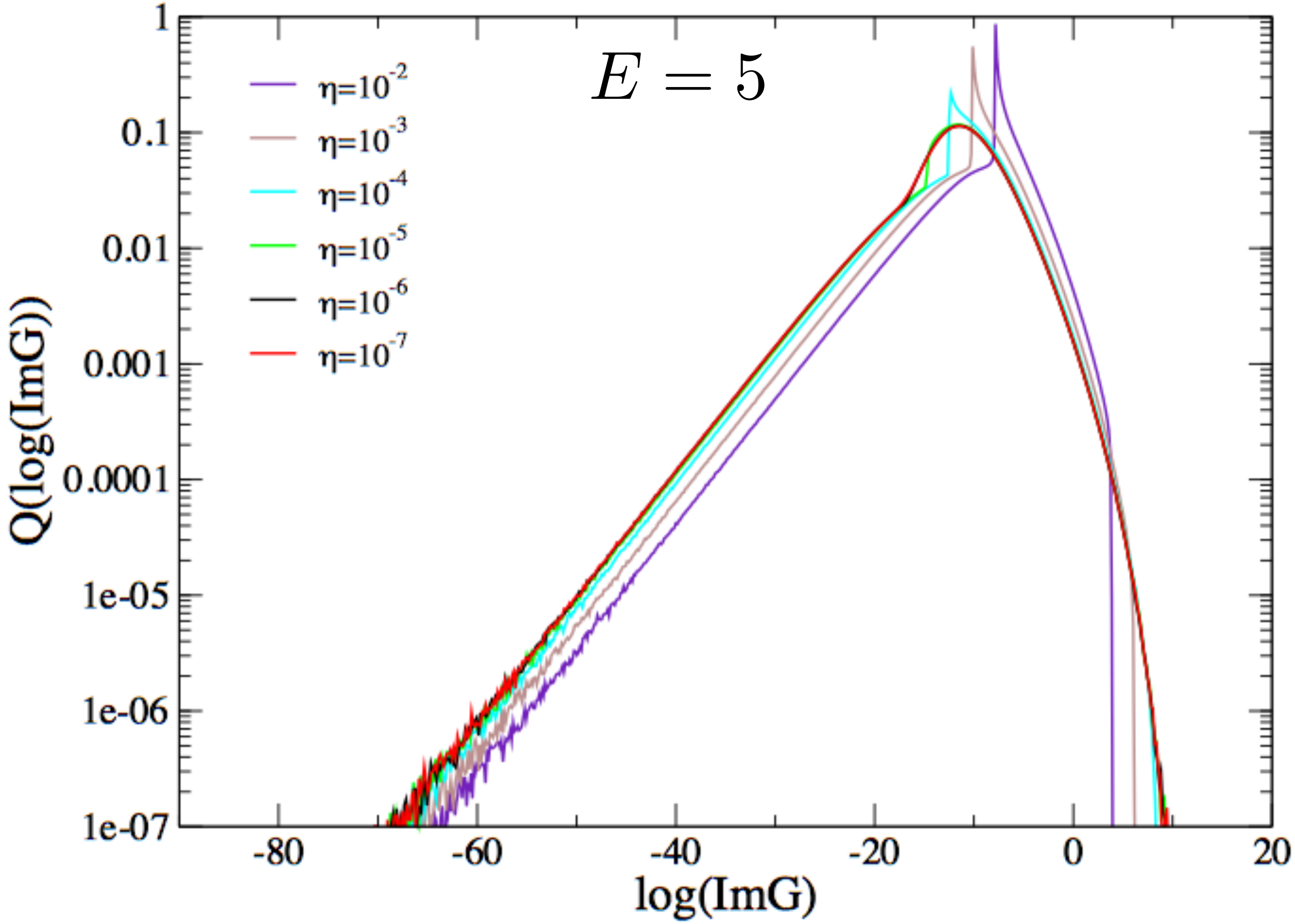
Self-consistent equation for $Q(G)$ (I)

★Extended phase

The distribution of $\mathrm{Im}G_{ii}(E - i\eta)$ is not singular in the limit $\eta \to 0^+$ ⟶ Stationary distribution



