The overlap matrix

$$\delta_{\mathcal{C}^\alpha,\mathcal{C}^\beta} = \begin{pmatrix} 1 & p_1 & p_2 & \cdots & p_n \\ p_1 & 1 & q_{12} & \cdots & q_{1n} \\ p_2 & q_{21} & 1 & \cdots & q_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_n & q_{n1} & q_{n2} & \cdots & 1 \end{pmatrix}$$

Overlap p_a with the reference configuration

Overlap q_{ab} among the free replicas

$\mathcal{C}_i^a = \mathcal{C}_i^0$ and $\mathcal{C}_i^b = \mathcal{C}_i^0 \longrightarrow \mathcal{C}_i^a = \mathcal{C}_i^b$

$\mathcal{C}_i^a = \mathcal{C}_i^0$ and $\mathcal{C}_i^b \neq \mathcal{C}_i^0 \longrightarrow \mathcal{C}_i^a \neq \mathcal{C}_i^b$

$\mathcal{C}_i^a \neq \mathcal{C}_i^0$ and $\mathcal{C}_i^b \neq \mathcal{C}_i^0 \longrightarrow$??????

example ($n = 5$):

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & ? & 0 & ? \\ 0 & 0 & ? & 1 & 0 & ? \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & ? & ? & 0 & 1 \end{pmatrix}$$

In the simplest approximation consists in assuming that $\mathcal{C}_i^a \neq \mathcal{C}_i^b$ in this case (justified in the Kaç limit for large M)

$$q_{ab}(i) = p_a(i) p_b(i)$$

The annealed approximation

$$\mathcal{S}_{rep}^{MF} = \sum_a \Big(-\frac{M\beta^2}{2} \sum_{\langle ij \rangle} p_a(i) p_a(j) + M \log 2 \sum_i p_a(i) \Big)$$

$$- M\beta^2 \sum_{ab} \sum_{\langle ij \rangle} p_a(i) p_b(i) p_a(j) p_b(j)$$

Going back to a spin model $\longrightarrow$ $p(i) = (1+\sigma_i)/2$

Random field random bond Ising model (with correlated disorder)

$$\mathcal{H}^{MF} = -\sum_{\langle ij \rangle} (J + J_{ij}) \sigma_i \sigma_j + \sum_i (H_{ext} - h_i) \sigma_i$$

$H_{ext} = Md(\beta_K^2 - \beta^2)/4 \propto M s_c$

$J = M\beta^2/8$

$\Delta_J^2 = M\beta^2$ $\qquad$ $\Delta_h^2 = M\beta^2 d/8$

$\overline{h_i h_j} = \overline{h_i J_{ij}} = M\beta^2/16$

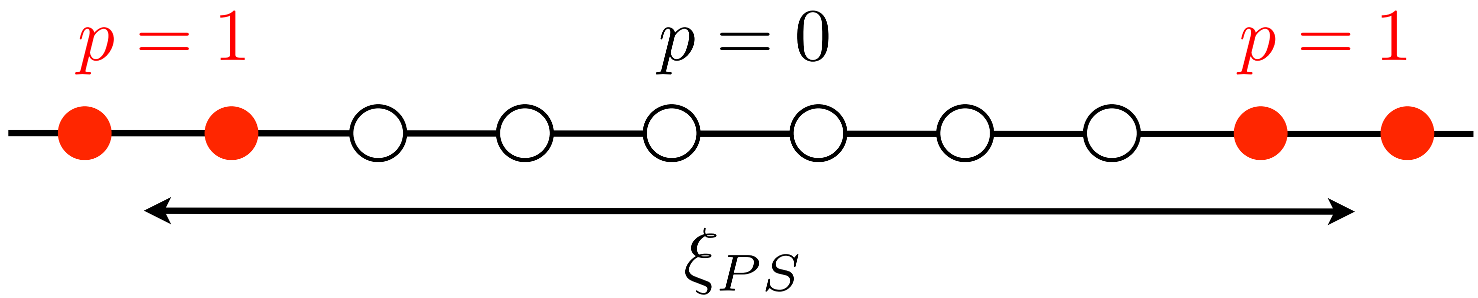
$\beta_K = \sqrt{2 \ln 2/d}$

Mean-field ($M \to \infty$)

critical temperature

Beyond the annealed computation

Consider $p_a(i) = p(i) \quad \forall a$ (first cumulant of the effective action)



Regions with $p=1$ closer than the point-to-set length → Locally decrease s_c → Induce a spontaneous RSB among the n replicas forced to have $p=0$

