

TDS

Y field eq: $(\square + m^2)\phi = J$

the general solution is the general solution of the homogeneous eq + particular solution

$$\phi_{\text{part}}(x) = i \int d^4y \mathcal{D}_{\text{ret}}(x-y) J(y)$$

indeed $(\square + m^2)\phi_{\text{part}} = i \int d^4y (\square + m^2) G_{\text{ret}}(x-y) J(y)$
 $= i \int d^4y (-i) \delta^{(4)}(x-y) J(y)$
 $= J(x)$ OK

Re: $i \int d^4y \mathcal{D}_{\text{ret}}(x-y) J(y)$
 $= i \int dy^0 \int d^3\vec{y} \theta(x^0 - y^0) (\Delta(x-y) - \Delta(y-x)) J(y)$
 $= i \int_{-\infty}^{x^0} dy^0 \int d^3\vec{y} (\Delta(x-y) - \Delta(y-x)) J(y)$

if $x^0 < -T$, $J=0$ in the whole domain of integration =

$\phi_{\text{part}}(x^0 < -T) = 0$ and doesn't spoil ϕ_c .

Y if $x^0 > T$

$$i \int_{-\infty}^{x^0} dy^0 \int d^3\vec{y} (\Delta(x-y) - \Delta(y-x)) J(y) = i \int_{-\infty}^{+\infty} dy^0 \dots$$

$$= i \int d^4 y \int \frac{d^3 p}{(2\pi)^3} \frac{J(p)}{2E_p} \left(e^{-iE_p(x^0-y^0) + i\vec{p}(\vec{x}-\vec{y})} - e^{iE_p(x^0-y^0) - i\vec{p}(\vec{x}-\vec{y})} \right) \quad (1)$$

$$= i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} \left(e^{-i p x} J(p) - e^{i p x} J^*(p) \right)$$

Fourier Transform with $p^0 = E_p$

all i all:

$$\varphi_2(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left\{ \underbrace{\left(a_{\vec{p}} + \frac{i}{\sqrt{2E_p}} J(p) \right)}_{b_{\vec{p}}} e^{-i p x} + \text{h.c.} \right\}$$

3) at $t \rightarrow \infty$, the source is 0, the Hamiltonian is

$$H = \int \frac{d^3 p}{(2\pi)^3} E_p b_{\vec{p}}^\dagger b_{\vec{p}}$$

$$= \int \frac{d^3 p}{(2\pi)^3} E_p \left(a_{\vec{p}}^\dagger - \frac{i}{\sqrt{2E_p}} J^*(p) \right) \left(a_{\vec{p}} + \frac{i}{\sqrt{2E_p}} J(p) \right)$$

$$\langle 0 | H | 0 \rangle = \int \frac{d^3 p}{(2\pi)^3} \frac{E_p}{2E_p} |J(p)|^2$$

$$= \int \frac{d^3 p}{(2\pi)^3} E_p \frac{N(p)}{2E_p}$$

$$\langle N \rangle = \int \frac{d^3 p}{(2\pi)^3} \frac{|J(p)|^2}{2E_p}$$

$$4) P(\omega) = \left| \langle 0 | 0 \rangle_{in} \right|^2$$

$$= \lim_{t \rightarrow (t_1, t_2) \in \mathbb{R}} |\langle 0 | e^{-iH_I t} | 0 \rangle|^2$$

$$= \left| \langle 0 | T e^{-i \int_{t_1}^{t_2} H_I dt} | 0 \rangle \right|^2$$

$$= \left| \langle 0 | T e^{-i \int d^4x J(x) \phi_2(x)} | 0 \rangle \right|^2$$

$$\langle 0 | T e^{-i \int d^4x J(x) \phi_2(x)} | 0 \rangle$$

$$= \langle 0 | 1 - i \int d^4x J(x) \phi_2(x) - \frac{1}{2} \int d^4x d^4y J(x) J(y)$$

$$T(\phi(x) \phi(y)) | 0 \rangle$$

$$= 1 - \frac{1}{2} \int d^4x d^4y \overline{\phi(x) \phi(y)} J(x) J(y)$$

$$= 1 - \frac{i}{2} \int d^4x d^4y J(x) J(y) \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2 - m^2 + i\epsilon}$$

$$= 1 - \frac{i}{2} \int \frac{d^4p}{(2\pi)^4} |J(p)|^2 \frac{1}{p^2 - m^2 + i\epsilon}$$



$$= 1 - \frac{i}{2} \int \frac{d^4p}{(2\pi)^4} |J(p)|^2 \frac{1}{(p_0 - E_p + i\epsilon)(p_0 + E_p - i\epsilon)}$$

$$= 1 - \frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \frac{|J(p)|^2}{2E_p} 2\pi i = 1 + \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \frac{|J(p)|^2}{2E_p}$$

$$= 1 - \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \frac{|J(p)|^2}{2E_p}$$

$$= 1 - \frac{\lambda}{2}$$

$$\begin{aligned} & \int J(x) J(y) \overline{\phi(x) \phi(y)} \\ &= \lambda \\ &= \int \frac{d^3p}{(2\pi)^3} \frac{|J(p)|^2}{2E_p} \end{aligned}$$

$$p(0) = \left| 1 - \frac{\lambda^2}{2} \right| = 1 - \lambda^2 + \mathcal{O}(\lambda^3)$$

5)

