AMORTISSEMENT DES PHONONS DANS UN GAZ DE FERMIONS SUPERFLUIDE

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PLAN DE l'EXPOSÉ

- Definition of the problem: phonon damping
- Phonon coupling from quantum hydrodynamics
- Convex phonon branch: Beliaev-Landau damping
- Unitary gas (convex branch) at $T = 0$: first correction to Beliaev damping
- Concave phonon branch: Landau-Khalatnikov damping
- Competing processes: BCS quasi-particles

CONTEXTE ET MOTIVATIONS

The system:

- Unpolarised spin-$1/2$ Fermi gas in the so-called BEC-BCS crossover. Zero-range $\uparrow - \downarrow$ interactions with s-wave scattering length a of arbitrary nonzero value.
- Realised in the lab with cold atoms and a magnetic Feshbach resonance.
- Recent experimental progress: spatially homogeneous gases can be prepared in flat bottom potentials. More fundamental questions can be addressed in the lab.

Low temperature limit:

- At zero temperature: BCS pair-condensed gas, entirely superfluid

- At low temperature

$$T \ll T_c,\ T \ll \Delta/k_B,\ T \ll mc^2/k_B$$

with Δ = pairing gap, c = sound velocity, one can ignore the BCS pair-breaking excitation branch and consider only the phononic excitation branch:

$$\omega_q \underset{q\to 0}{=} cq\left[1+\frac{\gamma}{8}\left(\frac{\hbar q}{mc}\right)^2+O(q^4\ln q)\right]$$

- The system then reduces to a weakly interacting gas of phonons (even if the underlying interparticle interactions are strong)
- What is the damping rate Γ_q of the phonon mode of wavevector q?

A universal and fondamental limit:

- **In this limit, all superfluids with short-range interparticle interactions reduce to a such a gas of phonons.**
- **Our results shall equally apply to liquid helium 4 and to the weakly interacting Bose gas.**
- **As we shall see, the dissipative dynamics depends quantitatively on the equation of state of the system and qualitatively on the sign of the curvature γ.**
- **In atomic Fermi gases, γ is tuned by changing a. In liquid helium, it is tuned by changing the pressure. In weakly interacting Bose gases, $\gamma > 0$ (Bogoliubov).**
- **In helium, Beliaev-Landau damping ($\gamma > 0$) observed, Landau-Khalatnikov damping ($\gamma < 0$) not yet. Beliaev damping seen by Davidson in a trapped BEC. In Fermi gases, damping seen in uniform gas by Zwierlein.**

COUPLAGE PHONON-PHONON

In the low-energy limit, given by quantum hydrodynamic theory of Landau and Khalatnikov (1949).

- **Two canonically conjugated fields: density $\hat{\rho}(\mathbf{r})$ and phase $\hat{\phi}(\mathbf{r})$ of the superfluid**
- **Hamiltonian**

$$\hat{H} = \int d^3r \left[\frac{1}{2} m \hat{\mathrm{v}} \cdot \hat{\rho} \hat{\mathrm{v}} + e_0(\hat{\rho})\right]$$

where $e_0(\rho)$ is the ground state energy density of the uniform system of density ρ and the superfluid velocity field is

$$\hat{\mathrm{v}}(\mathbf{r}) = \frac{\hbar}{m} \mathrm{grad}\, \hat{\phi}(\mathbf{r})$$

- **Corresponding Heisenberg equations of motion = Continuity and Euler's equations**

- **Linearize them around the spatially homogeneous solution:**

$$\hat{\rho}(\mathbf{r}) = \rho + \delta\hat{\rho}(\mathbf{r})$$
$$\hat{\phi}(\mathbf{r}) = \phi_0 + \delta\hat{\phi}(\mathbf{r})$$

