TD 6

1) $D_F^{(p)} = \frac{i}{p^2 - m^2 + i\varepsilon}$

$$D_F(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{-ipx} \frac{1}{p^2 - m^2 + i\varepsilon}$$

$$\mathcal{H}_I = \frac{\lambda}{4!} \phi_I^4$$

2) at leading order

$$\langle 0 | T \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) | 0 \rangle = \overbrace{\phi(x_1)\phi(x_2)}\, \overbrace{\phi(x_3)\phi(x_4)} + 2 \text{ perm.}$$

$= $ 

disconnected

3) at order λ^1 : D_1 $\overline{\overline{\quad}}$ 8 , $\mid\mid 8$, $\times 8$

D_2 $\underline{\underline{\quad 0 \quad}}$ $\overline{\overline{\quad 0 \quad}}$ $\times$ D_3

$$D_1 \to -\frac{i\lambda}{4!} \; 3 \left(\int d^4y \, D_F^2(0) \right) D_F(x_1 - x_2) D_F(x_3 - x_4)$$

$$D_2 \to \frac{-i\lambda}{4!} \; 4 \; 3 \left(\int d^4y \, D_F(x_1 - y) D_F(y - x_3) \right) D_F(x_2 - x_4)$$

$D_3 : \frac{-i\lambda}{4!}\, 4\cdot 3\cdot 2\cdot 1 \int d^4 y\, D_F(x_1-y) D_F(x_2-y) D_F(x_3-y) D_F(x_4-y)$

D_1 has bubble diags, disconnected

D_2 no bubble, disconnected

D_3 no bubble, connected.

4) $\quad 1 + 8$

$$1 \triangleq \frac{-i\lambda}{4!} 3 \cdot \int d^4 y\, D_F(0)^2$$

connected graphs: Need to divide by $1 - \frac{i\lambda}{8}\int d^4 y\, D_F(0)^2$
simplifies with diagram with bubble

5) $\quad 1 \quad \left(-\frac{i\lambda}{4!}\right) 4\cdot 3 = -\frac{i\lambda}{2}$



$\frac{1}{2}\left(-\frac{i\lambda}{4!}\right)^2 2\,4\,4\,3\,3 \qquad \frac{1}{2}\left(\frac{-i\lambda}{4!}\right)^2 2\,4\,3\,4\,3 \qquad \frac{1}{2}\left(\frac{-i\lambda}{4!}\right)^2 2\,4\,4\,3\,2$

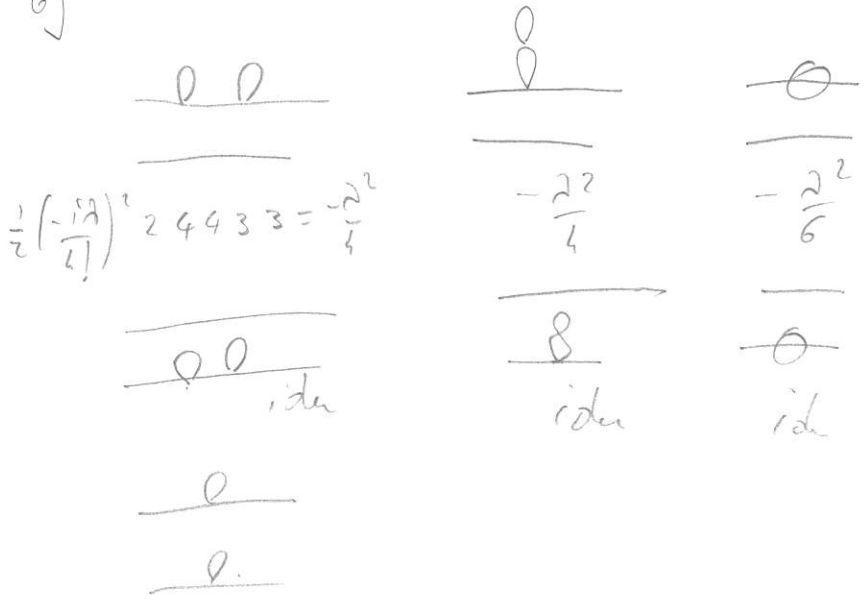
$-\frac{\lambda^2}{4} \qquad -\frac{\lambda^2}{4} \qquad -\frac{\lambda^2}{6}$

2-point connected graphs.