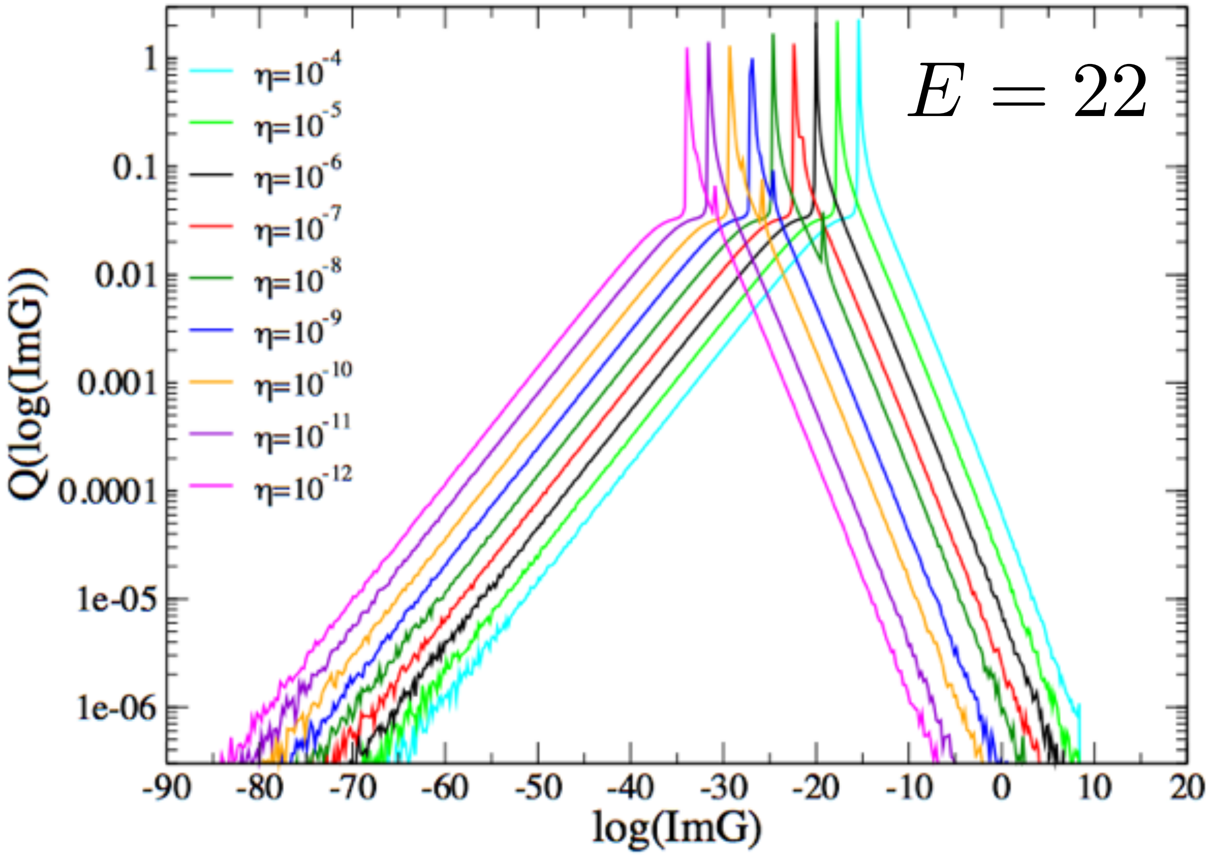
Self-consistent equation for $Q(G)$ (2)

★Localized phase

The distribution of $\mathrm{Im}G_{ii}(E-i\eta)$ is singular in the limit $\eta\to 0^+$

Almost all $\mathrm{Im}G_{ii}(E-i\eta)$ are of order η except very rare resonances

$$\tilde{Q}(\mathrm{Im}G)=\frac{1}{\eta}f\left(\frac{\mathrm{Im}G}{\eta}\right) \qquad \tilde{Q}(\mathrm{Im}G)=\frac{c\,\eta^{1-m}}{(\mathrm{Im}G)^{1+m}} \qquad m=\frac{1}{2}$$



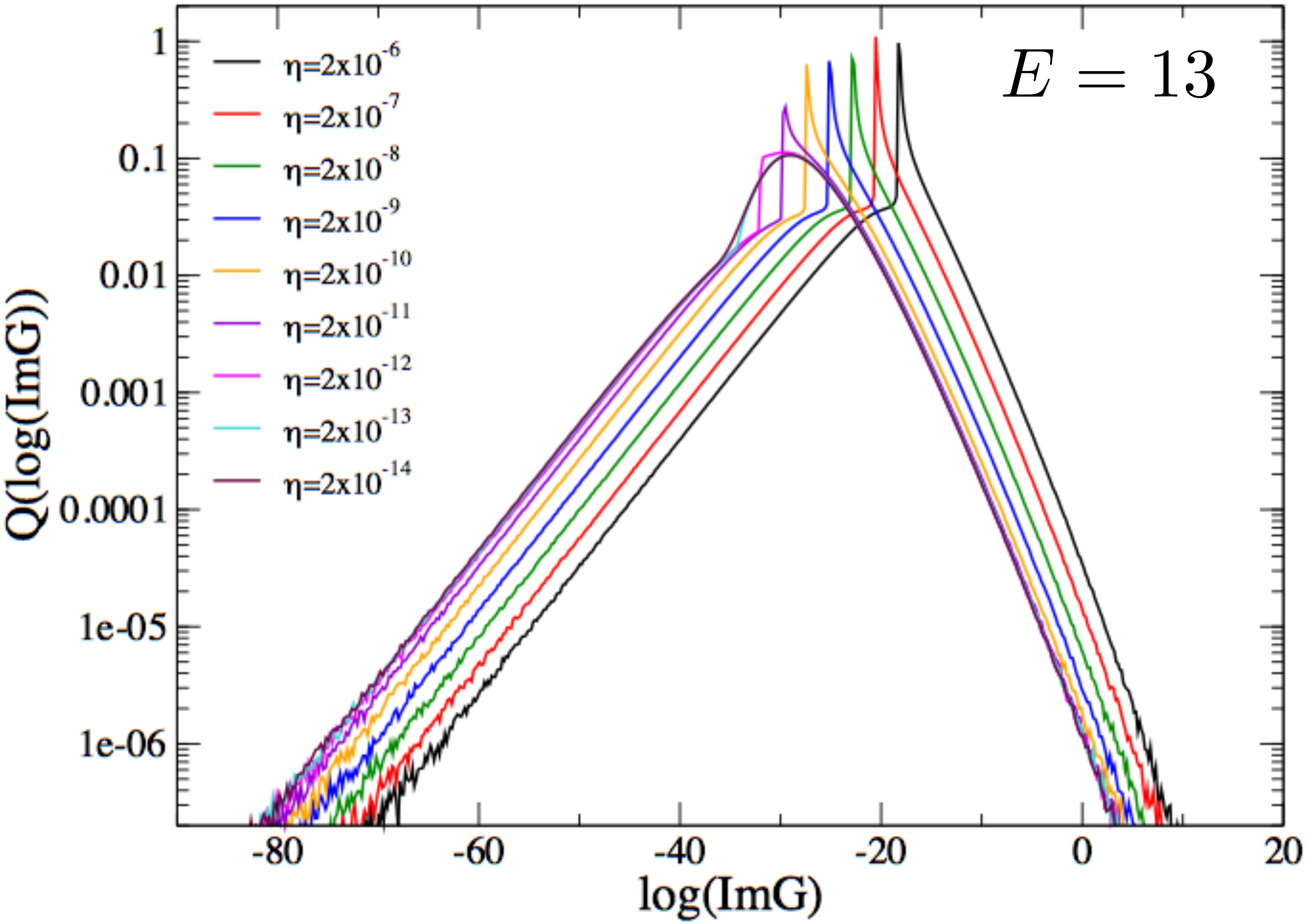
Self-consistent equation for $Q(G)$ (3)

