

$$\textcircled{1} \quad \gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3$$

$$\begin{aligned} \gamma^5 \gamma^0 &= i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^0 \\ &= -i \gamma^0 \gamma^1 \gamma^2 \gamma^0 \gamma^3 \\ &= +i \gamma^0 \gamma^1 \gamma^0 \gamma^2 \gamma^3 \\ &= -i \gamma^0 \gamma^2 \gamma^1 \gamma^2 \gamma^3 = -\gamma^0 \gamma^1 \end{aligned}$$

$$\begin{aligned} \gamma^5 \gamma^1 &= i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^1 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^1 \\ &= -i \gamma^1 \gamma^0 \gamma^1 \gamma^2 \gamma^3 \\ &= -\gamma^1 \gamma^5 \end{aligned}$$

$$\begin{aligned} (\gamma^5)^2 &= -\gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^0 \gamma^1 \gamma^2 \gamma^3 = (\gamma^0)^2 \gamma^1 \gamma^2 \gamma^3 \gamma^1 \gamma^2 \gamma^3 \\ &= -(\gamma^0)^2 (\gamma^1)^2 (\gamma^2)^2 (\gamma^3)^2 = 1 \end{aligned}$$

$$\begin{aligned} \text{2} \quad \gamma^\mu \gamma^\nu &= \gamma^\mu \gamma^5 \gamma^5 \gamma^\nu = -\gamma^\mu \gamma^5 \gamma^\nu \gamma^5 \\ &= -\gamma^\mu \gamma^\nu (\gamma^5)^2 = -\gamma^\mu \gamma^\nu \end{aligned}$$

pour 2k+1 matrice

$$\begin{aligned} \gamma^\mu \gamma^{\nu_1} \dots \gamma^{\nu_{2k+1}} &= \gamma^\mu \gamma^{\nu_1} \dots \gamma^{\nu_{2k}} \gamma^5 \gamma^{\nu_{2k+1}} \\ &= -\gamma^\mu \gamma^5 \gamma^{\nu_1} \dots \gamma^{\nu_{2k}} \gamma^{\nu_{2k+1}} \\ &= -\gamma^\mu \gamma^{\nu_1} \dots \gamma^{\nu_{2k+1}} \end{aligned}$$

$$\begin{aligned} \tau_{\mu} \gamma^{\mu} \gamma^{\nu} &= -\tau_{\mu} \gamma^{\nu} \gamma^{\mu} + 2\eta^{\mu\nu} \tau_{\mu} \mathbb{1} \\ &= -\tau_{\mu} \gamma^{\mu} \gamma^{\nu} + 8\eta^{\mu\nu} \end{aligned}$$

$$\tau_{\mu} \gamma^{\mu} \gamma^{\nu} = 4\eta^{\mu\nu}$$

$$\begin{aligned} \tau_{\mu} \gamma^{\mu} \gamma^{\nu} \gamma^{\rho} \gamma^{\sigma} &= -\tau_{\mu} \gamma^{\nu} \gamma^{\mu} \gamma^{\rho} \gamma^{\sigma} + 2\eta^{\mu\nu} \tau_{\mu} \gamma^{\rho} \gamma^{\sigma} \\ &= \tau_{\mu} \gamma^{\nu} \gamma^{\rho} \gamma^{\mu} \gamma^{\sigma} - 2\eta^{\mu\rho} \tau_{\mu} \gamma^{\nu} \gamma^{\sigma} + 8\eta^{\mu\nu} \eta^{\rho\sigma} \\ &= -\tau_{\mu} \gamma^{\nu} \gamma^{\rho} \gamma^{\sigma} \gamma^{\mu} + 2\eta^{\mu\rho} \tau_{\mu} \gamma^{\nu} \gamma^{\sigma} - 8\eta^{\mu\rho} \eta^{\nu\sigma} \\ 2\tau_{\mu} \gamma^{\mu} \gamma^{\nu} \gamma^{\rho} \gamma^{\sigma} &= 4(\eta^{\mu\nu} \eta^{\rho\sigma} - \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho}) \end{aligned}$$

$$3) \quad (\overline{\psi} \psi)^{\dagger} = \psi^{\dagger} (\gamma^0)^{\dagger} \psi = \overline{\psi} \psi. \text{ ok.}$$

$$\begin{aligned} (\overline{\psi} \gamma^{\mu} \psi)^{\dagger} &= \psi^{\dagger} (\gamma^{\mu})^{\dagger} \gamma^0 \psi = \psi^{\dagger} \gamma^0 \gamma^{\mu} \gamma^0 \psi \\ &= \overline{\psi} \gamma^{\mu} \psi. \text{ ok.} \end{aligned}$$

