

$$1) S = \int \bar{\psi} (i \not{\partial} - m) \psi$$

$$\delta S = \int -i \bar{\psi} (\not{\partial} - m) \delta \psi + \bar{\psi} (\not{\partial} - m) (i \delta \psi)$$

$$= \int -2i \bar{\psi} \not{\partial} \delta \psi$$

$$\delta S = -2i \bar{\psi} \not{\partial} \delta \psi$$

$$\begin{aligned} \partial_\mu \delta S &= -2i \bar{\psi} \not{\partial} \delta \psi = -2i \bar{\psi} \not{\partial} \delta \psi \\ &= i \bar{\psi} \not{\partial} \psi - i \bar{\psi} \not{\partial} \psi \end{aligned}$$

2)

$$\delta \psi = i \not{\partial} \psi$$

$$\delta \psi^\dagger = -i \psi^\dagger \not{\partial} = -i \psi^\dagger \not{\partial}$$

$$\delta \bar{\psi} = -i \psi^\dagger \not{\partial} = i \psi^\dagger \not{\partial} = i \bar{\psi} \not{\partial}$$

$$\begin{aligned} \delta S &= \int i \bar{\psi} \not{\partial} (i \not{\partial} \psi) + \bar{\psi} \not{\partial} (i \not{\partial} \psi) \\ &= \int -2i \bar{\psi} \not{\partial} \not{\partial} \psi = -2i \bar{\psi} \not{\partial} \not{\partial} \psi \end{aligned}$$

$$\delta S = -2i \bar{\psi} \not{\partial} \not{\partial} \psi$$

$$\partial_\mu \delta S = -2i \bar{\psi} \not{\partial} \not{\partial} \psi = -2i \bar{\psi} \not{\partial} \not{\partial} \psi$$

$$= -2i \bar{\psi} \not{\partial} \not{\partial} \psi + 2i \bar{\psi} \not{\partial} \not{\partial} \psi = -2i \bar{\psi} \not{\partial} \not{\partial} \psi$$

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$$3) S = \int \bar{\psi} (i \not{\partial} - m) \psi$$

$$\delta S = \delta \psi = \epsilon^\mu \partial_\mu \psi$$

$$\begin{aligned} \delta S &= \int \epsilon^\mu \partial_\mu \bar{\psi} + \bar{\psi} i \partial_\nu \epsilon^\mu \gamma^\nu \partial_\mu \psi \\ &= \int \partial_\mu \epsilon_\nu (-\not{\partial}^\nu \bar{\psi} + \bar{\psi} i \gamma^\nu \not{\partial}^\mu \psi) \end{aligned}$$

$$T^{\mu\nu} = -\not{\partial}^{\mu\nu} \bar{\psi} + i \bar{\psi} \gamma^\mu \not{\partial}^\nu \psi$$

$$\begin{aligned} \text{Eq of motion} \quad (i \not{\partial} - m) \psi &= 0 & \not{\partial} \psi &= -i m \psi \\ -i \partial_\nu \bar{\psi} \gamma^\nu - m \bar{\psi} &= 0 & \bar{\psi} \not{\partial} &= i m \bar{\psi} \end{aligned}$$

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= i \partial_\mu \bar{\psi} \gamma^\mu \not{\partial}^\nu \psi + i \bar{\psi} \gamma^\mu \partial_\mu \not{\partial}^\nu \psi \\ &= -m \bar{\psi} \not{\partial}^\nu \psi + i \bar{\psi} m \not{\partial}^\nu \psi = 0 \quad \text{OK} \end{aligned}$$

$$T^{10} \neq T^{01} !!!$$

$$S\psi = \omega_{\mu\nu} \left(x^\mu \partial^\nu - \frac{i}{2} S^{\mu\nu} \right) \psi$$

$$S\bar{\psi} = \omega_{\mu\nu} \left(x^\mu \partial^\nu \bar{\psi} + \frac{i}{2} \bar{\psi} \gamma^0 (S^{\mu\nu})^\dagger \gamma^0 \right)$$

