), how sand flows in an hourglass. Time has then been defined with reference to a characteristic time of a system taken as clock. This time unit is shown or assumed to be a constant, invariant as long as the clock is unchanged. The emblematic example is the period of a pendulum, which remains constant when friction damps the pendulum motion, provided the pendulum mass and length remain unchanged. More recently, clocks have been based on more intrinsic characteristic times, thought to provide an absolute unit: the period (inverse frequency) of vibration in a quartz clock, based on the sustained vibrations of an electrically excited crystal of quartz (piezo-electric effect), or the inverse frequency of the photon emitted by an electron when passing from an excited energy level to the ground one, in atomic clocks. In all these cases, time is a phenomenological notion. In classical physics, there is no time without a physical phenomenon, whose course allows measuring and in a way defining time. This operational view, ignoring any preexisting concept that could be formulated independently of the material world, is all the more accepted that it is supplemented with our own intuitive perception, considered as a matter of cognitive science or psychology that does not interfere with common physicists' practice.

4.2 The Concept of Time Scale

Turning to the concept of time scale, a first obvious distinction arises between *epistemic* scales and *intrinsic* scales. The first ones are the scales at which the system is observed or described. The latter are related to the characteristic times of the system, independently of the presence of an observer, for instance: the mean free time between two molecular collisions, the period of vibrations or oscillations, the quantity $1/\nu$ where the frequency ν is related to the energy difference between two atomic levels, the correlation time τ_c of a time-dependent quantity X as defined by the exponential decay of the correlation function $C(t) = < X_t\, X_0 > - < X_t > < X_0 > \sim A\ \mathrm{exp.}(-t/\tau_c)$ or the relaxation time τ_r defined by the exponential decay of a time-dependent observable x_t towards its equilibrium value x_0, namely $x_t - x_0 \sim A\ \mathrm{exp.}(-t/\tau_r)$. Generally a characteristic time is obtained according to a relation: *time* = *length* / *velocity* when a characteristic length and a characteristic velocity are known. In living systems, the replication time of the genome, the division time of a cell or the lifetime of an organism complete the previous enumeration (see in particular the Chap. 15 by Nicoglou in the this volume).

4.3 Time Scales: Epistemic vs Systemic Issues

Concurrently with the above two kinds of time scales, we distinguish two issues. A first, *epistemic*, issue is to appreciate how our understanding and description of a system depends on the scale at which it is observed. For instance, as explained in any textbook of statistical mechanics, the impact of air molecules on the wall of a container can be described, at small scale, as a succession of collisions of molecules against the wall, occurring at random times due to the randomness of the thermal motion of the molecules. At a coarser resolution, only the average effect of these collisions is perceptible, corresponding to a time-constant pressure.

A second issue, *intrinsic* to the system of interest, is to understand how the different processes taking place at different time scales are coupled. In *plain physical situations*, microscopic and macroscopic processes are weakly coupled. This means that it is possible to consider separately microscopic elements and macroscopic behaviors. The latter can be obtained by a mere averaging of the microscopic processes (over the system and over intermediary durations): the microscopic fluctuations average out, and what remains is the macroscopic evolution (this is sometimes technically more difficult in practice than in principle). In the above example, the issue is now to describe how the microscopic dynamics ruling the collisions of the molecules on the wall of the container and their kinetic energy will generate at the macroscopic scale the time-variations of pressure and temperature, described autonomously in the framework of classical thermodynamics, with no reference to the microscopic level.

Some situations, termed *"critical phenomena"*, do not obey the rule of separation of scales: details and events at all scales are strongly coupled and microscopic fluctuations have macroscopic repercussions. The hallmarks of such situations are the divergence of the range of temporal and spatial correlations, and singularities in the system's response to perturbations. The latter are typically revealed by a divergence to infinity of the system susceptibilities, meaning that a microscopic (spatially localized or small-amplitude) event is capable of triggering a macroscopic consequence, spreading in the whole system and over long durations

(Laguës and Lesne 2011). For instance, a very localized magnetic perturbation (single spin flip) or the application of a very small magnetic field have observable repercussions at all spatial and temporal scales in a ferromagnet at the critical point, whereas they have no macroscopically observable effect far from the critical point. Although a critical phenomenon occurs in a precisely defined situation (a single specific point in the parameter space), temporally long and spatially extended transients are observed around the critical point. More complicated phenomena, typically slow relaxation dynamics and long-range correlations in an extended domain of the parameter space, may arise in disordered systems in which the control parameter is no longer homogeneous in space (*Griffiths' phases*) (see e.g. Moretti and Muñoz 2013).