$-\frac{i\lambda}{2} \qquad -\frac{\lambda^2}{4} \qquad -\frac{\lambda^2}{4} \qquad -\frac{\lambda^2}{6}$

6)



$$\therefore \frac{1}{2}\left(\frac{-i\lambda}{4!}\right)^2 2\,4\,3\,4\,3 = -\frac{\lambda^2}{4}$$

All in all, in this sector:

$$= -\frac{i\lambda}{2} - \frac{i\lambda}{2} - \frac{\lambda^2}{4} - \frac{\lambda^2}{4} - \frac{\lambda^2}{4} - \frac{\lambda^2}{4} - \frac{\lambda^2}{6} - \frac{\lambda^2}{6} - \frac{\lambda^2}{4}$$

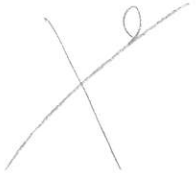
$$= \left(-\frac{i\lambda}{2}\right)^{12} \times (\quad)^{34}$$

3)

$\frac{1}{2}\left(\frac{-i\lambda}{4!}\right)^2$



$24\,4\,3\,2\,3 = -\frac{\lambda^2}{2}$

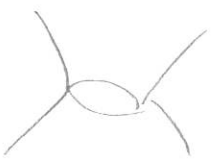


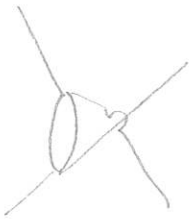


$\frac{1}{2}\left(\frac{-i\lambda}{4!}\right)^2$

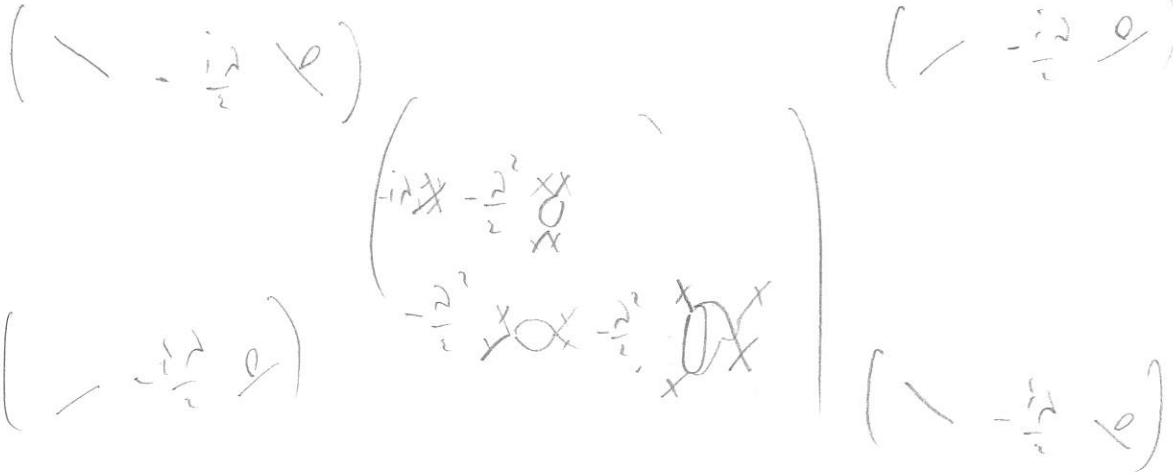


$24\,3\,4\,3\,2 = -\frac{\lambda^2}{2}$

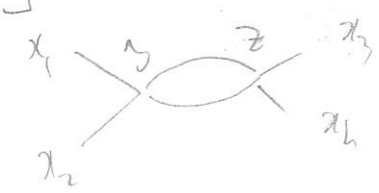




together with the first connected diagram. This can be interpreted as



4)



$$\int_{yz} D_F(x_1-y)\, D_F(x_2-y)\, D_F(x_3-z)\, D_F(x_4-z)\, D_F^2(y-z)$$

$$\int_{x_1 x_2 x_3 x_4} e^{i(p_1 x_1 - p_2 x_2 + p_3 x_3 + p_4 x_4)} \int_{q_1 \dots q_6} D_F(q_1) \dots D_F(q_6)\, e^{-iq_1(x_1-y)}\, e^{-iq_2(x_2-y)}\, e^{-iq_3(x_3-z)}\, e^{iq_4(x_4-z)}\, e^{-iq_5(y-z)}\, e^{-iq_6(y-z)}$$

