Ising-like effective theory for the glass transition

Marco Tarzia
LPTMC, Université Paris VI

Spin glasses: an old tool for new problems
Cargèse, August 25 - September 6 2014

with G. Biroli, C. Cammarota, and G. Tarjus

Introduction & motivation

Effective theory of the glass transition in terms the overlap $p(x)$ with an equilibrium configuration

- It allows to focus on the relevant field and on the "physical" order parameter and to obtain a more intuitive description of the glass transition

- It leads naturally to a scalar field theory in presence of quenched disorder, which is easier to handle than the original replica field theory. It can be studied using standard tools of statistical physics (nonperturbative RG, numerical simulations, ...) cfr Gille's lecture

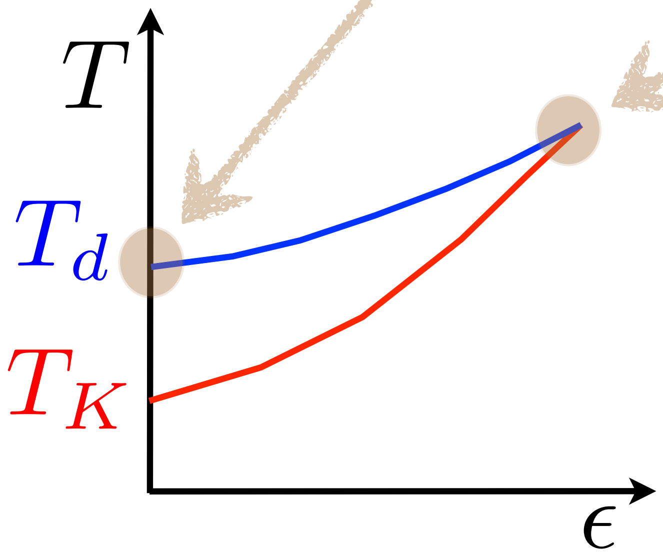
- It allows to go beyond mean-field theory and RFOT, and to study the nature and the critical properties of the critical points. It naturally allows to identify the possible mechanisms that could destroy the glass transition in finite dimensions

Known results: Self-induced disorder and RFIM

- Analysis of perturbation theory of the Replica Field Theory

Critical overlap fluctuations close to the dynamical transition in the β-regime are in the same universality class of the spinodal point of the RFIM

Franz, Parisi, Ricci-Tersenghi, Rizzo



The terminal critical point in the T-ϵ phase diagram is in the same universality class of the RFIM

Franz & Parisi; Biroli, Cammarota, Tarjus, MT

The same result holds for the continuous glass transition found at the terminal point of the random pinning phase diagram

Cammarota & Biroli; Nandi & Biroli

- The distribution of the overlap fluctuations has been computed in numerical simulations of glass forming systems and have been interpreted in terms of an effective RFIM

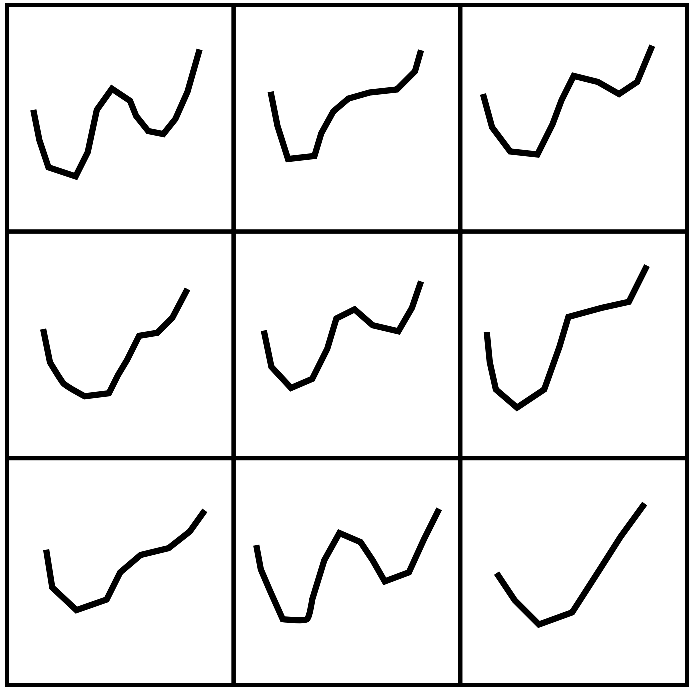
Stevenson & al

Intuituve arguments

cfr Giulio's lecture and Silvio's talk

The equilibrium reference configuration acts as a random field

Local fluctuations of the Franz-Parisi potential due to the density fluctuations of the reference configuration



↓

Local fluctuations of the configurational entropy and of the surface tension

↓

Random field random bond Ising model

overlap p
configurational entropy s_c → magnetization m
height of the barrier

magnetization m
magnetic field h
ferromagnetic coupling J

What do we want/need to compute?

