First steps towards the study of finite-dimensional non-perturbative fluctuations beyond the mean-field theory of glasses

Marco Tarzia

Thèse d'habilitation à diriger des recherches

Laboratoire de Physique Théorique de la Matière Condensée
Université Pierre et Marie Curie

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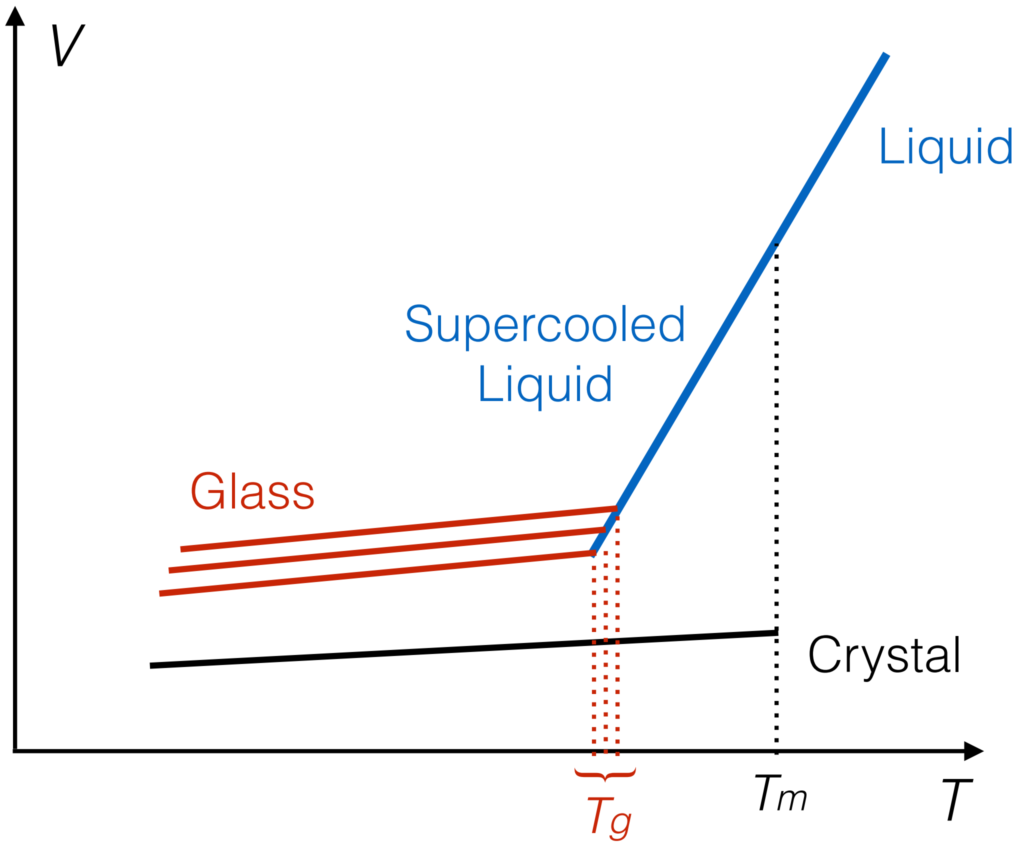
The glass transition

"The deepest and most interesting unsolved problem in solid state theory is probably the theory of the nature of glass and the glass transition."

[Anderson '95]



The relaxation times becomes 15 orders of magnitude larger than the microscopic one:
Amorphous solid

No remarkable sign of any structural change

Ubiquitous phenomena in a wide variety of different systems (molecular liquids, colloids, grains, computer science, quantum systems, …)