- **Dispersion relation:**

$$\omega_q = cq \quad \text{with} \quad mc^2 = \rho\frac{d\mu_0}{d\rho}$$

- **Modal expansion:**

$$\delta\hat{\rho}(\mathbf{r}) = \frac{1}{\mathcal{V}^{1/2}}\sum_{\mathbf{q}\neq 0}\left(\frac{\hbar q\rho}{2mc}\right)^{1/2}(\hat{b}_{\mathbf{q}} + \hat{b}^{\dagger}_{-\mathbf{q}})e^{i\mathbf{q}\cdot\mathbf{r}}$$

$$\delta\hat{\phi}(\mathbf{r}) = \frac{-i}{\mathcal{V}^{1/2}}\sum_{\mathbf{q}\neq 0}\left(\frac{mc}{2\hbar\rho q}\right)^{1/2}(\hat{b}_{\mathbf{q}} - \hat{b}^{\dagger}_{-\mathbf{q}})e^{i\mathbf{q}\cdot\mathbf{r}}$$

where $\hat{b}_{\mathbf{q}}$ annihilates a phonon of wavevector $\mathbf{q}$.

- Inserting this expansion in this Hamiltonian gives cubic, quartic, etc, phonon-phonon coupling.

Curvature γ of the phonon branch:

- Crucial: determines the leading resonant processes for phonon damping.
- $\gamma > 0$: $\phi \leftrightarrow \phi\phi$ Beliaev-Landau
- $\gamma < 0$: three-phonon processes and $\phi \leftrightarrow \phi\phi\phi$ forbidden by energy-momentum conservation. Leading process is $\phi\phi \leftrightarrow \phi\phi$ Landau-Khalatnikov process.
- To get γ one needs a more microscopic theory (no measurement available yet). From Anderson's RPA (Combescot, Kagan, Stringari, 2006), we calculated γ in 2016:

$$\gamma > 0 \text{ iff } 1/(k_{\mathrm{F}}a) > -0.144$$

and $\gamma = 0.084$ at unitarity. Only an approximation.

$\gamma > 0$: AMORTISSEMENT BELIAEV-LANDAU

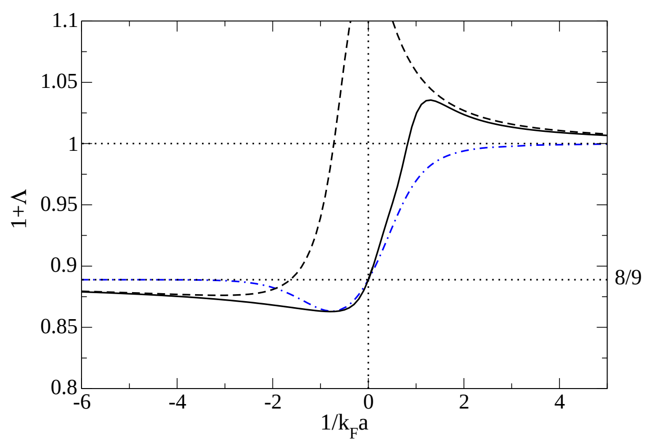
Beliaev coupling amplitude from quantum hydrodynamics on the energy shell:

$$\mathcal{A}(\mathrm{q};\mathrm{k},\mathrm{k}'=\mathrm{q}-\mathrm{k}) = \frac{3}{\sqrt{32}}(1+\Lambda)(\check{\omega}_q\check{\omega}_k\check{\omega}_{k'})^{1/2}$$

with $\check{\omega} = \hbar\omega/mc^2$ **and**

$$\Lambda = \frac{\rho}{3}\frac{d^2\mu}{d\rho^2}\left(\frac{d\mu}{d\rho}\right)^{-1}$$

deducible from measured equation of state.



Fermi Golden rule for direct and inverse processes with Bose amplification factors and δ of energy conservation. Integration over the direction of $\mathbf{k}$: $u = \mathbf{k}\cdot\mathbf{q}/kq$

$$\int_{-1}^{1} du\, \delta(u-u_0) = 1$$

with $-1 < u_0 < 1$ for $\gamma > 0$ and $u_0 \to 1$ when $q \to 0$. For $\gamma < 0$, $u_0 > 1$.