★Crossover region

$\eta > \eta_c$ ⟶ "singular" power-law tails $1/2 < m \lesssim 1$

$\simeq$ non-ergodic system

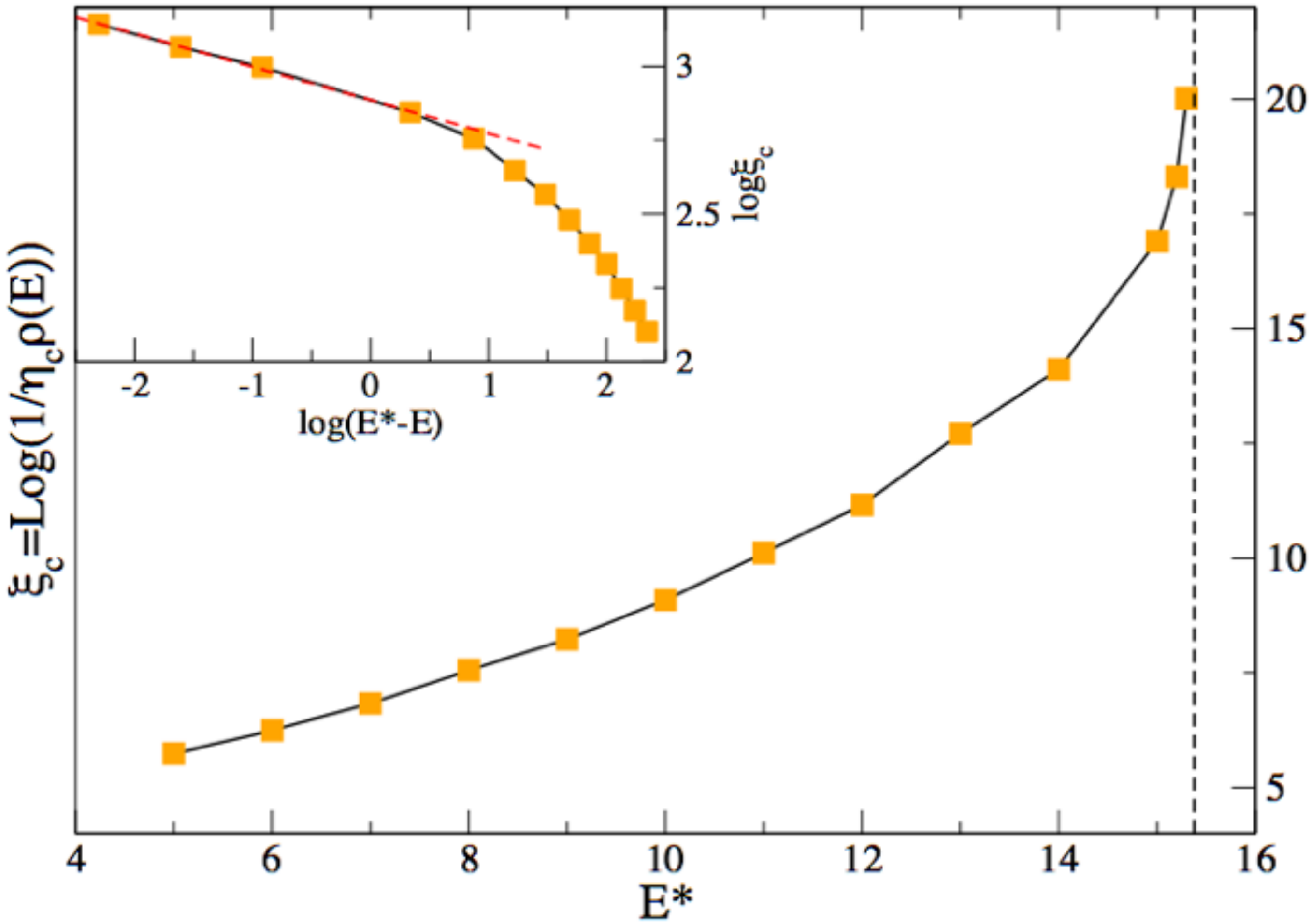
$\eta < \eta_c$ ⟶ stationary distribution $\rightarrow$ ergodicity