integrals over $x_1 \to q_1 = -p_1$ ①

$x_2 \to q_2 = -p_2$

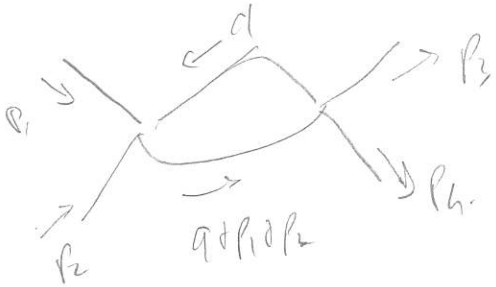
$x_3 \to q_3 = p_3$

$x_4 \to q_4 = p_4$

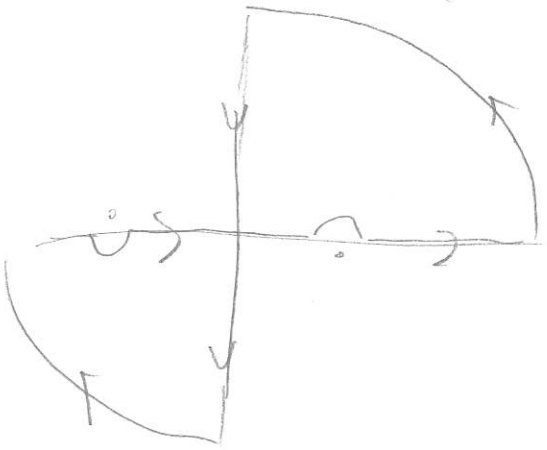
$y \to \delta(q_1 + q_2 - q_5 - q_6)$

$z \to \delta(q_3 + q_4 + q_5 + q_6)$

$$= (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4)\, D_F(-p_1)\, D_F(p_2)\, D_F(p_3)\, D_F(p_4) \int \frac{d^4 q_5}{(2\pi)^4} D_F(q_5)\, D_F(-p_1 - p_2 - q_5)$$



Need to compute $\int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m^2 + i\varepsilon} \frac{1}{(q+p)^2 - m^2 + i\varepsilon}$



Consider the integral over the previous slide. It is 0 because no pole. The arc of circle gives us what

$$\int_{-\infty}^{+\infty} dq_0 \, f(q_0) = i \int_{-\infty}^{+\infty} dq_E \, f(i q_E) = i \int_{-\infty}^{+\infty} dq_E \frac{1}{(-q_E^2 - \vec{q}^2 - m^2 + i\varepsilon)^2}$$

the ε plays no role $\to$ $i \int_{-\infty}^{+\infty} \frac{1}{(q_E^2 + \vec{q}^2 + m^2)^2}$

NLO in dd, we need the calculation

$$i\int \frac{d^4q}{(2\pi)^4(q^2+m^2)^2} = i\frac{2\pi^{\frac{4}{2}}}{(2\pi)^4}\int_0^\infty \frac{q^3 dq}{(q^2+m^2)^2}$$

$$= \frac{i}{16\pi^2}\int_0^\infty \frac{x\,dx}{(x+m^2)^2}$$

$$= \frac{i}{16\pi^2}\int_0^{\Lambda^2} \frac{x\,dx}{(x+m^2)^2} = \frac{i}{16\pi^2}\int_0^{\Lambda^2} \frac{1}{(x+m^2)} - \frac{m^2}{(x+m^2)^2}$$

$$= \frac{i}{16\pi^2}\left[\ln\frac{\Lambda^2+m^2}{m^2} + m^2\left(\frac{1}{\Lambda^2+m^2} - \frac{1}{m^2}\right)\right)$$

$$= \frac{i}{16\pi^2}\left[\ln\frac{\Lambda^2+m^2}{m^2} - \frac{\Lambda^2}{\Lambda^2+m^2}\right]$$

$$\sim \frac{i}{16\pi^2}\left[\ln\Lambda^2\right]$$

in d dim

$$i\frac{1}{(2\pi)^d}\int d^dq\,\frac{1}{(q^2+m^2)^2} = \frac{i\,2\pi^{d/2}}{(2\pi)^d\Gamma(\frac{d}{2})}\int q^{d-1}dq\,\frac{1}{(q^2+m^2)^2}$$

$$= \frac{i}{(4\pi)^{d/2}\Gamma(\frac{d}{2})}\int \frac{x^{\frac{d-2}{2}}\,dx}{(x+m^2)^2}$$

$$= \frac{i}{(4\pi)^{d/2}\Gamma(\frac{d}{2})}\,m^{\frac{d-4}{2}}\int \frac{x^{\frac{d-2}{2}}}{(x+1)^2}\,dx$$

$$a = \frac{d-2}{2}$$

$$b = 2$$

$$\frac{i}{(4\pi)^{d/2}} n^{\frac{d-4}{2}} \frac{\Gamma\left(\frac{d}{2}\right)\Gamma\left(-1-\frac{d-2}{2}+2\right)}{\Gamma(2)}$$

$$= \frac{i}{(4\pi)^{d/2}} n^{\frac{d-4}{2}} \Gamma\left(\frac{4-\delta}{2}\right)$$

$$= \frac{i}{(4\pi)^{2-\varepsilon}} n^{-\varepsilon} \Gamma(+\varepsilon)$$

$$\Gamma(\varepsilon) \sim \frac{1}{\varepsilon}$$

$$\sim \frac{i}{16\pi^2} \frac{1}{\varepsilon}.$$

не обсчитывается $\frac{1}{\varepsilon} \sim \ln^2$