A very unconventional transition

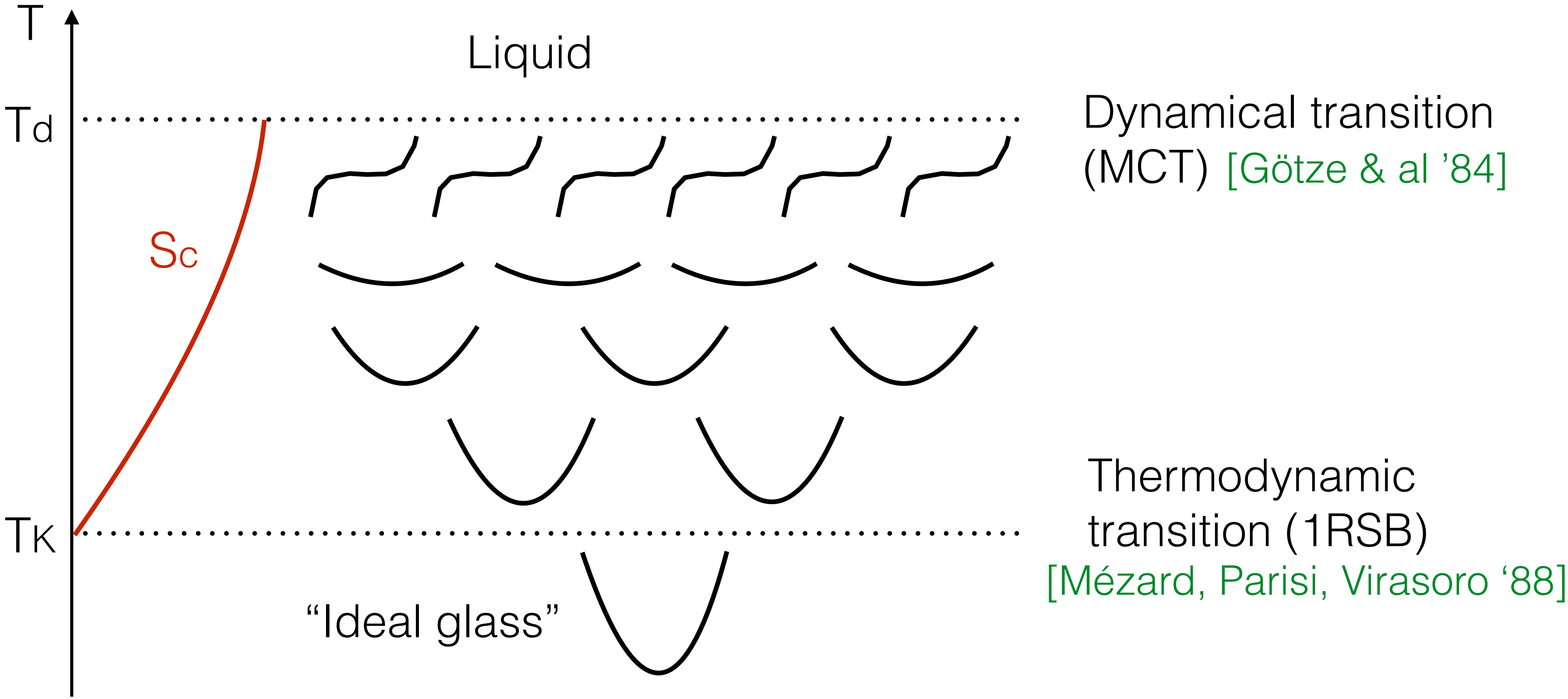
- **The relaxation times increases dramatically fast** as the temperature is lowered (the activation energy grows as T is decreased)
- **Usual two-point correlation functions are featureless.** A snapshot of a glass configuration looks just like a liquid one
- Is the glass transition a **collective phenomenon?** Is it associated to a **phase transition?**
- And if yes, of which kind?
    - Dynamical or thermodynamical?
    - Order parameter? Which kind of symmetry breaking?
    - Which is the nature of the long-range order?
    - What are the relevant (statics and/or dynamics) length scales? How are they related to the growing time scales?