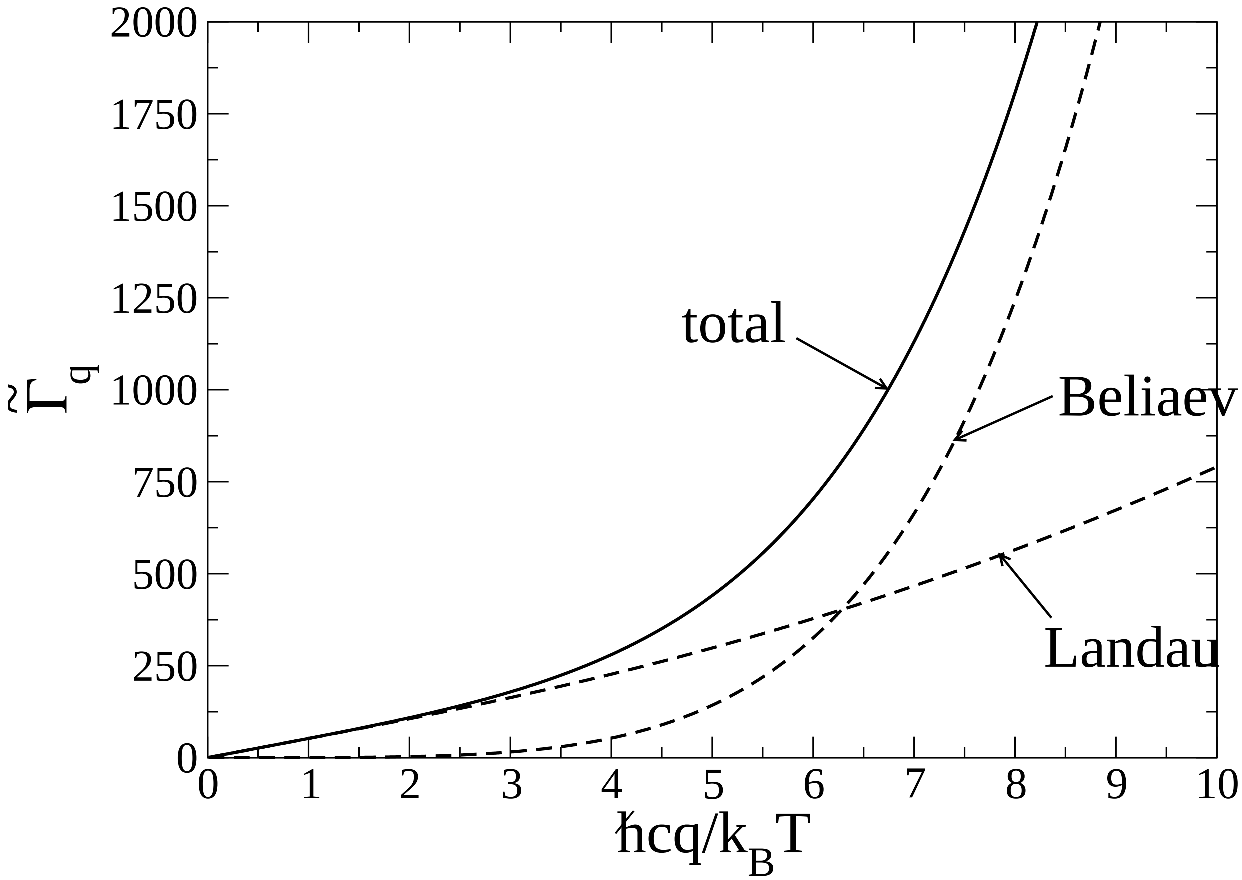
Integration over wavenumber k doable analytically. Exact low-temperature equivalent at fixed $\tilde{q} = \hbar c q / k_B T$:

$$\boxed{\Gamma_q \underset{T\to 0}{\sim} \frac{9(1+\Lambda)^2}{32\pi}\frac{mc^2}{\hbar\rho}\left(\frac{mc}{\hbar}\right)^3\left(\frac{k_B T}{mc^2}\right)^5 \tilde{\Gamma}(\tilde{q})}$$

$$\tilde{\Gamma}(\tilde{q}) = \frac{\tilde{q}^5}{30} + 96[\zeta(5) - g_5(e^{-\tilde{q}})] - 48\tilde{q}g_4(e^{-\tilde{q}}) + 8\tilde{q}^2[\zeta(3) - g_3(e^{-\tilde{q}})]$$

where $g_\alpha(z)$ is the usual Bose function.