$$(\overline{\psi} \gamma^{\mu} \gamma^{\nu} \psi)^{\dagger} = \psi^{\dagger} (\gamma^{\nu})^{\dagger} (\gamma^{\mu})^{\dagger} \gamma^0 \psi.$$

$$\begin{aligned} \gamma^{\nu\dagger} &= \gamma^{\nu} \gamma^0 \gamma^{\nu} = -\gamma^{\nu} \gamma^0 \gamma^{\nu} = -\gamma^{\nu} \gamma^0 \gamma^{\nu} \\ &= \gamma^{\nu} \gamma^0 \gamma^{\nu} \end{aligned}$$

$$\hookrightarrow (\overline{\psi} \gamma^{\mu} \gamma^{\nu} \psi)^{\dagger} = \psi^{\dagger} \gamma^{\nu} \gamma^0 \gamma^{\mu} \psi = \overline{\psi} \gamma^{\mu} \gamma^{\nu} \psi \text{ ok}$$

unde Noeth

$$\psi \rightarrow \psi + \frac{i}{2} \omega_{\mu\nu} S^{\mu\nu} \psi$$

$$S^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu]$$

$$\begin{aligned} (S^{\mu\nu})^\dagger &= -\frac{i}{4} [\gamma^{\nu\dagger}, \gamma^{\mu\dagger}] = -\frac{i}{4} \gamma^0 [\gamma^\nu, \gamma^\mu] \gamma^0 \\ &= \frac{i}{4} \gamma^0 S^{\mu\nu} \gamma^0 \end{aligned}$$

~~$$\delta \bar{\psi} \psi = \bar{\psi} \gamma^0 S^{\mu\nu} \gamma^0 \psi + \psi^\dagger \gamma^0 S^{\mu\nu} \gamma^0 \psi$$~~

$$\psi^\dagger \rightarrow \psi^\dagger + \frac{i}{2} \omega_{\mu\nu} \psi^\dagger (S^{\mu\nu})^\dagger$$

$$= \psi^\dagger + \frac{i}{2} \omega_{\mu\nu} \psi^\dagger \gamma^0 S^{\mu\nu} \gamma^0$$

$$\bar{\psi} \rightarrow \bar{\psi} + \frac{i}{2} \omega_{\mu\nu} \bar{\psi} S^{\mu\nu}$$

$$\delta \bar{\psi} \psi = \frac{i}{2} \omega_{\mu\nu} (\bar{\psi} S^{\mu\nu} \psi - \bar{\psi} S^{\mu\nu} \psi) = 0$$

$$\begin{aligned} \delta \bar{\psi} \gamma^S \psi &= \frac{i}{2} \omega_{\mu\nu} \bar{\psi} S^{\mu\nu} \gamma^S \psi - \frac{i}{2} \omega_{\mu\nu} \bar{\psi} \gamma^S S^{\mu\nu} \psi \\ &= 0 \end{aligned}$$

$$\delta \bar{\psi} \gamma^\mu \psi = \frac{i}{2} \omega_{\beta\delta} \bar{\psi} S^{\beta\delta} \gamma^\mu \psi - \frac{i}{2} \bar{\psi} S^{\beta\delta} \gamma^\mu \psi S^{\beta\delta} \quad (20)$$

$$= -\frac{i}{2} \omega_{\beta\delta} \bar{\psi} [\gamma^\mu, S^{\beta\delta}] \psi$$

$$[\gamma^\mu, S^{\beta\delta}] = \frac{i}{4} [\gamma^\mu, \gamma^\beta \gamma^\delta] = \frac{i}{4} [\gamma^\mu, \gamma^\beta] \gamma^\delta - \frac{i}{4} \gamma^\beta [\gamma^\mu, \gamma^\delta]$$

$$= \frac{i}{4} (\gamma^\mu \gamma^\beta \gamma^\delta - \gamma^\beta \gamma^\delta \gamma^\mu) - \frac{i}{4} (\gamma^\mu \gamma^\delta \gamma^\beta - \gamma^\delta \gamma^\beta \gamma^\mu)$$

$$= \frac{i}{4} (2\eta^{\mu\beta} \gamma^\delta - \gamma^\beta \gamma^\mu \gamma^\delta - \gamma^\delta \gamma^\mu \gamma^\beta + 2\eta^{\mu\delta} \gamma^\beta)$$

$$- \frac{i}{4} (\gamma^\beta \gamma^\mu \gamma^\delta - \gamma^\mu \gamma^\beta \gamma^\delta)$$