While in plain systems, fast microscopic events accumulate and produce collectively a slow macroscopic evolution, the converse may also occur in some systems displaying what has been called *"self-organized criticality"*: a slow accumulation of elementary events relaxes into a sudden and fast collective event, characterized by power-law distributions of the various observables (e.g. the size and duration of the macroscopic events). This concept has been introduced to summarize the complex behavior of avalanches observed in continuously fed sand piles, earthquakes, species extinctions and cascading failures in a network (Bak 1996). Despite all the work and interest it has attracted, the concept of self-organized criticality is still lacking operational power beyond that of an elegant description, and some macroscopic power-law behaviors could have a different origin.

However, the study of criticality and self-organized criticality has underlined the importance of investigating the time scales of the observed phenomenon, in both describing its characteristic features and understanding the underlying mechanisms. For instance, the response of a system may differ according to the frequency f at which its parameters or the applied forces are varied, compared to the (inverse) characteristic times of the system (Yordanov et al. 2011). A system with a reaction time τ does not react if a control parameter changes periodically with a frequency $f >> 2\pi/\tau$; on the contrary, it smoothly follows the changes occuring at a frequency $f << 2\pi/\tau$. An oscillator with natural frequency f_0, displays a huge amplification, called a resonance, if $f \sim f_0$. Scales also matter in any interaction: the scale at which the interaction takes place determines the range of the integration over microscopic structures and processes, which occurs during the interaction. Think e.g. of the interaction between two molecules (integration over the atoms constituting the molecules), the solid friction of a sliding object along a slope (integration of the details of the two contacting surfaces, from the molecular scale to the scale of the object), or the attraction between two planets. This interaction range determines the relevant variables of the simplest effective description. For instance, in first approximation, the interacting planets are described as point particles fully characterized by their mass, and not by their countless components.

Up to now, we have considered scales, which have a clear-cut definition (as seen above). In living systems, the relevant decomposition of the system in nested layers has to follow the different *levels of organization*, rather than scales: one distinguishes an enzyme, a cell, an organ, an organism, even if some organisms (e.g. bacteria) may be smaller than some cells (e.g. human cells). Although these levels somehow follow the scales, their delineation is more qualitative and is associated with a functional or physiological description of the system. Here again, the duality between epistemic decomposition and intrinsic decomposition into nested levels of organization is a delicate issue, which has to be clarified to avoid misinterpretations.

4.4 Relation Between Temporal and Spatial Scales

In the above examples, the systems also display a space dimension, hence not only time scales but also spatial scales are relevant. In physics, fast processes generally correspond to spatially microscopic processes, and slow ones to spatially macroscopic ones. For instance, atomic vibrations are very fast compared to the oscillations of a rope. This intuitive thought is quantitatively supported for linear systems (e.g. wave propagation) by *dispersion relations*, which express the wavelength of the phenomenon as a monotonously increasing (and currently simple) function of its period. (Linear systems are rare, but actually, many systems can be locally linearized, i.e. approximated by a close linear system, exactly as a curve can be locally approximated by its tangent line).

This correspondence between temporal and spatial scales does not hold in living systems. For instance, a spike (elementary activity event) in a neuron involves the transfer of only a few ions in a region of nanometric scale in space, however it extends over about a millisecond in time, far above the usual molecular time scales. Moreover, phenomena like neural coordination or synchronization yield macroscopic events having the same time scale as elementary ones.

4.5 Time Scales, Time Representation and Modeling Choices

The relevant representation of time depends on the observation/description time scale compared to the intrinsic scales of the system. It is actually a modeling choice. Currently three representations of the time variable are encountered:

1. *An integer label qualitatively describing the qualitative ordering of a sequence of events*. An example of such a "qualitative time" is provided by generations, in population dynamics, whatever the duration separating an individual and its offsprings. Another example is a cell, which has a priori its own clock for growth and division. In some cases, e.g. behavioral sequences, only the ordering is kept and the duration of events is eliminated. Due to individual variability and context dependence, what is relevant to understand the behavioral processes is indeed the qualitative succession of events.

