Table 1.1: Main results from second quantification.

Many-body states	$\|\alpha_1 \cdots \alpha_n) = \|\alpha_1\rangle \otimes \cdots \otimes \|\alpha_n\rangle$
	$\|\alpha_1 \cdots \alpha_n\} = \frac{1}{\sqrt{n!}} \sum_P \zeta^P \|\alpha_{P(1)} \cdots \alpha_{P(n)})$
	$\|\alpha_1 \cdots \alpha_n\rangle = \frac{1}{(\prod_{i=1}^p n_{\alpha_i}!)^{1/2}} \|\alpha_1 \cdots \alpha_n\}$
Closure relation	$\sum_{n=0}^{\infty} \frac{1}{n!} \sum_{\alpha_1 \cdots \alpha_n} \|\alpha_1 \cdots \alpha_n\}\{\alpha_1 \cdots \alpha_n\| = 1$
Creation and annihilation operators	$\hat{\psi}_\alpha^\dagger \|\alpha_1 \cdots \alpha_n\} = \|\alpha\alpha_1 \cdots \alpha_n\}$
	$\hat{\psi}_\alpha \|\alpha_1 \cdots \alpha_n\} = \sum_{i=1}^n \zeta^{i-1} \delta_{\alpha,\alpha_i} \|\alpha_1 \cdots \hat{\alpha}_i \cdots \alpha_n\}$
	$\hat{\psi}_\alpha^\dagger \|\alpha_1 \cdots \alpha_n\rangle = \sqrt{n_\alpha + 1} \|\alpha\alpha_1 \cdots \alpha_n\rangle$
	$\hat{\psi}_\alpha \|\alpha_1 \cdots \alpha_n\rangle = \frac{1}{\sqrt{n_\alpha}} \sum_{i=1}^n \zeta^{i-1} \delta_{\alpha,\alpha_i} \|\alpha_1 \cdots \hat{\alpha}_i \cdots \alpha_n\rangle$
(Anti)commutation relations	$[\hat{\psi}_\alpha, \hat{\psi}_{\alpha'}]_{-\zeta} = [\hat{\psi}_\alpha^\dagger, \hat{\psi}_{\alpha'}^\dagger]_{-\zeta} = 0$
	$[\hat{\psi}_\alpha, \hat{\psi}_{\alpha'}^\dagger]_{-\zeta} = \delta_{\alpha,\alpha'}$
Field operators	$\hat{\psi}_\sigma(\mathbf{r}) = \sum_\alpha \langle \mathbf{r}, \sigma \| \alpha \rangle \hat{\psi}_\alpha$
	$\hat{\psi}_\sigma^\dagger(\mathbf{r}) = \sum_\alpha \langle \alpha \| \mathbf{r}, \sigma \rangle \hat{\psi}_\alpha^\dagger$
	$[\hat{\psi}_\sigma(\mathbf{r}), \hat{\psi}_{\sigma'}(\mathbf{r}')]_{-\zeta} = [\hat{\psi}_\sigma^\dagger(\mathbf{r}), \hat{\psi}_{\sigma'}^\dagger(\mathbf{r}')]_{-\zeta} = 0$
	$[\hat{\psi}_\sigma(\mathbf{r}), \hat{\psi}_{\sigma'}^\dagger(\mathbf{r}')]_{-\zeta} = \delta_{\sigma,\sigma'} \delta(\mathbf{r} - \mathbf{r}')$
One-body operator	$\hat{V}^{(1)} = \sum_{\alpha,\alpha'} \langle \alpha \| \hat{V} \| \alpha' \rangle \hat{\psi}_\alpha^\dagger \hat{\psi}_{\alpha'}$
	$= \int d^d r d^d r' \sum_{\sigma,\sigma'} \langle \mathbf{r}, \sigma \| \hat{V} \| \mathbf{r}', \sigma' \rangle \hat{\psi}_\sigma^\dagger(\mathbf{r}) \hat{\psi}_{\sigma'}(\mathbf{r}')$
Two-body operator	$\hat{V}^{(2)} = \frac{1}{2} \sum_{\alpha_1 \cdots \alpha_2'} (\alpha_1 \alpha_2 \| \hat{V} \| \alpha_1' \alpha_2') \hat{\psi}_{\alpha_1}^\dagger \hat{\psi}_{\alpha_2}^\dagger \hat{\psi}_{\alpha_2'} \hat{\psi}_{\alpha_1'}$

1.2.3 Green functions

1.2.3.1 Retarded Green function

When studying the quantum mechanics of a single particle in Sec. 1.1.1, we introduced the propagator

$$G^R(\mathbf{r}, \mathbf{r}'; t) = -i\Theta(t) \langle \mathbf{r} | e^{-i\hat{H}t} | \mathbf{r}' \rangle = -i\Theta(t) \sum_n e^{-i\epsilon_n t} \varphi_n(\mathbf{r}) \varphi_n^*(\mathbf{r}') \tag{1.183}$$

© N. Dupuis, 2023