Experiments and simulations are very hard to perform because of the extremely large equilibration time which would be needed

Pre-asymptotic effects hamper the validation of the correct theory

(A very intricate) mean-field theory

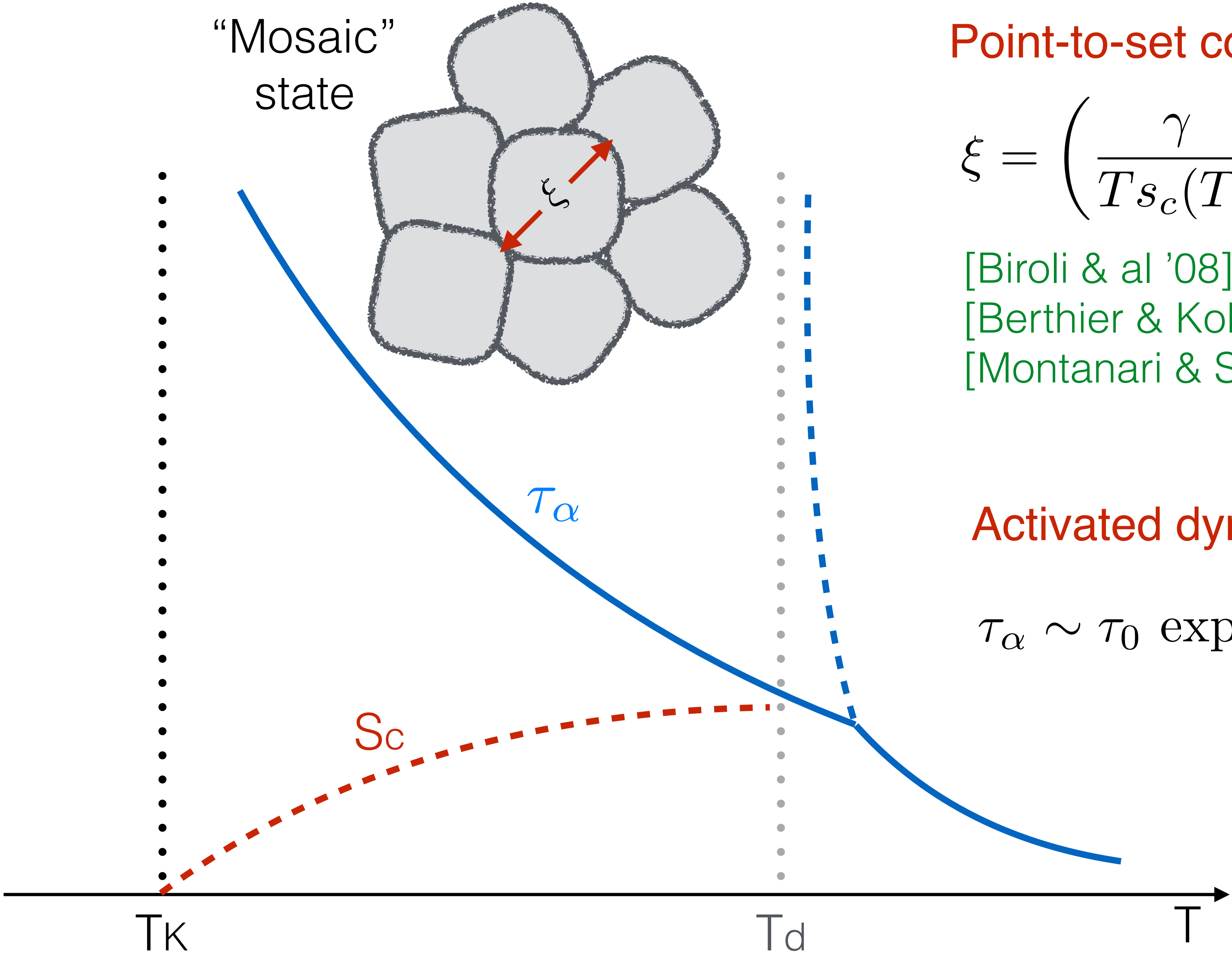
Complex free-energy landscape [Goldstein '69]