Characteristic crossover size N_c

$$\eta \gg \frac{1}{N\rho(E)} \Rightarrow N_c = \frac{1}{\eta_c \rho(E)}$$

$$\ln N_c \propto \ln N_m$$



N_c grows extremely fast and becomes huge close to $E^\star$

$$N_c \sim C e^{A(E^\star - E)^{-\delta}}$$

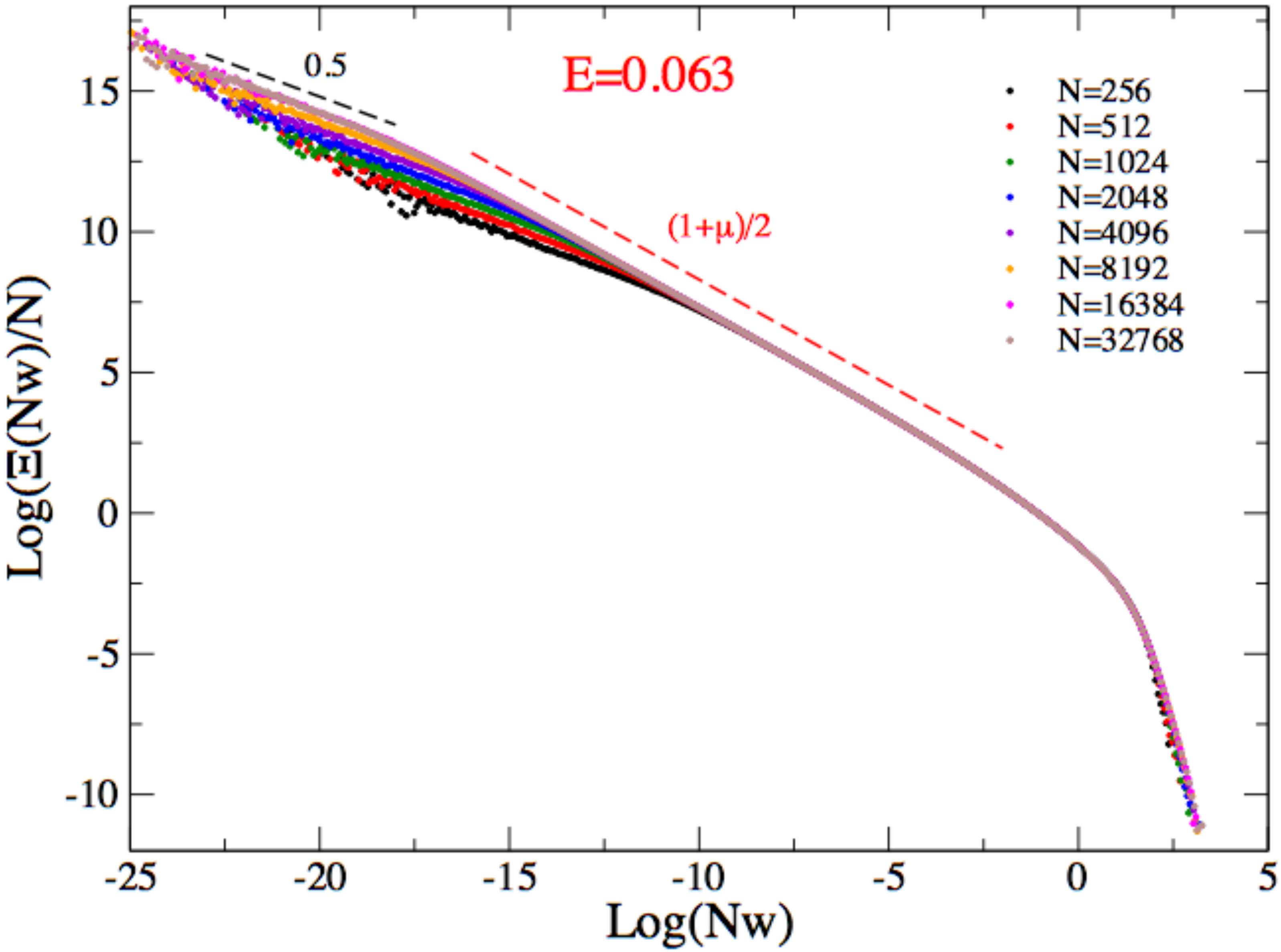
Mirlin & Fyodorov '92