TAUX BELIAEV-LANDAU RÉDUIT



GAZ UNITAIRE, $T = 0$: 1ERE CORRECTION À BELIAEV

Leading contribution $\Gamma_q \propto \check{q}^5$ with $\check{q} = \hbar q/mc$. Subleading one is $\propto \check{q}^7$. First attempt by Bighin, Salanich, Marchetti and Toigo (2015) is incomplete.

We find four sources of corrections:

1. curvature of the spectrum, involves γ assumed > 0
2. correction to Beliaev $\phi \leftrightarrow \phi\phi$ coupling amplitude. Can be obtained from the Son and Wingate effective field theory using conformal invariance of the unitary gas. We find on the energy shell

$$\mathcal{A}(\mathrm{q};\mathrm{k},\mathrm{k}') = \frac{\sqrt{2}}{3}(\breve{\omega}_q\breve{\omega}_k\breve{\omega}_{k'})^{1/2}\left[1 - \frac{7\gamma}{32}(\breve{\omega}_q^2 + \breve{\omega}_k^2 + \breve{\omega}_{k'}^2)\right]$$

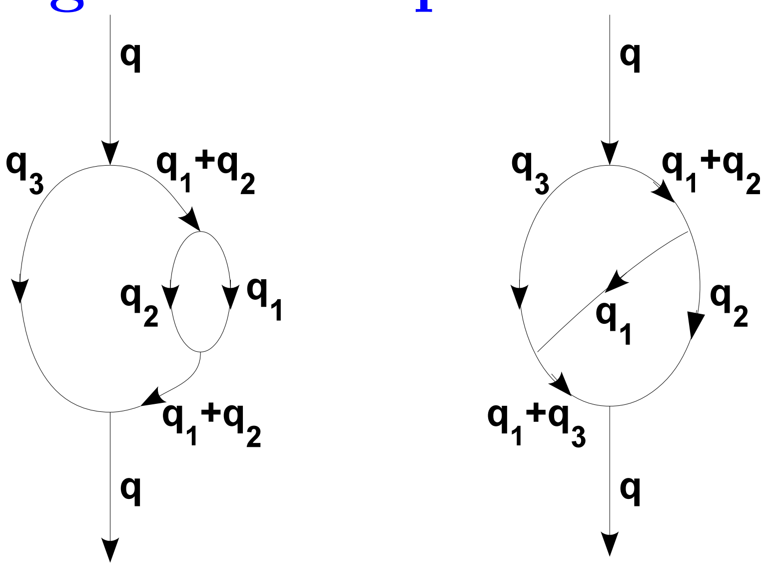
It unexpectedly involves the same combination of beyond-hydrodynamic parameters c_1 and c_2 as in the spectrum, $\gamma \propto 2c_1 + 3c_2$, contrarily to the modal amplitudes.

3. We are calculating the decay rate of a resonance (at fixed total momentum, discrete state $|\mathbf{q}\rangle$ coupled to a continuum $|\mathbf{k}, \mathbf{q}-\mathbf{k}\rangle$). Qualitatively, the Dirac δ acquires a nonzero width $\propto q^5$. Angular integral becomes

$$\int_{-1}^{1} du \frac{\check{q}^4/\pi}{(u-u_0)^2+(\check{q}^4)^2} \underset{\check{q}\to 0}{=} 1 + C\check{q}^2 + o(\check{q}^2)$$

because $1 - u_0 \approx \check{q}^2$.

4. Higher order processes: $\phi \leftrightarrow \phi\phi\phi$



Only the Beliaev process with a single loop correction (itself of the Beliaev nature) to the virtual phonons angular eigenfrequency contributes at order q^7.