$$= \frac{i}{4} (\eta^{\mu\beta} \gamma^\delta - \gamma^\beta \gamma^\mu \gamma^\delta - \gamma^\delta \gamma^\mu \gamma^\beta + \eta^{\mu\delta} \gamma^\beta)$$

$$\delta \bar{\psi} \gamma^\mu \psi = \frac{1}{4} \omega_{\beta\delta} (\eta^{\mu\beta} \gamma^\delta - \gamma^\beta \gamma^\mu \gamma^\delta - \gamma^\delta \gamma^\mu \gamma^\beta + \eta^{\mu\delta} \gamma^\beta)$$

V_τ

$$= \frac{1}{4} (\omega^\tau_\sigma V^\sigma) \quad \text{OK!}$$

$$1) \pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}^*$$

$$[\phi(x), \pi_\phi(y)] = i \delta(x-y)$$

$$[\phi^*(x), \pi_{\phi^*}(y)] = i \delta(x-y)$$

$$\pi_{\phi^*} = \dot{\phi}$$

$$\begin{aligned} 2) Q &= \int d^3x \ i (\dot{\phi}^* \phi - \phi^* \dot{\phi}) \\ &= \int d^3x \ i (\pi_\phi \phi - \pi_{\phi^*} \phi^*) \end{aligned}$$

$$[Q, \phi(x)] = \int d^3y \ i \left[(\pi_\phi(y) \phi(y) - \pi_{\phi^*}(y) \phi^*(y)), \phi(x) \right]$$

$$= \phi(x)$$

$$[Q, \phi^*(x)] = -\phi^*(x)$$

$$[Q, \pi_\phi(x)] = -\pi_\phi(x)$$

$$[Q, \pi_{\phi^*}(x)] = \pi_{\phi^*}(x)$$

Transf. under phase

(4)

$$H = \int d^3x \, 2\dot{\phi}^* \dot{\phi} - (\dot{\phi}^* \dot{\phi} - \partial_i \phi^* \partial_i \phi - V(\phi^* \phi))$$

$$= \int d^3x \, \pi_{\phi^*} \pi_{\phi} + \partial_i \phi^* \partial_i \phi + V(\phi^* \phi)$$

$$= \int d^3x \, \pi_{\phi^*} \dot{\phi} + \pi_{\phi} \dot{\phi} - \mathcal{L} \quad \Rightarrow \text{Legendre transform}$$

$$[\phi, \phi(x)] = -i \pi_{\phi^*}(x) = -i \dot{\phi}^*$$

$$[H, \phi^*(x)] = -i \pi_{\phi}(x) = -i \dot{\phi}$$

$$[H, \pi_{\phi}(x)] = i (-\Delta \phi^* + \phi^* V'(\phi^* \phi)) = -i \ddot{\phi}^* = -i \dot{\pi}_{\phi^*}$$

$$[H, \pi_{\phi^*}(x)] = i (-\Delta \phi + \phi V'(\phi^* \phi)) = -i \ddot{\phi} = -i \dot{\pi}_{\phi^*}$$

$$P^i = \int d^3x \quad \dot{\phi}^* \partial_i^{i} \phi + \partial_i^{i} \phi^* \dot{\phi}$$
$$= - \int d^3x \quad \Pi_{\phi} \partial_i^{i} \phi + \partial_i^{i} \phi^* \Pi_{\phi^*}$$

$$[P^i, \phi(x)] = i \partial_i \phi$$

$$[P^i, \phi^*(x)] = i \partial_i \phi^*$$

$$J^{\mu\nu} = \int d^3x \quad J^{0\mu\nu} = \int d^3x \quad (x^{\mu} T^{0\nu} - x^{\nu} T^{0\mu})$$

$$J^{ij} = \int d^3x \quad (x^i T^{0j} - x^j T^{0i})$$

$$= - \int d^3x \quad (x^i (\dot{\phi}^* \partial_j \phi + \dot{\phi} \partial_j \phi^*)$$
$$- x^j (\dot{\phi}^* \partial_i \phi + \dot{\phi} \partial_i \phi^*))$$

$$J^{0i} = \int d^3x \quad (x^0 T^{0i} - x^i T^{00})$$
$$= \int d^3x \quad (-t (\Pi_{\phi} \partial_i \phi + \Pi_{\phi^*} \partial_i \phi^*)$$
$$- x^i (\Pi_{\phi} \Pi_{\phi} + \partial_i \phi^* \partial_i \phi + V(\phi^* \phi)))$$

This is Hermitian