A variational Ground-State approximation

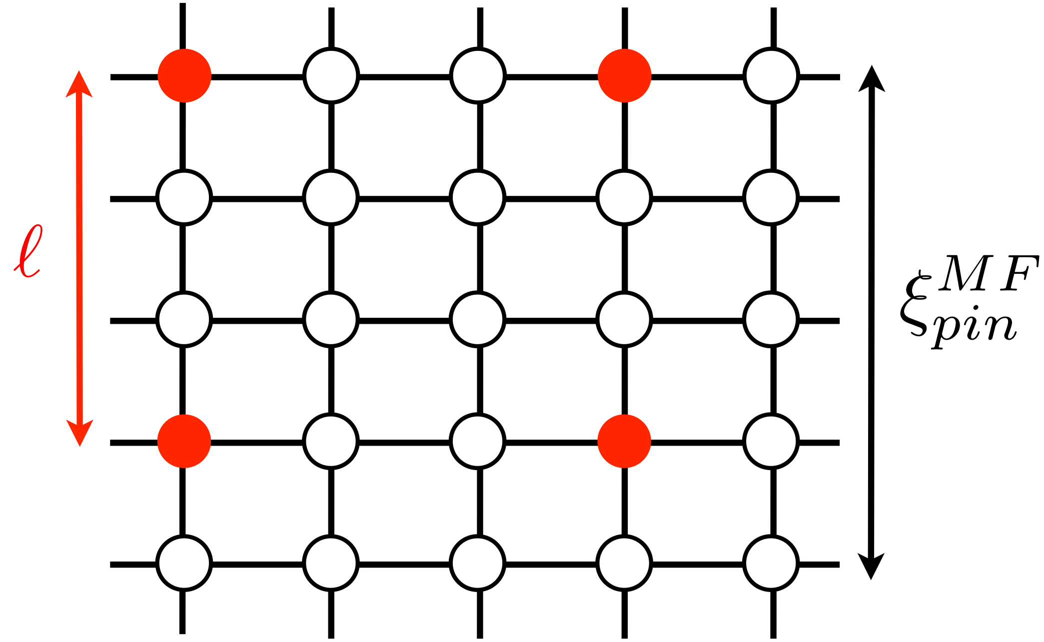
$$p(i) = 0 \rightarrow q_{ab}(i) = \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 0 & \boxed{1} & & 0 & \\ \vdots & & \boxed{1} & & \\ \vdots & 0 & & \boxed{1} & \\ 0 & & & & \end{pmatrix} \updownarrow m$$

combinatorial factor

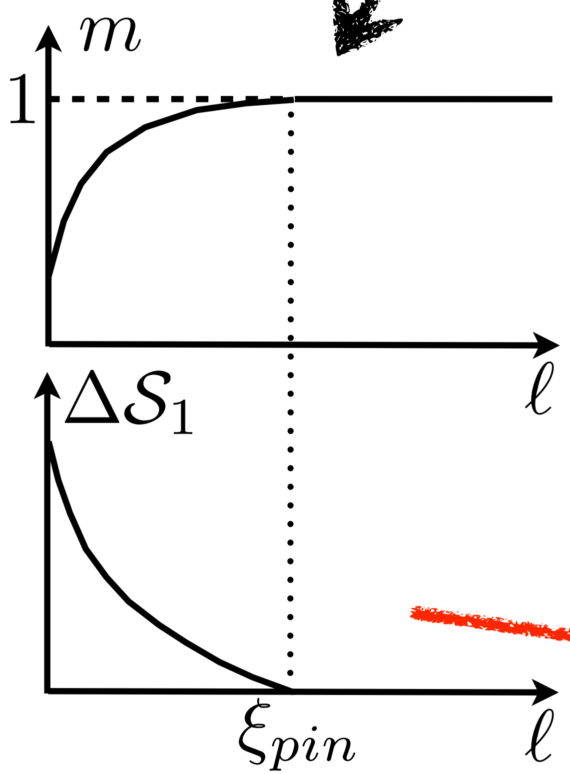
$$\binom{2^M - 1}{n/m}$$

The periodic pinning

Set $p(x) = 1$ on the vertices of a d-dimensional hyper-cube of size ℓ and $p(x) = 0$ elsewhere



Use the variational ansatz and optimize over m



$$\mathcal{S}_1[p(i)] = \mathcal{S}_1^{MF}[p(i)] + \Delta\mathcal{S}_1[p(i)]$$

$$\xi_{pin}^{MF} = \left(\frac{\beta_K^2 - \beta^2}{\beta_K^2}\right)^{\frac{1}{d}}$$

Additional effective interaction among the $p(i)$

Effective long-range antiferromagnetic interaction

Approximate ansatz for the effective interaction → possibly long-range pair interaction $K(r)$ + external field ΔH

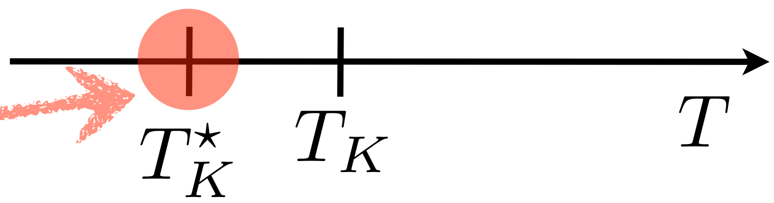
$$\mathcal{H} = -\sum_{\langle ij \rangle}(J + J_{ij})\sigma_i\sigma_j + \sum_{i,j} K(|i-j|)\,\sigma_i\sigma_j + \sum_i (H_{ext} + \Delta H + h_i)\sigma_i$$

$$K(r) \simeq \frac{Mc}{r^{2d}}\,\theta(\xi_{pin}^{MF} - r)$$

ΔH lowers the transition temperature with respect to mean-field (by renormalizing the configurational entropy)

$$\beta_K \longrightarrow H_{ext}(\beta_K^\star) + \Delta H(\beta_K^\star) = 0 \qquad \beta_K^\star > \beta_K$$