$$\begin{aligned} S^{10} &= \frac{i}{4} (\gamma^1 \gamma^0 - \gamma^0 \gamma^1) \\ \gamma^0 (S^{10})^\dagger \gamma^0 &= \frac{i}{4} \gamma^0 (\gamma^0 \gamma^1 - \gamma^1 \gamma^0) \gamma^0 \\ &= -\frac{i}{4} (\gamma^0 \gamma^1 - \gamma^1 \gamma^0) = \frac{i}{4} (\gamma^1 \gamma^0 - \gamma^0 \gamma^1) = S^{10} \end{aligned}$$

$$S\bar{\psi} = \omega_{\mu\nu} \left(x^\mu \partial^\nu \bar{\psi} + \frac{i}{2} \bar{\psi} S^{\mu\nu} \right).$$

$$\begin{aligned} \delta S &= \int d^4x \left[\omega_{\mu\nu} \left(x^\mu \partial^\nu \bar{\psi} + \frac{i}{2} \bar{\psi} S^{\mu\nu} \right) \psi (i \not{\partial} - m) \psi \right. \\ &\quad \left. + \bar{\psi} (i \not{\partial} - m) \omega_{\mu\nu} \left(x^\mu \partial^\nu - \frac{i}{2} S^{\mu\nu} \right) \psi \right] \end{aligned}$$

$$\begin{aligned} &= \int d^4x \left[\omega_{\mu\nu} x^\mu \partial^\nu \mathcal{L} + \omega_{\mu\nu} \frac{i}{2} \bar{\psi} [S^{\mu\nu}, (i \not{\partial} - m)] \psi \right. \\ &\quad \left. + i \bar{\psi} \gamma^\rho \partial_\rho \omega_{\mu\nu} \left(x^\mu \partial^\nu - \frac{i}{2} S^{\mu\nu} \right) \psi \right. \\ &\quad \left. + i \bar{\psi} \gamma^\rho \omega_{\mu\nu} \partial^\nu \psi \right] \end{aligned}$$

$$\begin{aligned} &= \int d^4x \left[-\partial^\nu \omega_{\mu\nu} x^\mu \mathcal{L} + i \bar{\psi} \gamma^\rho \partial_\rho \omega_{\mu\nu} \left(x^\mu \partial^\nu - \frac{i}{2} S^{\mu\nu} \right) \psi \right. \\ &\quad \left. - \frac{i}{2} \omega_{\mu\nu} \bar{\psi} [S^{\mu\nu}, \gamma^\rho] \partial_\rho \psi + i \bar{\psi} \gamma^\rho \omega_{\mu\nu} \partial^\nu \psi \right] \end{aligned}$$

definition R.g

$$-\frac{i}{2} \omega_{\mu\nu} \bar{\psi} (\gamma^\mu \partial^\nu \psi - \gamma^\nu \partial^\mu \psi) + i \omega_{\mu\nu} \bar{\psi} \gamma^\mu \partial^\nu \psi = 0$$

$$SS = \int \partial_\rho \omega_{\mu\nu} \left(-\frac{1}{2} \eta^{\nu\rho} x^\mu \psi + \frac{1}{2} \eta^{\mu\rho} x^\nu \psi \right. \\ \left. + \frac{i}{2} \bar{\psi} \gamma^\rho (x^\mu \partial_\nu - x^\nu \partial_\mu) \psi + \frac{1}{2} \bar{\psi} \gamma^\rho S^{\mu\nu} \psi \right) \quad (8)$$

$$M^{\mu\nu} = x^\mu T^{\nu\rho} - x^\nu T^{\mu\rho} + \bar{\psi} \gamma^\rho S^{\mu\nu} \psi$$

$$\partial_\rho M^{\mu\nu} = T^{\nu\rho} - T^{\mu\rho} + \bar{\psi} \overleftarrow{\gamma} S^{\mu\nu} + \bar{\psi} [\gamma^\rho S^{\mu\nu}] \partial_\rho \psi \\ = i \bar{\psi} \gamma^\mu \partial^\nu \psi - i \bar{\psi} \gamma^\nu \partial^\mu \psi + \bar{\psi} S^{\mu\nu} \not{\partial} \psi \quad \text{on the eq. of m.} \\ + i \bar{\psi} (\gamma^\nu \partial^\mu \psi - \gamma^\mu \partial^\nu \psi) = 0 \quad \text{OK}$$

$$M^{\alpha\beta} = x^\alpha T^{\beta\rho} - x^\beta T^{\alpha\rho} + \bar{\psi} \gamma^\rho S^{\alpha\beta} \psi$$