Final exact expansion at unitarity (if $\gamma > 0$):

$$\Gamma_q \underset{\check{q}\to 0}{=} \frac{2mc^2}{9\pi\hbar}\left(\frac{mc}{\hbar\rho^{1/3}}\right)^3 \frac{\check{q}^5}{30}\left[1 - \frac{25}{112}\gamma\check{q}^2 + \frac{22\sqrt{3}\xi^{3/2}}{1701\gamma}\check{q}^2 + o(\check{q}^2)\right]$$

where the Bertsch parameter $\xi = \mu/\epsilon_F \simeq 0.376$ was measured by Zwierlein et al. (2012). With $\gamma = \gamma_{RPA} = 0.084$ the overall correction is positive.

$\gamma < 0$: AMORTISSEMENT LANDAU-KHALATNIKOV

As qualitatively understood by Landau and Khalatnikov:

- The leading process is $\phi\phi \leftrightarrow \phi\phi$: $\mathbf{q} + \mathbf{q}' \leftrightarrow \mathbf{k} + \mathbf{k}'$.
- The effective coupling $\mathcal{A}_{\rm eff}(\mathbf{q}, \mathbf{q}'; \mathbf{k}, \mathbf{k}')$ is the sum of the direct coupling (quartic terms in $\hat{H}$) and of the indirect coupling generated by off-resonant three-phonon processes (cubic terms in $\hat{H}$) treated to second order in perturbation theory (6 diagrams).
- The integral over $\mathbf{q}'$ and $\mathbf{k}$ diverges for a linear spectrum for aligned wavevectors: so including the curvature term is crucial, and the integral is dominated by almost-aligned-wavevectors configurations.

On a quantitative level:

- Landau and Khalatnikov only calculated the decay rate in the low-$\tilde{q}$ and the high-$\tilde{q}$ limits.

- They claim that a single diagram dominates in these limits.
- We disagree with this statement. We find that all diagrams have similar contributions, that destructively interfere so our $\mathcal{A}_{\rm eff}$/resp. rate is subleading with respect to Landau-Khalatnikov by one/two order(s) in $\tilde{q}$.
- Our conclusion results from a systematic $k_B T/mc^2 \to 0$ expansion at fixed $\tilde{q}$, after rescaling of the angles θ of $\mathbf{q}'$ and $\mathbf{k}$ with respect to $\mathbf{q}$ as follows

$$\theta = \frac{k_B T}{mc^2}|\gamma|^{1/2}\tilde{\theta}$$

- The result for $\gamma < 0$:

$$\boxed{\frac{\hbar\Gamma_q}{mc^2} \underset{T\to 0}{\overset{\text{fixed}\,\tilde{q}}{\sim}} \frac{81(1+\Lambda)^4}{256\pi^4|\gamma|}\left(\frac{k_B T}{mc^2}\right)^7\left(\frac{mc}{\hbar\rho^{1/3}}\right)^6\tilde{\Gamma}(\tilde{q})}$$

where the universal function $\tilde{\Gamma}(\tilde{q})$ is a quadruple integral with simple limiting behaviors