Choose an equilibrium reference configuration $\mathcal{C}_0$ at random (according to $e^{-\beta\mathcal{H}(\mathcal{C}_0)}/Z$)

Compute the probability that a copy of the system has an overlap profile $p(x)$ with the reference configuration:

$$e^{-\mathcal{S}[p(x)|\mathcal{C}_0]} \propto \sum_{\mathcal{C}} e^{-\beta\mathcal{H}(\mathcal{C})}\delta[p(x) - Q_x(\mathcal{C},\mathcal{C}_0)]$$

The cumulants of $\mathcal{S}[p(x)|\mathcal{C}_0]$ can be computed through an expansion in free replica sums: Tarjus & Tissier (cfr Gille's lecture)

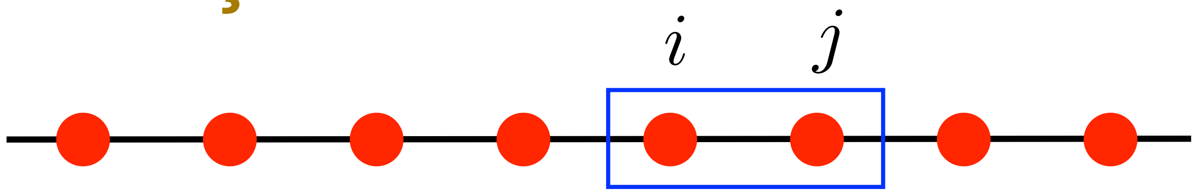
$$e^{-\mathcal{S}_{rep}[\{p_a(x)\}]} = \overline{e^{-\sum_{a=1}^{n}\mathcal{S}[p(x)|\mathcal{C}_0]}}^{\mathcal{C}_0}$$

$$\mathcal{S}_{rep}[\{p_a(x)\}] = \sum_{a=1}^{n}\mathcal{S}_1[p_a(x)] - \frac{1}{2}\sum_{a,b=1}^{n}\mathcal{S}_2[p_a(x),p_b(x)] + \cdots$$

$$\mathcal{S}_1[p(x)] = \overline{\mathcal{S}[p(x)|\mathcal{C}_0]}\,; \qquad \mathcal{S}_2[p_1(x),p_2(x)] = \overline{\mathcal{S}[p_1(x)|\mathcal{C}_0]\mathcal{S}[p_2(x)|\mathcal{C}_0]}^{c}$$

The Kaç version of the Random Energy Model

Franz, Parisi, Ricci-Tersenghi



2^M configurations on each site: $\mathcal{C}_i = \{1, \ldots, 2^M\}$

$$\mathcal{H} = \sum_{\langle ij \rangle} E_{\langle ij \rangle}(\mathcal{C}_i, \mathcal{C}_j)$$

Random energy on each link $\langle ij \rangle$

$E_{\langle ij \rangle} = E(\mathcal{C}_i, \mathcal{C}_j)$ iid Gaussian

$$\overline{E(\mathcal{C}_i, \mathcal{C}_j)} = 0$$

$$\overline{E(\mathcal{C}_i, \mathcal{C}_j) E(\mathcal{C}'_i, \mathcal{C}'_j)} = M \delta_{\mathcal{C}_i, \mathcal{C}'_i} \delta_{\mathcal{C}_j, \mathcal{C}'_j}$$

Compute the replicated action $\alpha = 0, \ldots, n \longrightarrow n+1$ replicas

$$e^{-\mathcal{S}_{rep}[\{p_a(i)\}]} \propto \overline{\sum_{\{\mathcal{C}_i^\alpha\}} \exp\left(-\beta \sum_{\langle ij \rangle, \alpha} E_{\langle ij \rangle}(\mathcal{C}_i^\alpha, \mathcal{C}_j^\alpha) \right) \prod_{a,i} \delta_{p_a(i), q(\mathcal{C}_i^0, \mathcal{C}_i^a)}}$$

$$= \sum_{\{\mathcal{C}_i^\alpha\}} \exp\left(\frac{\beta^2 M}{2} \sum_{\langle ij \rangle} \sum_{\alpha, \beta} \delta_{\mathcal{C}_i^\alpha, \mathcal{C}_i^\beta} \delta_{\mathcal{C}_j^\alpha, \mathcal{C}_j^\beta} \right) \prod_{a,i} \delta_{p_a(i), q(\mathcal{C}_i^0, \mathcal{C}_i^a)}$$