$N > N_c$ → Ergodic system, GOE statistics

$N < N_c$ → Non-ergodic system, Poisson statistics

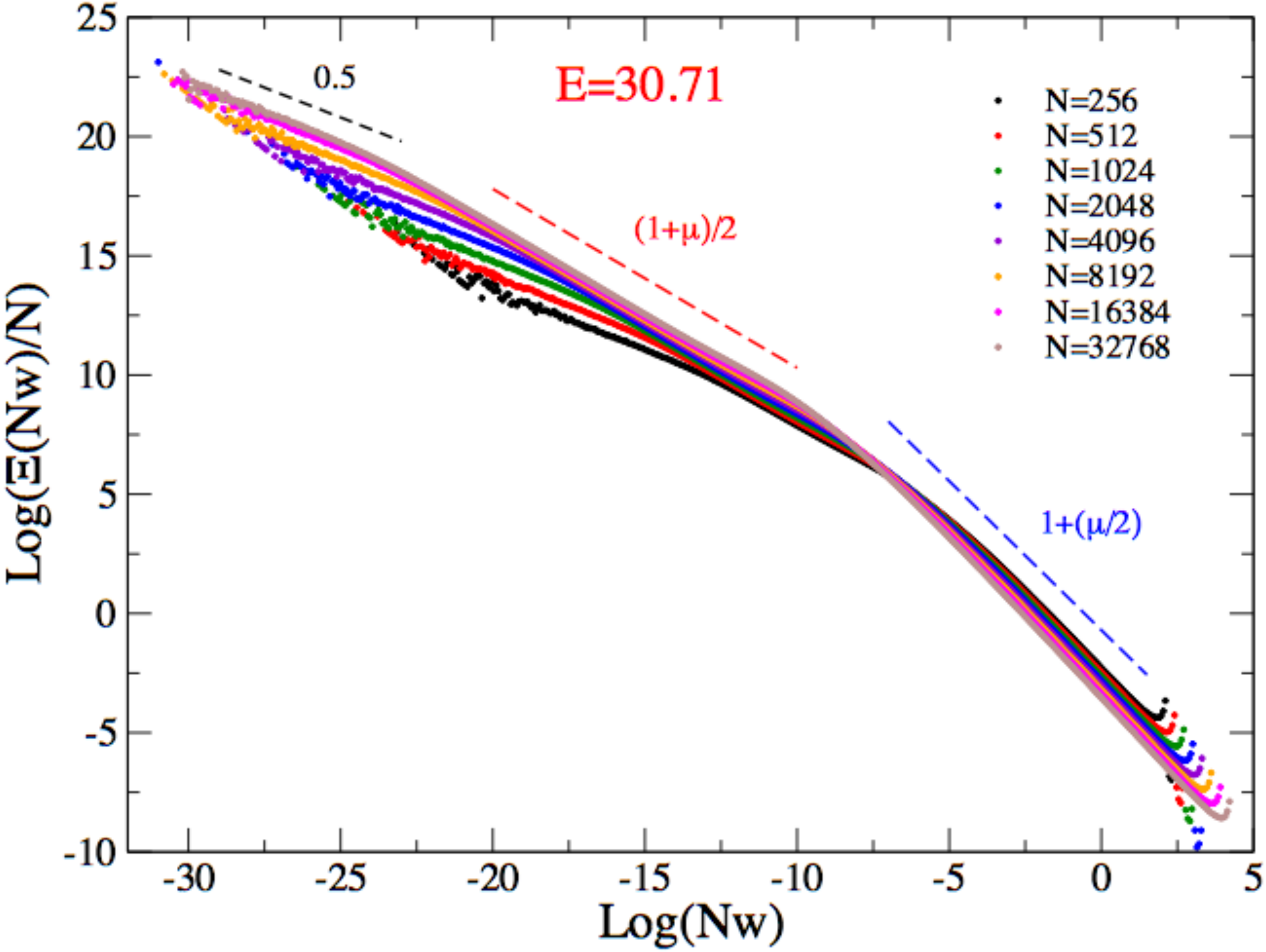
Wave-functions statistics $\mu \in (0,1)$ (I)

Extended regime



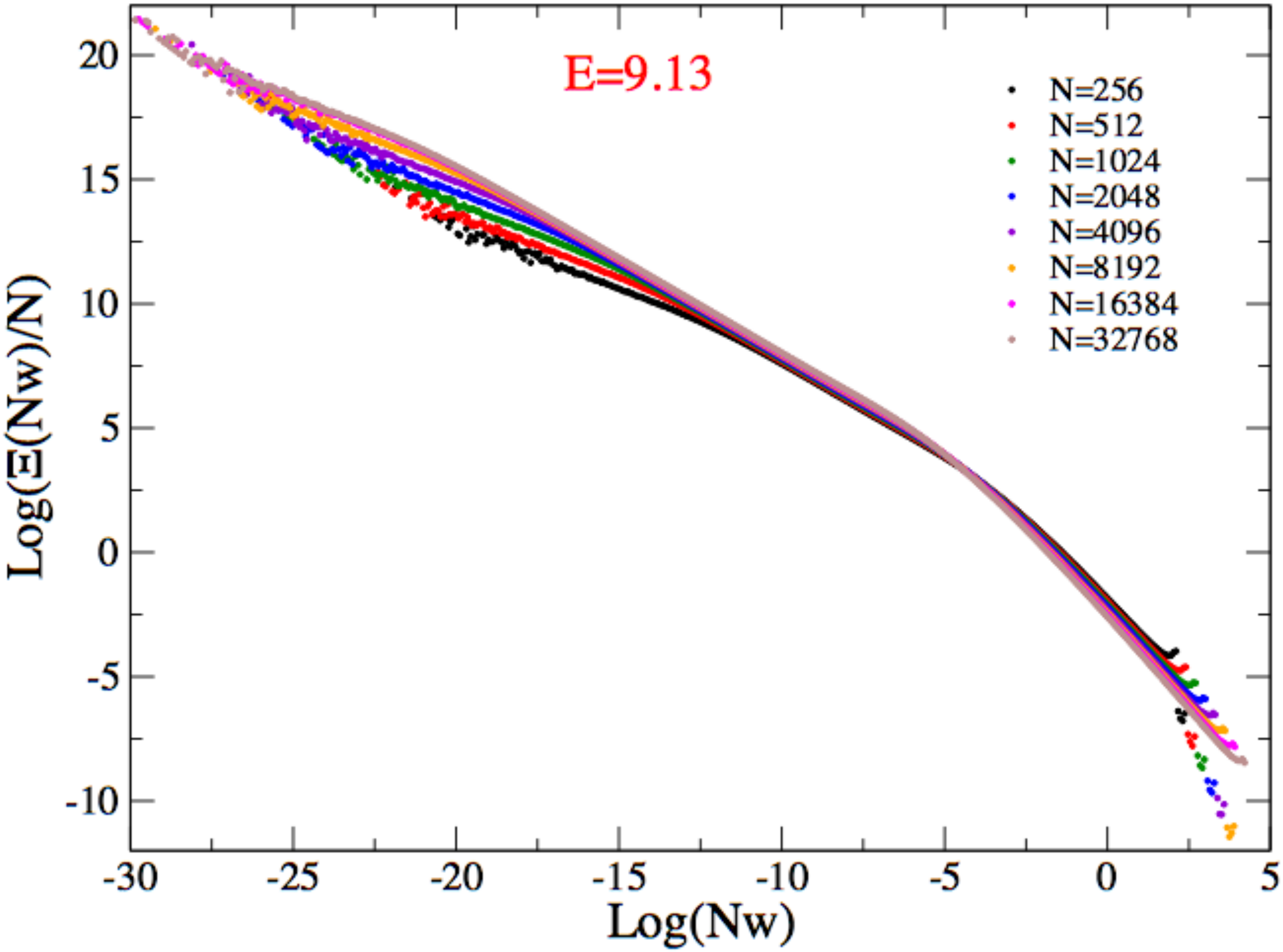
Wave-functions statistics $\mu \in (0,1)$ (2)

