

Quantum Field Theory I

Master ICFP 2026-2027

Problem Set 5

October 6, 2026

1 Quantum field coupled to a classical source

The aim of this exercise is to study the simplest interaction of all: the interaction to a classical source.

We consider a real Klein-Gordon field which is coupled linearly to a classical, spacetime dependent, real source J . The Lagrangian density is:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} m^2 \Phi^2 + J\Phi$$

The source is switched on at time $-T$ and switched off at time T , with $T > 0$.

In the regime where the source has not been switched on yet ($t \leq T$), we decompose the field in annihilation and creation operators:

$$\Phi_{<}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_{\vec{p}} e^{-ip \cdot x} + a_{\vec{p}}^\dagger e^{ip \cdot x} \right)$$

1. Write the solution of the field equation at all time by making use of the retarded propagator.
2. Write the field at time $t > T$ as

$$\Phi_{>}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(b_{\vec{p}} e^{-ip \cdot x} + b_{\vec{p}}^\dagger e^{ip \cdot x} \right)$$

and relate the operators a and b . The operators b and $b^\dagger$ closely resemble the out operators of the lecture. However in the lecture we shifted the field such that the operators a and b annihilate the vacuum. We will not do this shift so $b_{\vec{p}}|0\rangle \neq 0$.

3. What is the hamiltonian of the theory at $t > T$? Starting from the vacuum, compute the average energy created during the process. Guess the probability density of creating a particle of momentum $\vec{p}$. Compute the average number of created particles $\langle N \rangle$.

In the following, we will consider the same problem with the formalism developed in the lecture.

4. Show that the probability of creating no particle is

$$P(0) = \left| \langle 0 | T \exp(i \int d^4x \Phi_I(x) J(x)) | 0 \rangle \right|$$

5. Evaluate the previous expression in an expansion in powers of J . Show that the result is $P(0) = 1 - \lambda + \mathcal{O}(J^4)$ where λ is the result obtained above for $\langle N \rangle$.
6. Represent the terms obtained in the previous question as a Feynman diagram. Now represent the whole perturbation series of $P(0)$ in terms of Feynman diagrams. Show that this series exponentiates so that it can be resummed exactly: $P(0) = e^{-\lambda}$.
7. Compute the probability density that the source creates one particle of momentum $\vec{k}$. Deduce the probability of creating 1 particle, irrespective of its momentum. First do the calculation at leading order and then generalize to all orders.
8. Compute the probability of creating n particles. You should obtain a Poisson distribution.
9. Compute $\langle N \rangle$ and ΔN .