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à tous les ordres: $\frac{(-i)^{2n}}{2n!}$  $(2n-1)(2n-3) \dots 3 \cdot 1$

$$(-i)^n \lambda^n \frac{(2n-1)(2n-3) \dots 3 \cdot 1}{2n(2n-1)(2n-2) \dots 3 \cdot 2 \cdot 1}$$

$$(-i)^n \frac{\lambda^n}{2^n} \frac{1}{n!}$$

resonance: $e^{-\frac{\lambda^2}{2}}$

$$p(0) = \left| \langle 0 | e^{+i \int \phi^2} | 0 \rangle \right|^2 = e^{-2}$$

6) $\langle \vec{p} | T e^{+i \int \phi^2(x)} | 0 \rangle$

$$= \int d^4x e^{ipx} (\Box_x + m^2) \langle 0 | T \phi(x) \left(1 + i \int \phi^2(y) + \dots \right) | 0 \rangle$$

$$= - \int d^4x d^4y (m^2 - p^2) e^{ipx} J(y) \langle 0 | T \phi(x) \phi(y) | 0 \rangle$$

$$= - \int d^4x d^4y \int \frac{d^4q}{q} (m^2 - p^2) e^{ipx} \tilde{D}(y) \frac{i e^{-iq(x-y)}}{q^2 - m^2 + i\epsilon} \frac{d^4q}{(2\pi)^4}$$

$$= +i \int \frac{d^4q}{q} J(p)$$

$$\left\{ \int d^4x d^4y e^{ipx} (\Box_x + m^2) \phi(x) \phi(y) J(y) \right. \\ \left. = -i J(p) \right.$$

at higher order

$$\langle \vec{p} | T e^{i \int \phi(x) dx} | 0 \rangle$$

$$= \frac{(i)^{2n+1}}{(2n+1)!} i \int d^4 x e^{i p x} (D_n(x)) \langle 0 | T \phi(x) \phi(x_1) \dots \phi(x_{2n+1}) | 0 \rangle$$

$$\int d^4 x_1 \dots d^4 x_{2n+1} J(x_1) \dots J(x_{2n+1}).$$

$$= \frac{(-1)^{n+1}}{(2n+1)!} \int d^4 x d^4 x_1 \dots d^4 x_{2n+1} (m^2 - p^2) D(x-x_1) D(x_1-x_2) \dots$$

$$D(x_{2n}-x_{2n+1})$$

$$\times (2n+1)(2n-1) \dots B.1 J(x_1) \dots J(x_{2n+1})$$

$$= \frac{(-1)^{n+1}}{2^n n!} (-i) J(p) \lambda^n$$

$$= i (-1)^n \frac{\lambda^n}{2^n n!} J(p)$$

$$\text{resums to } i J(p) e^{-\frac{\lambda}{2}}$$

Probe for having 1 particle:

$$P(1) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} |b(p)|^2 e^{-\lambda} = \lambda e^{-\lambda}.$$

n particles

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$$\langle p_1 \dots p_n | \overline{\Phi} e^{i \int d\omega_1 \phi(\omega_1) \bar{J}(\omega_1)} | 0 \rangle$$

$$\frac{(i)^{2m+n}}{(2m+n)!} (i)^n \int d\omega_1 \dots d\omega_n d\omega_{2m+1} \dots d\omega_{2m+n} (\square_{\omega_1+n^2}) \dots (\square_{\omega_{2m+n}})$$

$$\langle 0 | \bar{J}(\omega_1) \dots \bar{J}(\omega_n) \phi(\omega_{2m+1}) \dots \phi(\omega_{2m+n}) | 0 \rangle$$

$$= \frac{(-1)^{m+n}}{(2m+n)!} (-i \bar{J}(p_1)) \dots (-i \bar{J}(p_n)) \lambda^m \frac{(2m+n)(2m+n-1) \dots (2m+1)}{(2m-1)(2m-3) \dots 3 \cdot 1}$$

$$= i^n \bar{J}(p_1) \dots \bar{J}(p_n) \frac{(-\lambda)^m}{2^m m!}$$

resumes to $i \bar{J}(p_1) \dots \bar{J}(p_n) e^{-\frac{\lambda}{2}}$

Proof of creating n particles: integrate over momenta

but beware: $|p, p\rangle$ is the same state as $|p, p\rangle$

If we integrate $\int d^3 p_1 \dots d^3 p_n$, we count several times

the same state $\rightarrow$ divide by $n!$

$$P(n) = \frac{1}{n!} \int \frac{d^3 p_1}{(2\pi)^3} \dots \frac{d^3 p_n}{(2\pi)^3} \frac{1}{2\epsilon_1 \dots 2\epsilon_n} |\bar{J}(p_1)|^2 \dots |\bar{J}(p_n)|^2 e^{-\lambda}$$

$$= \frac{\lambda^n}{n!} e^{-\lambda}$$

This is a Poisson dist. tail.

$$\sum_n P_n = 1 \text{ as it should}$$

$$\langle N \rangle = \sum_{n=0}^{\infty} \frac{n \lambda^n}{n!} e^{-\lambda}$$

$$= \sum_{n=1}^{\infty} \frac{\lambda^n}{(n-1)!} e^{-\lambda} = \lambda \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} = \lambda$$

this is what we found before

$$\langle N(N-1) \rangle = \sum_{n=2}^{\infty} \frac{\lambda^n}{(n-2)!} e^{-\lambda} = \lambda^2$$

$$= \langle N^2 \rangle - \langle N \rangle = \langle N^2 \rangle - \lambda$$

$$\langle N^2 \rangle = \lambda^2 + \lambda$$

$$\Delta N = \lambda$$