Propagator

$$1) \int D(k-y) T(k) d^d x$$

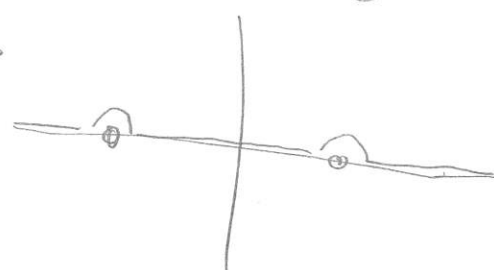
$$(D+m^2) \int D(k-y) T(k) d^d y = - \int i \delta(k-y) T(k) = -i T(k)$$

$$U(k) = i \int D(k-y) T(k) d^d y$$

$$2) \int d^d x e^{ip(x-y)} (D+m^2) D(x-y) = -i$$

$$(m^2 - p^2) \tilde{D}(p) = -i$$

$$\tilde{D}(p) = \frac{i}{p^2 - m^2}$$

$$D_R(x) = \int \frac{d^d p}{(2\pi)^d} \frac{i}{p^2 - m^2} e^{-ipx^0 + i\vec{p}\vec{x}}$$


if $x^0 < 0$ close the contour by above. No pole $G_R = 0$.

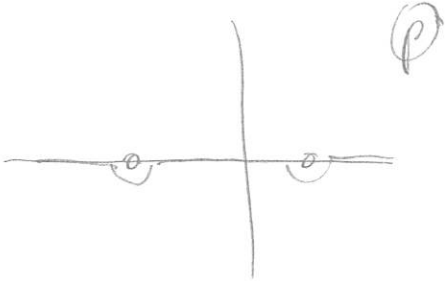
if $x^0 > 0$ close the contour by below.

$$\begin{aligned} \int \frac{dp^0}{2\pi} \frac{i}{p_0^2 - (p^2_{\vec{p}})} e^{-ip^0 x^0} &= -i \int \frac{dp^0}{2\pi} \frac{i e^{-ip^0 x^0}}{(p_0 + i\epsilon)(p_0 - i\epsilon)} \\ &= \frac{i(-2\pi i)}{2\pi} \left[\frac{e^{-iE_p x^0}}{2E_p} - \frac{e^{iE_p x^0}}{2E_p} \right] \end{aligned}$$

$$D_R(n) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{e^{-iE_p x^0 + i\vec{p}\vec{x}}}{2E_p} - \frac{e^{iE_p x^0 + i\vec{p}\vec{x}}}{2E_p}$$

$$= (\Delta(n) - \Delta(-n))$$

$$D_R(n) = \Theta(x^0) (\Delta(n) - \Delta(-n))$$

$$D_A(n) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2} e^{-ip^0 x^0 + i\vec{p}\vec{x}}$$


if $x^0 > 0$ close by below $\rightarrow$ no pole 0

if $x^0 < 0$ close by above

$$\int \frac{dp^0}{2\pi} \frac{i}{p^2 - m^2} e^{-ip^0 x^0} = \int \frac{d\omega}{2\pi} \frac{i}{(\omega + \sqrt{\omega^2 + m^2})(\omega - \sqrt{\omega^2 + m^2})} e^{-i\omega x^0}$$

$$= \frac{(2\pi i)i}{2\pi} \left(\frac{e^{-iE_p x^0}}{2E_p} - \frac{e^{iE_p x^0}}{2E_p} \right)$$

$$= - \frac{e^{-iE_p x^0}}{2E_p} + \frac{e^{iE_p x^0}}{2E_p}$$

$$D_A(n) = \Theta(-x^0) \int \frac{d^3\vec{p}}{(2\pi)^3} - \frac{e^{-iE_p x^0 + i\vec{p}\vec{x}}}{2E_p} + \frac{e^{iE_p x^0 + i\vec{p}\vec{x}}}{2E_p}$$

$$= \Theta(-x^0) (-\Delta(n) + \Delta(-n))$$

Feynman Propag:



$x^0 < 0$ close by above, keep the pole in $-p_0$

$$\int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m^2} e^{-i p^0 x^0} = \frac{i(m i)}{-2E_p 2\pi} e^{i E_p x^0} = \frac{e^{i E_p x^0}}{2E_p}$$

$$\mathcal{D}_F = \Delta(-x)$$

$x^0 > 0$ close by below, keep pole in $+p_0$

$$\int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m^2} e^{-i p^0 x^0} = \frac{i(-2\pi i)}{2E_p 2\pi} e^{-i E_p x^0} = \frac{e^{-i E_p x^0}}{2E_p}$$

$$\mathcal{D}_F = \Delta(x)$$

$$\mathcal{D}_F = \Theta(x^0) \Delta(x) + \Theta(-x^0) \Delta(-x)$$

$$\begin{aligned} \S \quad \mathcal{D}_F - \mathcal{D}_R &= \Theta(x^0) \Delta(x) + \Theta(-x^0) \Delta(-x) - \Theta(x^0) (\Delta(x) - \Delta(-x)) \\ &= (\Theta(x^0) + \Theta(-x^0)) \Delta(-x) \\ &= \Delta(-x) \end{aligned}$$

$$\begin{aligned} \mathcal{D}_F - \mathcal{D}_A &= \Theta(x^0) \Delta(x) + \Theta(-x^0) \Delta(-x) - \Theta(-x^0) (-\Delta(x) + \Delta(-x)) \\ &= (\Theta(x^0) + \Theta(-x^0)) \Delta(x) \\ &= \Delta(x) \end{aligned}$$

$$(\Box_{+n^2}) \Delta(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{-i(E_p t - \vec{p} \cdot \vec{x})}$$

$$= \int \frac{d^4 p}{(2\pi)^4} \frac{\Theta(p^0)}{1} e^{-i p x} \delta(p^2 - m^2)$$

$$(\Box_{+n^2}) \Delta = \int \frac{d^4 p}{(2\pi)^4} \frac{\Theta(p^0)}{1} (-p^2 + m^2) \delta(p^2 - m^2)$$

$= 0$

$$\begin{aligned} \S \quad T(\phi(x) \phi(y)) &= \phi_+(x) \phi_-(y) \Theta(x^0 - y^0) + \phi_-(y) \phi_+(x) \Theta(y^0 - x^0) \\ &= (\phi_+(x) + \phi_-(x)) (\phi_+(y) + \phi_-(y)) \Theta(x^0 - y^0) \\ &\quad + (\phi_+(y) + \phi_-(y)) (\phi_+(x) + \phi_-(x)) \Theta(y^0 - x^0) \end{aligned}$$

$$\begin{aligned} \circ \quad \phi(x) \phi(y) &= \phi_+(x) \phi_+(y) + \phi_-(x) \phi_-(y) + \phi_+(x) \phi_-(y) \\ &\quad + \phi_-(y) \phi_+(x) \end{aligned}$$

$$\begin{aligned} T(\phi(x) \phi(y)) - \circ \phi(x) \phi(y) &= \phi_+(x) \phi_-(y) \Theta(x^0 - y^0) + \phi_+(y) \phi_-(x) \Theta(y^0 - x^0) \\ &\quad - \phi_-(x) \phi_+(y) \Theta(y^0 - x^0) - \phi_-(y) \phi_+(x) \Theta(x^0 - y^0) \\ &= [\phi_+(x), \phi_-(y)] \Theta(x^0 - y^0) + [\phi_+(y), \phi_-(x)] \Theta(y^0 - x^0) \end{aligned}$$

First commutator:

$$\begin{aligned} [\phi_+(x), \phi_-(y)] &= \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \frac{1}{\sqrt{2E_p} \sqrt{2E_q}} [a_p e^{-i p x}, a_q^\dagger e^{i q y}] \\ &= \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \frac{1}{\sqrt{2E_p} \sqrt{2E_q}} e^{-i(p x - q y)} (2\pi)^3 \delta(p - q) \end{aligned}$$

$$= \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip(x-z)} = \Delta(x-z)$$

(S/D)

$$\begin{aligned} T(\phi(x)\phi(y)) - \phi(x)\phi(y) &= \theta(x^0 - y^0) \Delta(x-z) + \theta(y^0 - x^0) \Delta(y-x) \\ &= D_F \end{aligned}$$