- *p*-spin model, Potts glass, … [Derrida '81; Gross & Mézard '84]
- Lattice glass models [Biroli & Mézard '01; Tarzia & al '03]
- Lennard-Jones particles, soft and **hard spheres** [Kurchan & al '13] [Charbonneau & al '14]
  [Mézard & Parisi '99]

The Random First-Order Transition Scenario

[Kirkpatrick, Thirumalai, Wolynes '87 - '89]



Point-to-set correlation length

$$\xi = \left(\frac{\gamma}{T s_c(T)} \right)^{\frac{1}{d-\theta}}$$

[Biroli & al '08]
[Berthier & Kob '12]
[Montanari & Semerjian '06]

Activated dynamics

$$\tau_\alpha \sim \tau_0 \exp \left(\frac{c\, \xi^\psi}{k_B T} \right)$$

Finite-dimensional fluctuations beyond RFOT

- Technically challenging
  - Intricate nature of the order parameter
  - Huge degeneracy of metastable states
  - The behavior in finite dimensions of most of the paradigmatic models from which RFOT originates is drastically different

- Conceptually difficult

  Infinite life-time of metastable states in mean-field. The very process characterizing slow and glassy dynamics (activated jumps from one metastable state to another) are absent in mean-field theory, which therefore cannot explain by itself the growth of time and length scales

- Several important recent analytical results
  - Instantons [Dzero & al '05; Franz & Montanari '07]
  - Kaç models [Franz '06]

The standard theoretical framework to explore the critical properties of the system beyond mean-field is the Renormalization Group approach

A new kind of criticality (1)

Two important differences with respect to the standard theory of phase transitions and critical phenomena

1. Disorder →

- The order parameter is a $n \times n$ overlap matrix (with $n \to 0$): **no nonperturbative definition of the functional integral**
- Follow the flow of an infinite number of coupling constants: **Functional RG?**
- **New kind of fixed points** describing the low-temperature phase?

In some cases (random manifold and RFIM) these issues have been solved
[Balents & Fisher '93; Chauve, Le Doussal, Rosso, Wiese '01 - '04]
[Tarjus & Tissier '04 - '12]

However for glasses these problems are still very far from being overcome (**RS *vs* RSB**)

An extra complication: Most of the RFOT disordered spin models have a very weak transition even in infinite dimensions. Standard techniques such as high-*T* expnsion are useless [Cammarota & al '13]

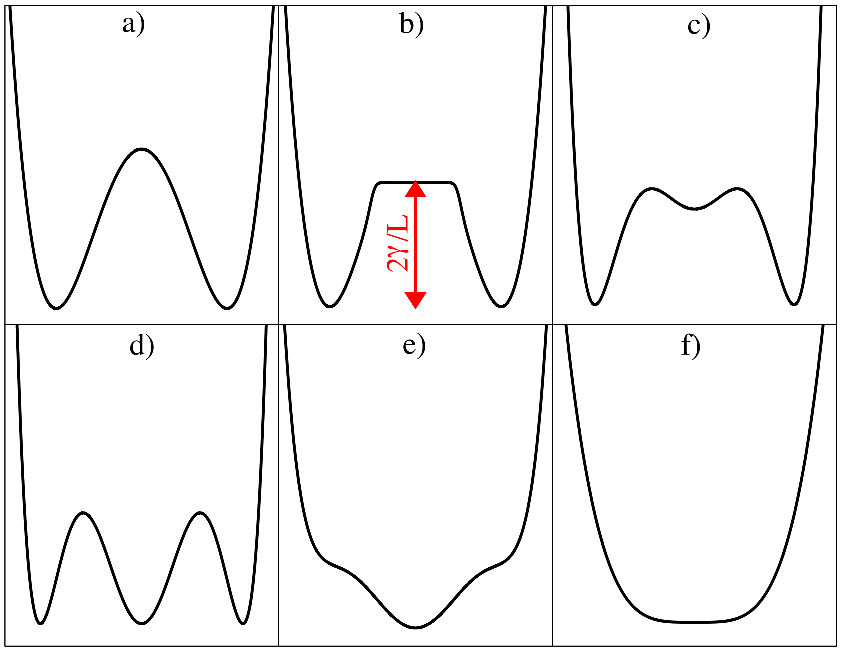
A new kind of criticality (2)