Localized regime



Wave-functions statistics $\mu \in (0,1)$ (3)

Crossover region



Multifractality

Moments of the wave-functions amplitudes (generalized IPRs)

$$\langle \Upsilon_q \rangle = \left\langle \sum_{i=1}^{N} [w_n(i)]^q \right\rangle \propto N^{-\tau(q)}$$

$$\begin{cases} \tau(q) = q - 1 & \text{ergodic states} \\ \tau(q) \neq q - 1 & \text{“multifractal” states} \end{cases}$$

Number $\mathcal{N}(\alpha)$ of sites having amplitude scaling as $N^{-\alpha}$ behaves as $\mathcal{N}(\alpha) \sim N^{f(\alpha)}$

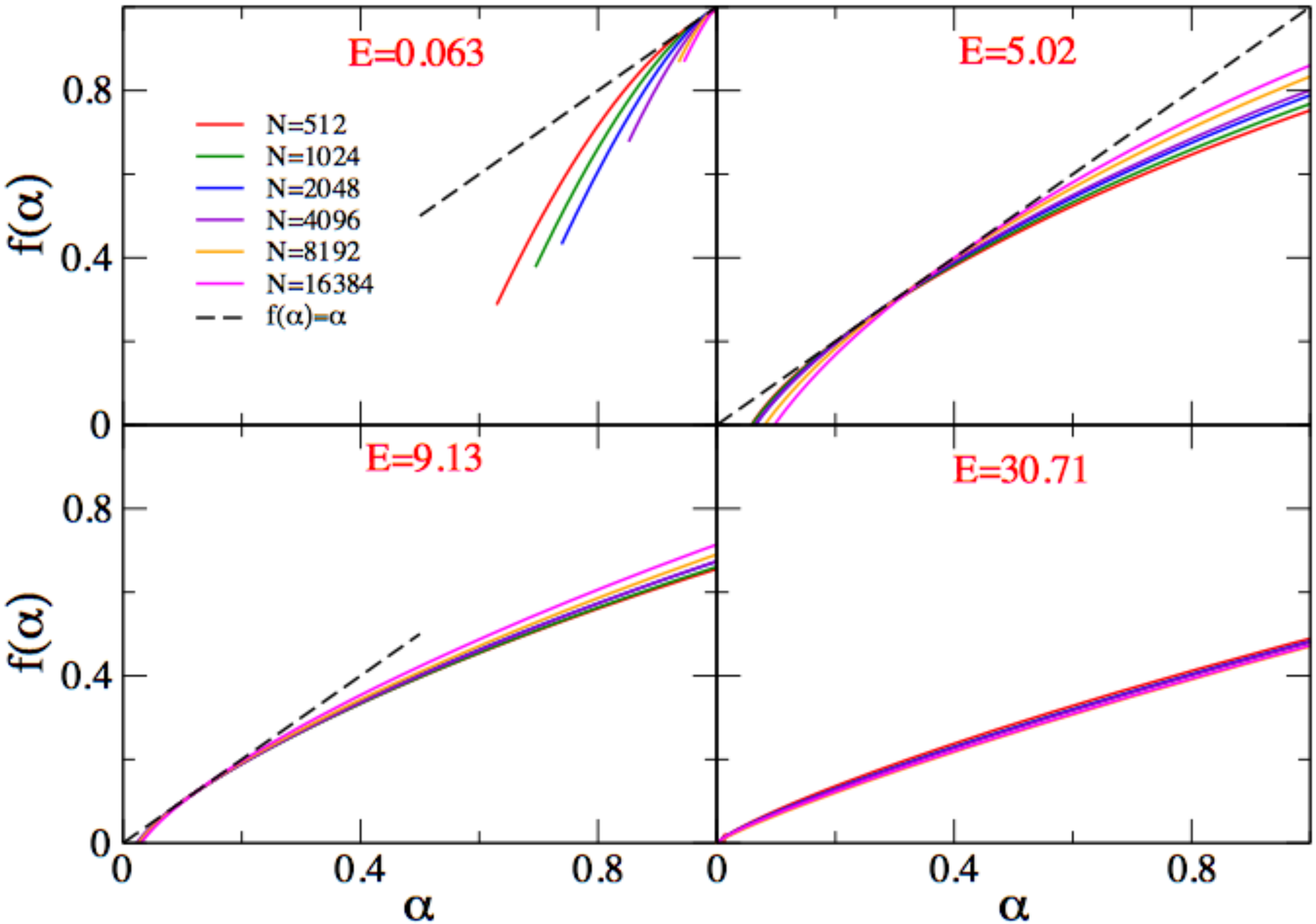
Saddle point computation of $\langle \Upsilon_q \rangle$ ⟶ Legendre transform formula