2. *An integer variable n measuring a number of time steps*. This amounts to describe the evolution of the system by considering the succession of its states at discrete times $n\Delta t$, where Δt is a fixed unit of reference. If the time step Δt is much smaller than the characteristic time(s) of the system, the discretization is valid with no significant loss of information. If on the contrary the time step is much larger than some characteristic time of the system, the associated processes are not well captured and the discretized dynamics will only reflect properly the slow trends. For instance, a description of the climate dynamics with a step equal or larger than a year will ignore seasonal variations. A more intrinsic (hence more robust) way to discretize the evolution of a system, known as the "Poincaré section method", is possible when the dynamics is close to a periodic regime (limit cycle). One considers an hypersurface of codimension 1 in the state space (e.g. a plane if the state space is of dimension 3), and replaces the study of a trajectory by the study of the sequence of its intersections with the hypersurface; for example, this hypersurface can be defined as the set of points where an observable quantity takes a given value. The time step is now irregular, of duration vaying from one to another, and the discretization amounts to consider a sequence of events. Adaptive time-steps are also encountered in discrete mechanics, where this flexibility is required to preserve both the geometric properties and energy conservation in the use of a discrete variational principle (see Chap. 9 by Ardourel and Barberousse in this volume).
3. *A real time variable t varying continuously*. One should however remember that using a continuous time variable t in a physical model is in fact a mathematical idealization: indeed, the model and associated equations have been derived under some assumptions on the time resolution (the lower time scale of the description), meaning that faster processes have been neglected. Contrary to the mathematical notion, the so-called infinitesimal time increment dt has a physical meaning and should rather be denoted Δt. Similarly the notion of time derivative dx/dt is an idealization (hence an approximation) of a finite rate $\Delta x/\Delta t$.

Also the choice of the framework used to describe the dynamics (i.e. the evolution rules) depends on the relative magnitudes of intrinsic and description scales. They condition for instance the choice of a deterministic or a stochastic dynamic model. The evolution will be described as deterministic if dt is far larger than the correlation time, so that fluctuations average out, or at the opposite (reversible molecular dynamics) if dt is far smaller than the typical time between two successive collisions experienced by a molecule with the other ones (mean free time). In between, fluctuations cannot be ignored and a stochastic model would be more relevant. An important issue is to establish inter-relations between these representations of time and modeling choices (Lesne 2007).

4.6 The Notion of Equilibrium

Seemingly intuitive and commonsense, the notion of equilibrium is in fact complex. An operational definition would be: *a system is in equilibrium if its characteristics do not evolve along time*. In practice, this formulation shows that equilibrium is a relative notion. The definition has to be refined by the prescription of scales. Namely, a system in equilibrum experiences no perceptible changes at a given resolution (lower bound on the time scale) over a given duration (upper bound on the time scale). In plain words, fast processes have evened out and slow ones have not yet begun. Also the resolution on the observable characteristics has to be defined, namely, what is the value of a significant change that can be detected by the observer. The notion of equilibrium is thus deeply related to space and time scales.

The notion of equilibrium involves time in a dual way. Its above definition implicitely refers to the absence of significant dynamic processes; for instance the equilibrium of a (mechanical) scales means that it no longer oscillates unless an additional weight is introduced. However, the very notion of time is related to a rate of change (see above our ways of measuring time); time would be an irrelevant notion for a system in perfect equilibrium. Luckily, this paradox vanishes on its own since there is no such thing as a perfect equilibrium: relaxation becomes infinitely slow near the equilibrium point; surrounding noise unavoidably disturbs the equilibrium; slowly evolving environment modifies what is the equilibrium state; thermal motion implies that equilibrium is an irrelevant notion at molecular scales. In practice, it is safer to speak of "*stationary state*", possibly defined on statistical quantities, and always in reference to prescribed scales and resolutions.

A puzzling statement remains: what is the meaning of the usual definition of an adiabatic evolution in thermodynamics (also encountered under the name of quasi-stationary approximation in numerous studies of complex dynamics) as being "*a succession of equilibrium states*". Answering this question requires a mathematical formulation of time scale separation (Lesne 2006).

4.7 Multiple Time Scales and Quasi-stationary Approximation

A clue to recognize whether a model captures the coexistence of processes at different time scales is the presence of a small parameter ε in the equations of evolution, hinting at the presence of contributions acting on a longer term than the others. Then, the main step is to identify fast and slow variables. Typically, the fast variables describe the microscopic events while the emerging macroscopic trends, corresponding to average or aggregated behaviors, are generally described by slowly varying quantities.