2. Metastability

- Take into account the **complexity of the rugged free-energy landscape** and the huge degeneracy of metastable states
- Characterize the **disappearance of metastable states** on the scale of the point-to-set length **via nonperturbative fluctuations** (rare, localized events, non-uniform in space)

A toy model with metastability and (strong) nonperturbative fluctuations

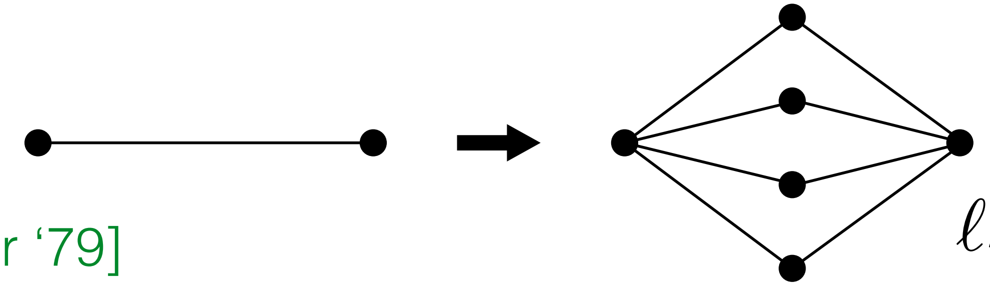
$$\mathcal{H}[\varphi] = \int_0^L \mathrm{d}x \left[\frac{c}{2} \left(\frac{\mathrm{d}\varphi(x)}{\mathrm{d}x} \right)^2 + \frac{r}{4} (\varphi^2 - 1)^2 \right]$$



Failure of any simple truncation scheme of the NPRG to recover the low-temperature physics ($\xi \propto e^{S/T}$) due to strong nonlinear effects [Rulquin & al '16]

Real-space RG analysis of the RFOT

Migdal-Kadanoff RG on hierarchical lattices
[Kadanoff '75; Migdal '76; Berker '79]