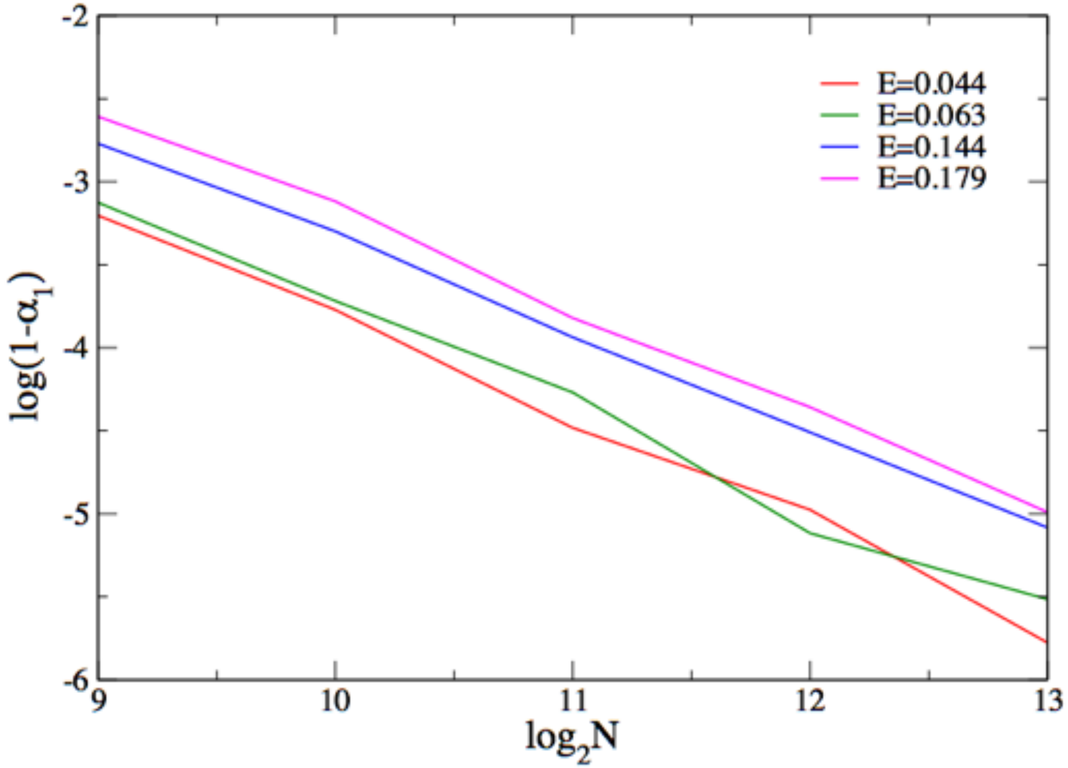
$$\begin{cases} \tau(q) = q\alpha - f(\alpha) \\ f'(\alpha) = q \end{cases}$$