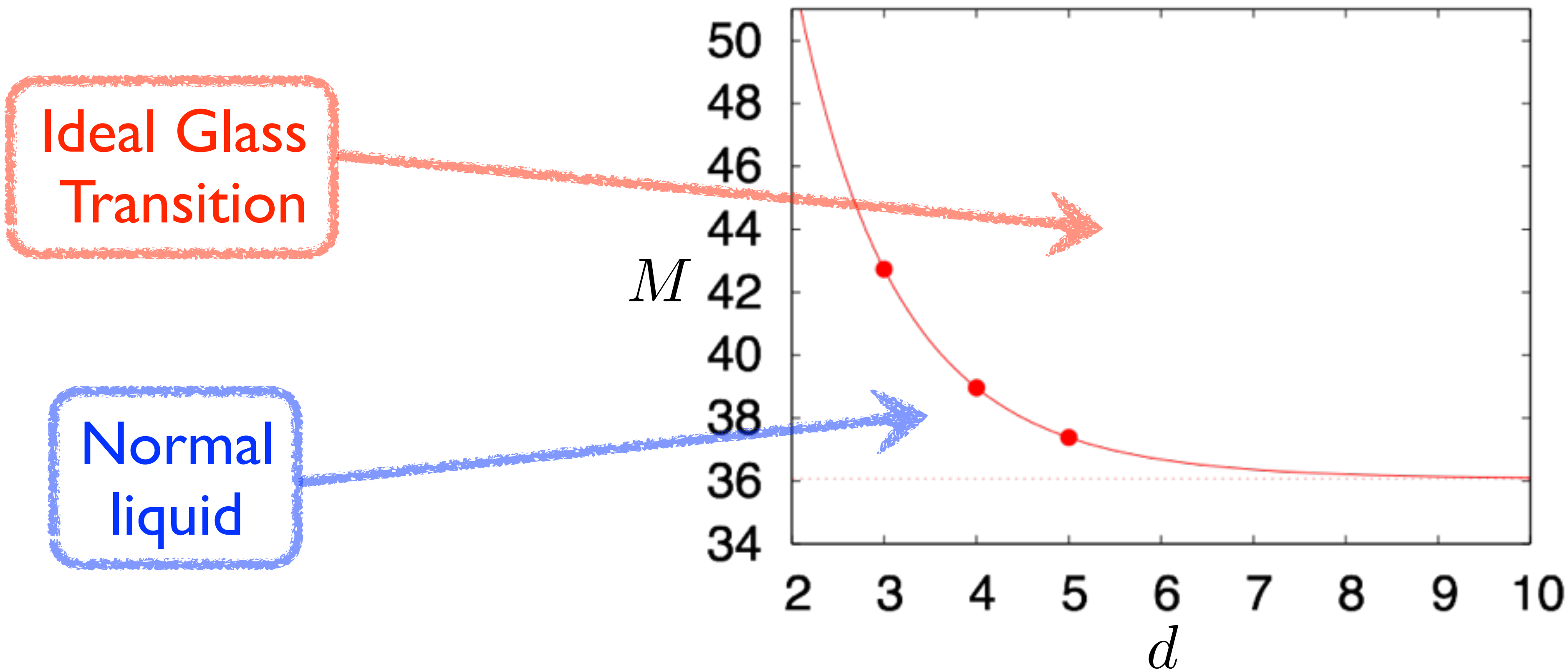
Study the properties of the critical point using the effective Hamiltonian



Estimation of the disorder at the critical point

Numerical simulations in $d=3$ ⟶ $\frac{\sqrt{\Delta_h^2}}{Jd} \simeq 0.4$

- Neglect the random bonds (ok for large M)
- Neglect correlations between n.n. random fields and random bonds
- Use the result of $d=3$ in higher dimensions
- $(Jd)_{eff} = Jd - \frac{1}{2}\int r^{d-1}K(r)\mathrm{d}r$



Conclusions & Perspectives

- First steps towards the derivation of an Ising-like effective theory for the glass transition close to T_K
- Random field random bond Ising model (with correlated disorder) + long-range antiferromagnetic interaction
- Go beyond RFOT. Use the effective Hamiltonian to study the properties of the original model (length scales, critical points, ...)
- Identify mechanisms which may destroy the Ideal glass transition

- Check the robustness of the results with respect to other geometry of the overlap profile
- Extend the computation to the second cumulant of the effective action (possibly long-range correlated disorder?)
- Extend the calculation to other models (similar results are found for a generic effective Ginzburg-Landau replicated action)
- Improve the variational ground state approximation (low temperature expansion?)