Let us denote x the fast variables reflecting the system micro-evolution, and Y the slowly varying observables accounting for the system macro-evolution. The equations of evolution can be written $[dx/dt = f(x, Y), dY/dt = \varepsilon\, g(x, Y)]$ where ε is the small parameter measuring the scale separation between the characteristic times at which x and Y evolve. At a fine temporal resolution, for ε small enough, the quantity x evolves significantly during one time step $\Delta t = 1$, while Y remains constant. This justifies to invoke a *quasi-static approximation* (for Y) in which the second equation of evolution reduces to $dY/dt = 0$ and what remains is an equation for x where Y is a constant parameter – one also speaks of *parametric approximation* (for x).

At a coarser temporal resolution, the time variable becomes $T = \varepsilon t$. The equations of evolution are now written $[\varepsilon\, dx/dT = f(x, Y), dY/dT = g(x, Y)]$. During one macro-step $\Delta T = 1$, x completes its evolution at fixed $Y = Y(T)$ and reaches the asymptotic state $x^*[Y(T)]$ satisfying $f(x^*(Y),Y) = 0$. Plugging this asymptotic value turns the equation of evolution of $Y(T)$ into the closed equation $dY/dT = g(x^*(Y), Y)$, where the variation of x^* is *slaved* to Y (Haken 1996). This procedure is termed *quasi-stationary approximation* (for x) since it amounts to replace the full equation of evolution $\varepsilon\, dx/dT = f(x, Y)$ by $f(x,Y) = 0$. Illustrations are given in (Nayfeh 1973; Lesne 2006).

If the evolution is a sustained oscillation, the fast variable x self-averages into a quantity $\underline{x}[Y(T)]$ during one time macro-step $\Delta T = 1$, and the quasi-stationary approximation amounts to replacing x by $\underline{x}[Y(T)]$ in the evolution of Y, yielding a closed equation $dY/dT = g(\underline{x}(Y), Y)$. This reflects the fact that the macroscopic features are not influenced by all the microscopic fluctuations, but are only sensitive to their average and emerging trends.

The quasi-stationary approximation means that the fast processes reach their stationary state before the slower processes begin to evolve, and so on in a nested way from fast small-scale processes to slow macroscopic contributions. In this respect, the slow evolution is described as a succession of equilibria of the fast degrees of freedom, and the above quasi-stationary approximation is the operational way to formulate this idea. A useful spin-off is the derivation of macroscopic evolutions, i.e. closed equations of evolution for a (usually small) number of macroscopic variables, from the knowledge of the microscopic dynamics (Givon et al. 2004; Castiglione et al. 2008).

4.8 The Multiple-Scale Method: Several Independent Times

Let us mention a thought-provoking method to simplify the above decoupling procedure. The idea is to introduce jointly several time variables $t_0 = t$, $t_1 = \varepsilon t$ (slower by a factor of ε since $\Delta t_1 = 1$ for $\Delta t = 1/\varepsilon$), $t_2 = \varepsilon^2 t$ (slower by a factor of ε^2 since $\Delta t_2 = 1$ for $\Delta t = 1/\varepsilon^2$), and to treat them as independent time variables to dissect the processes at different scales. The time derivative becomes

$d/dt = \partial/\partial t_0 + \varepsilon\partial/\partial t_1 + \varepsilon^2\partial/\partial t_2$ and the solution $x(t)$ is searched as a function of the form $X(t_0, t_1, t_2)$. Scale-decoupling approximation here amounts to consider t_0, t_1, and t_2 as formally *independent variables*, with fast variations captured with time variable t_0, slow variations captured with time variable t_1, and slower trends captured with time variable t_2. However, considering independent time variables is only a computational trick, equivalent to (but more concise than) nested quasi-stationary approximations. Ultimately, the solution is obtained under the form $x(t) = X(t, \varepsilon t, \varepsilon^2 t)$ depending on the single original time variable t. This method has been successfully applied to complex evolutions with superimposed time scales, depending on a small parameter ε in a singular way ("singular" insofar as the solution for $\varepsilon = 0$ is qualitatively different from the solution for $\varepsilon > 0$ (whatever small) hence displays a singularity as a function of ε in $\varepsilon = 0$). An example is the treatment of the so-called secular terms in the 3-body problem in astronomy: these terms arise when investigating e.g. the motion of a planet under both the (dominant) influence of the Sun and the (seemingly marginal) influence of other planets. They induce slow variations perceptible only over centuries (hence the name "secular") and cannot be accounted for in a perturbative way starting from the easy case where only the Sun influences the planet motion (Nayfeh 1973; Lesne 2006; Castiglione et al. 2008).