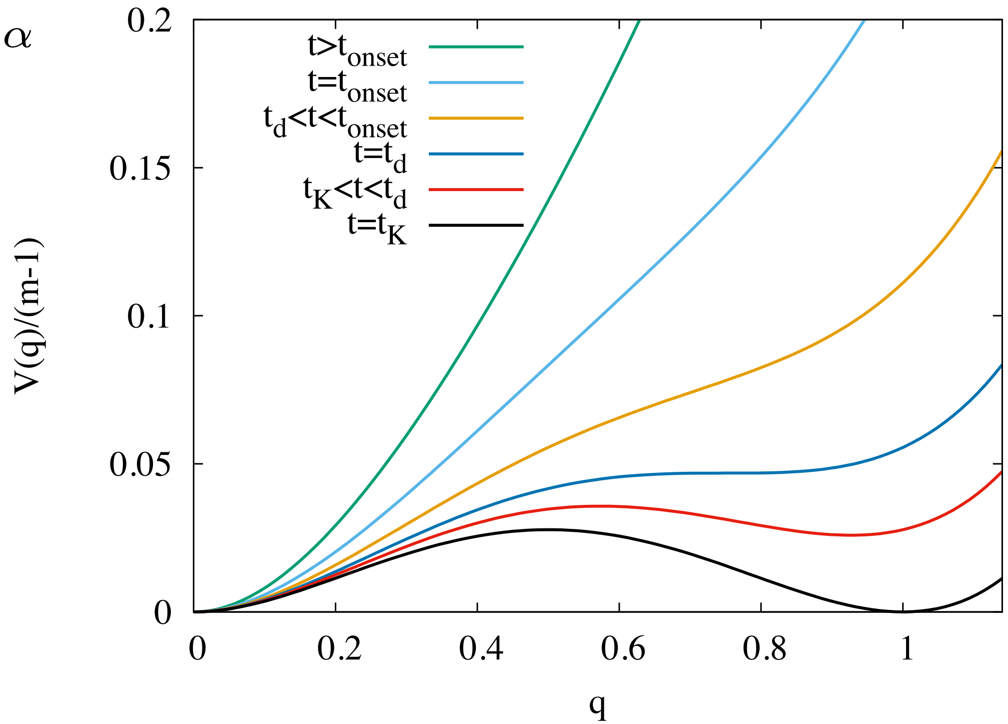
$\ell_n = 2^n$

Ginzburg-Landau functional in the "universality class" of RFOT

$$\mathcal{H}[\{q_{\alpha\beta}\}] = \int \mathrm{d}\mathbf{x} \left\{ \frac{c}{2} \sum_{\alpha \neq \beta} \left(\nabla q_{\alpha\beta}(\mathbf{x}) \right)^2 + V[q_{\alpha\beta}(\mathbf{x})] \right\}$$

$$V[q_{\alpha\beta}] = \sum_{\alpha \neq \beta} U[q_{\alpha\beta}] - \frac{u}{3} \sum_{\alpha\beta\gamma \neq} q_{\alpha\beta} q_{\beta\gamma} q_{\gamma\alpha}$$

$$U[q] = \frac{t}{2} q^2 - \frac{u+w}{3} q^3 + \frac{y}{4} q^4$$



Local coarse-grained overlap order parameter matrix $q_{\alpha\beta}(\mathbf{x})$

Exact RG flow equation

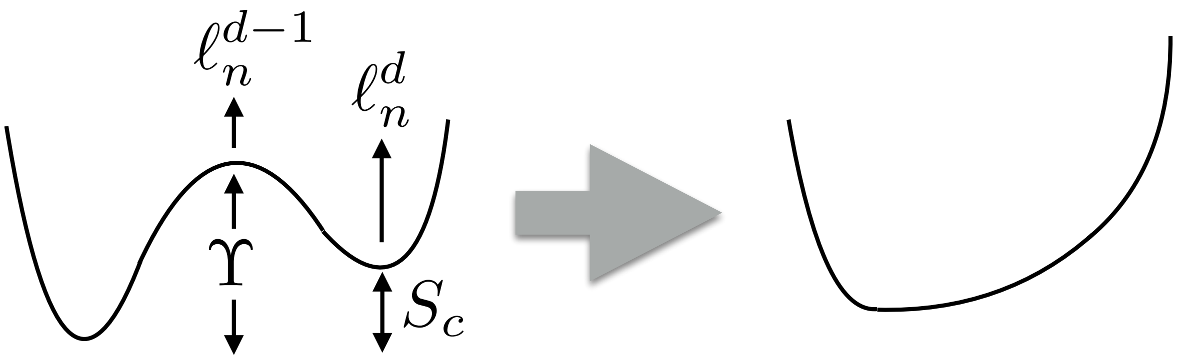
[Cammarota & al '11]

$$W_n[q^{(i)}_{\alpha\beta}, q^{(j)}_{\alpha\beta}] = \ln \left[\int \mathrm{d}q^{(k)}_{\alpha\beta}\, e^{W_{n-1}[q^{(i)}_{\alpha\beta}, q^{(k)}_{\alpha\beta}] + V[q^{(k)}_{\alpha\beta}] + W_{n-1}[q^{(k)}_{\alpha\beta}, q^{(j)}_{\alpha\beta}]} \right]^b$$

$$W_0[q^{(i)}_{\alpha\beta}, q^{(j)}_{\alpha\beta}] = \frac{c}{2} \sum_{\alpha\neq\beta} [q^{(i)}_{\alpha\beta} - q^{(j)}_{\alpha\beta}]^2$$

Saddle-point approximation (RS and RSB ansatz)

