Quantum Field Theory I
Master ICFP 2026-2027
Problem Set 7

October 19, 2026

1 Feynman propagator for the Dirac field

You proved in the lecture that the Feynman propagator for Dirac fermions is

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon} e^{-ip(x-y)}$$

1. Show that

$$D_F(x-y) = (i\not{\partial}_x + m) D(x-y) \tag{1.1}$$

2. Conclude that $(i\not{\partial}_x - m) D_F(x-y) = i\delta^{(4)}(x-y)$.

3. Show that

$$D_F(x-y) = D(x-y)(-i\overleftarrow{\not{\partial}}_y + m) \tag{1.2}$$

4. Conclude that $D_F(x-y)(-i\overleftarrow{\not{\partial}}_y - m) = i\delta^{(4)}(x-y)$.

2 Yukawa theory at tree level

We consider a theory with Dirac fermions interacting through the exchange of a bosonic particle. The lagrangian density is

$$\mathcal{L} = \frac{1}{2}\bar{\psi}(i\not{\partial} - m)\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 - g\phi\bar{\psi}\psi$$

1. Using the LSZ reduction formulae for the Dirac fields in real space, write the S matrix for

(a) an in state with one fermion of momentum p_1, spin s_1 and one antifermion with momentum p_2 spin s_2

(b) an out state with one fermion of momentum p_3, spin s_3 and one antifermion with momentum p_4 spin s_4

2. We only consider the connected part of the S matrix. What is the lowest order in perturbation theory? Draw the associated Feynman diagrams.

3. Compute the contributions of these two diagrams.

4. Compute now the modulus square of the scattering amplitude. To simplify the expression, average over the spins of the incoming particles (this corresponds to a physical situation where the incoming particle beams are unpolarized) and sum over final spin states (this corresponds to detector which are insensitive to the spin of outgoing particles). To simplify things, work in the limit of small m and use the properties

$$\sum_s u_s(\vec{b})\bar{u}_s(\vec{p}) = \not{p} + m$$
$$\sum_s v_s(\vec{b})\bar{v}_s(\vec{p}) = \not{p} - m$$

5. Express your final result in the center of mass frame, with an incoming particle momentum $\vec{p}$ and an outgoing particle of momentum $\vec{p}'$ which makes an angle θ with $\vec{p}$. Show that the previous expression presents a pole for some value of $|\vec{p}$.