Spectrum of fractal dimensions

$\mu \in (0,1)$

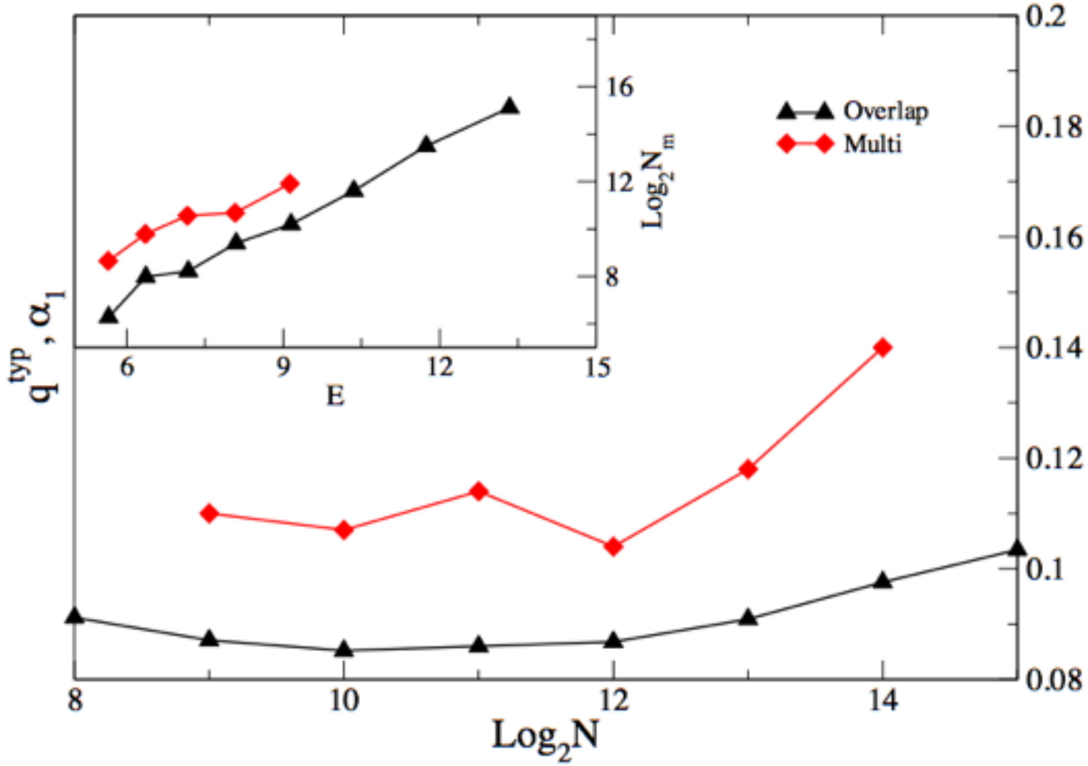
De Luca & al '14





$\alpha_1 \rightarrow 1$
at small energies

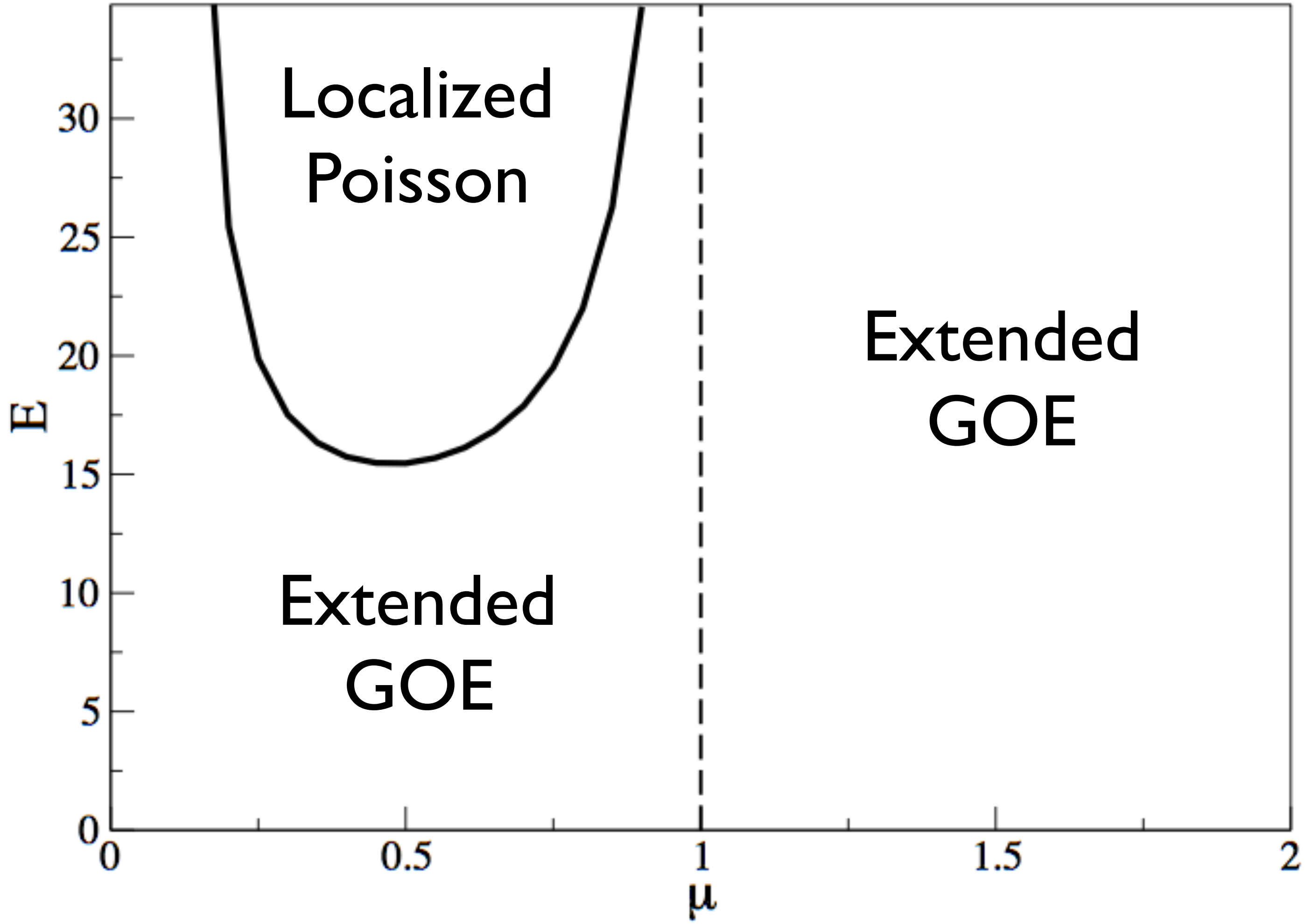
α_1 is non-monotonic
in the crossing region



Conclusions

★Phase diagram of Lévy Matrices

- Localization transition
- Level statistics
- Properties of the wave-functions



★Explain and clarify the numerical issues of previous works

★Crossover regime

Huge characteristic size diverging at the transition below which the system is non-ergodic