4.9 Time's Arrow

The long debated issue of time's arrow and the apparent inconsistency between the Second Principle of the thermodynamics and the micro-reversibility of molecular dynamics is also a matter of scales. The Second Principle, presented in any textbook of basic thermodynamics, is a macroscopic feature rooted in a statistical argument for a large number of molecules. It is not expected to – and does not – hold for a system of a few molecules (this is rarely underlined in textbooks, which deal with thermodynamic properties, observed in macroscopic systems). There is no such time's arrow in small microscopic systems. For a macroscopic system like a gas in a compartment, the explanation of the time's arrow runs as follows. Considering the expansion of a gas initially confined in a small compartment, the number of reverse trajectories, starting from an expanded gas configuration and leading back to a confined one (i.e. belonging to a very constrained subset of the configuration space), is infinitely small compared to the number of plain trajectories, starting from the expanded gas configuration and leading to another expanded configuration (see e.g. Castiglione et al. 2008). There is thus an (almost) infinitely small probability to observe the spontaneous confinement of an expanded configuration, the smaller the larger the number of molecules in the considered system. Similarly, there is an (almost) infinitely small number of trajectories leading from the confined configuration to another confined configuration, compared to the number of trajectories leading from the confined configuration to an expanded configuration: in practice, at our macroscopic scale, we always observe an expansion of the gas. However, we here see that the Second Principle describes an *improbability* to observe backward evolutions, not their *impossibility*. The reasoning relies on the large size of the system, and fails for a microscopic system composed of a few molecules.

This argument solves the time's arrow issue for the spontaneous evolution of a closed system, that is, the relaxation from a prepared state to an equilibrium state. It does not answer the deeper issue of time's arrow in open non-equilibrium systems (non-equilibrium meaning here "externally driven in a steady state with non-vanishing fluxes"). Fluctuation theorems have established the probability distribution of the time-averaged irreversible entropy production (Kurchan 1998; Cohen and Gallavotti 1999; Evans and Searles 2002) but many questions are still open, motivating on-going studies in the non-equilibrium statistical physics community.

In this context, the question of the "nature of time" is mostly irrelevant for (classical) physicists. They handle time as an effective and operational notion, like the scientists of the nineteenth century handled the notion of temperature: they devised several kinds of thermometers and made efficient thermodynamic machines without understanding the "nature" of temperature.

4.10 Time Scales and Evolution Theory

We have seen that the response of a dynamic system to a perturbation depends on the time scale of this perturbation, compared to characteristic times of the system. This feature is highly relevant not only in the physical world but also in the living world. Typically, living systems evolve robustness to fast transient changes (homeostasis), and adaptation to sustained (directional) changes. Evolutionary strategies are thus to be examined according to the time scale of environmental variation.

More generally, a scale-decoupling argument is invoked in the very notion of genetic equilibrium, assuming that the changes in allelic frequencies from one generation to another are faster than the occurrence of novel mutations, itself faster than the modification of the surrounding ecosystem. If this decoupling applies, the evolution of a species can be seen as a succession of genetic equilibria, each equilibrium corresponding to the local result of natural selection, that is, an optimized distribution of genotypes in the environment present at that time, while the passage from one equilibrium to another is due to mutations, i.e. a change in the set of possible genotypes or/and changes in the environment. Actually, it has been presumed for a long time that populational, ecological and evolutionary processes occur at very different time scales. However, it has recently been realized that ecological and (macro)evolutionary processes can be fast, and scale decoupling is not necessarily valid. If environmental changes or mutations rates are too fast, the above quasi-stationary approximation fails and what is observed for a species is a succession of transient and unpredictable allelic distributions. The decoupling approximation remains most often useful in a first step to identify the different processes and clarify their consequences (see Chap. 14 by Huneman in this volume).

The discussion about quasi-stationary approximation could also enlighten an operational notion of species. I would suggest that considering a species as a quasi-equilibrium and evolution of species as an adiabatic evolution would reconcile the existence of separate species at a given moment and the gradual phenomenon of speciation. Here again nothing relevant could be said without explicitly mentioning the time scale(s) at which processes are discussed.

4.11 Conclusion

In classical physics and natural sciences, time is not a unique absolute concept, but rather a relative and operational one. A physicist has no access to the *nature* of systems and phenomena, in particular the nature of time. The relevant issues are rather time representation and the choice of a proper time variable. In this operational perspective, I have argued that a central notion is that of time scales. It allows clarifying paradoxical issues like the notion of equilibrium, scale separation, decoupling slow and fast processes or even the time's arrow predicted by the Second Principle of the thermodynamics. In a biological context, time scales are especially important in evolutionary studies and the very definition of a species or the fitness of an individual in a population.

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