Standard behavior observed in a conventional first-order transition!!
(nucleation of droplets) [Fisher & Berker '98]



$$\xi \sim (T - T_K)^{-1}$$

$$\ln \tau_\alpha \sim (T - T_K)^{-(d-1)}$$

The point-to-set corresponds to the length scale over which the metastable states disappear and the free-energy becomes convex

Issues and limitations of the MKRG

- The choice of the hierarchical lattice could be questioned

  Recent results on the REM on the Dyson hierarchical lattice
  [Castellana & al '10]

- The Ginzburg-Landau functional is a crude approximation

  Most of the disordered RFOT spin models shows a drastically different behavior in finite dimensions [Brangian, Kob, Binder '02 - '03]

  Unusual fragility of the RFOT scenario to short-range fluctuations
  [Cammarota & al '13]

  Promising results on the MKRG performed directly on spin models with a strong RFOT character [Angelini & Biroli '17]

- The most severe shortcoming is the saddle-point approximation. It misses a fundamental physical properties of glassy systems: **The self-induced disorder** [Franz & al '11]

  e.g., the transition persists down to $d=2$

Effective theories of the glass transition

Identify the effective theory, which describes the fluctuations of the physical order parameter, **the overlap field with an equilibrium reference configuration** [Monasson '95; Franz & Parisi '95]

- More intuitive description of the glass transition

- **Circumvent the explicit description in terms of metastable states**

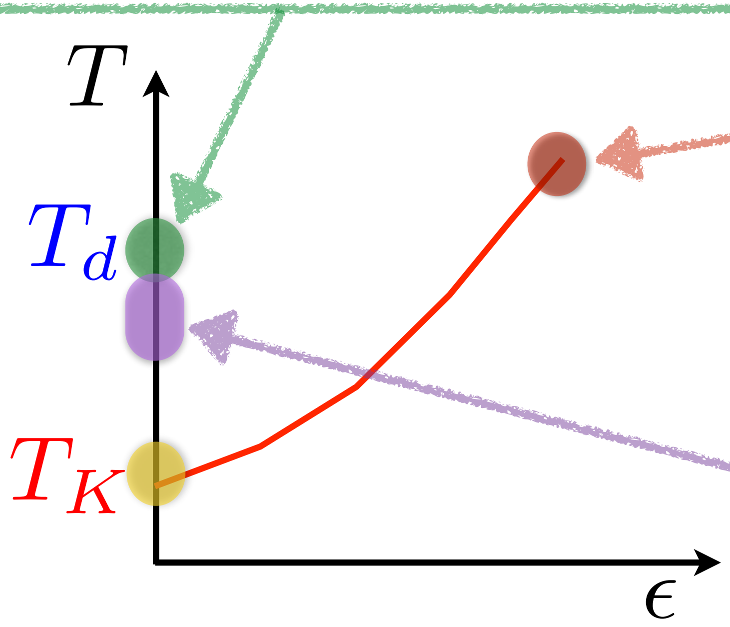
- It naturally leads to a **scalar field theory with quenched disorder** (which can be studied by all nonperturbative means at our disposal, e.g., NPRG, numerical simulations, …)

- Study fluctuations beyond mean-field theory, and identify the possible mechanisms which could modify or destroy the glass transition

RFIM-like criticality: (some) Known results

- Analysis of the **perturbation theory of the replica field theory**

The critical overlap fluctuations close to the MCT transition are in the same universality class as the spinodal of the RFIM [Franz & al '11]



The terminal critical point in the T-ϵ plane belongs to the universality class of the standard ϕ^4-RFIM [Franz & Parisi '13; Biroli & al '14]

Dynamical field theory for the MCT crossover corresponds to a RFIM in presence of an external field [Rizzo '16]

- The effective description in terms of a 2-state picture with disorder is also supported by recent **numerical simulations** [Berthier '13; Berthier & Jack '15; Stevenson & al '08]

- The relevance of RFIM-like criticality in glass-forming systems emerge also in the context of the **Dynamic Facilitation** [Franz & Sellitto '13; Jack, Garrahan, Turner '15 - '16; Ikeda & al '16]