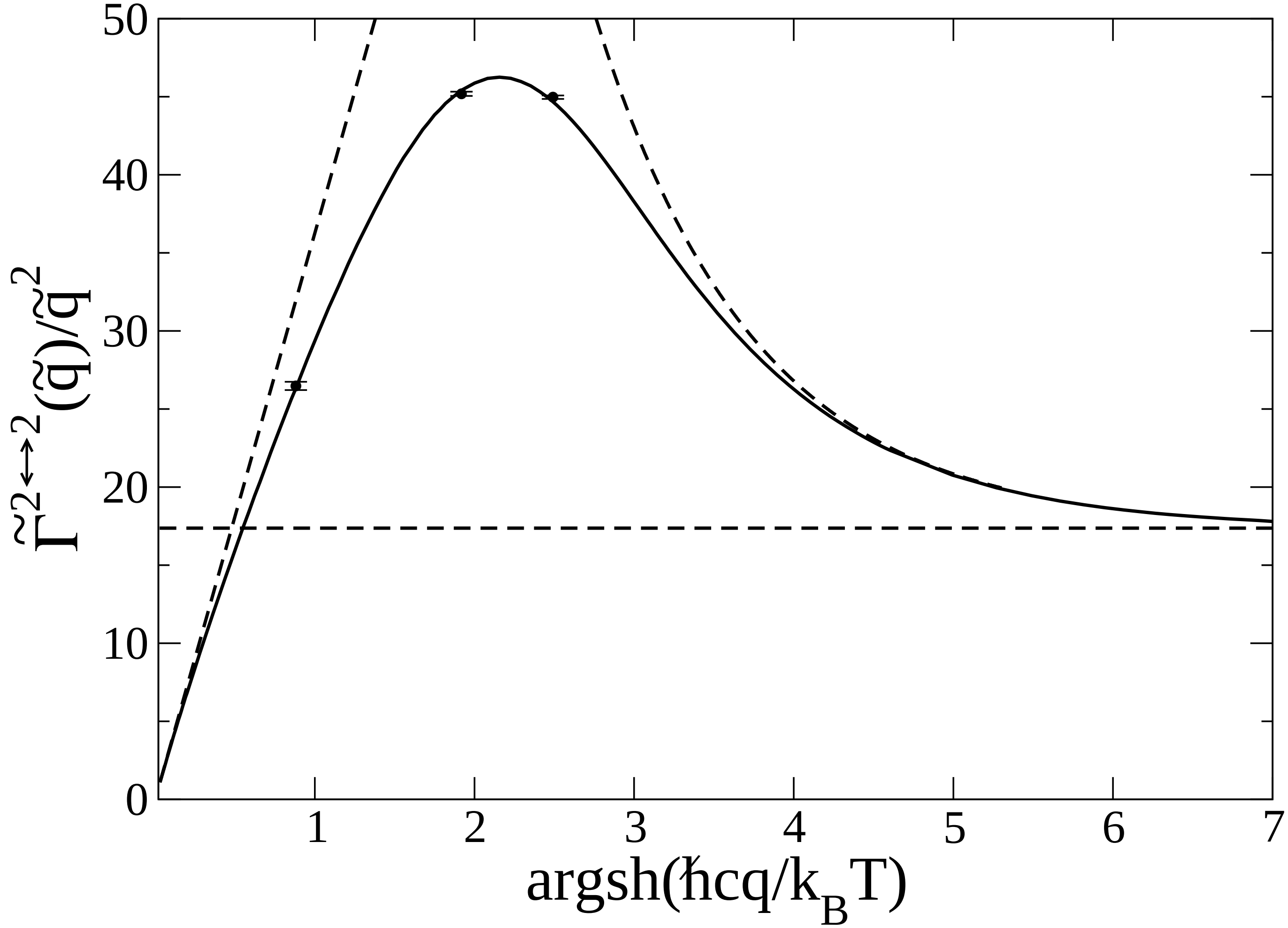
$$\boxed{\tilde{\Gamma}(\tilde{q}) \underset{\tilde{q}\to 0}{\sim} \frac{16\pi^5}{135}\tilde{q}^3} \quad \text{and} \quad \boxed{\tilde{\Gamma}(\tilde{q}) \underset{\tilde{q}\to +\infty}{\sim} \frac{16\pi\zeta(5)}{3}\tilde{q}^2}$$

- Understanding the scaling with temperature:

$$\Gamma \approx \int d^3q' d^3k |\mathcal{A}|^2 \delta(\omega_q + \omega_{q'} - \omega_k - \omega_{k'})$$

If the 4 vectors are aligned, the angular frequency difference $\Delta\omega$ vanishes for a linear spectrum, due to momentum conservation, and is $\approx q^3 \approx T^3$ for a curved spectrum. So

$$\Gamma \approx [T^3 \times \underbrace{T^2}_{\text{solid angle}}]^2 \left|\frac{T^{3/2} \times T^{3/2}}{T^3}\right|^2 \frac{1}{T^3} = T^7$$

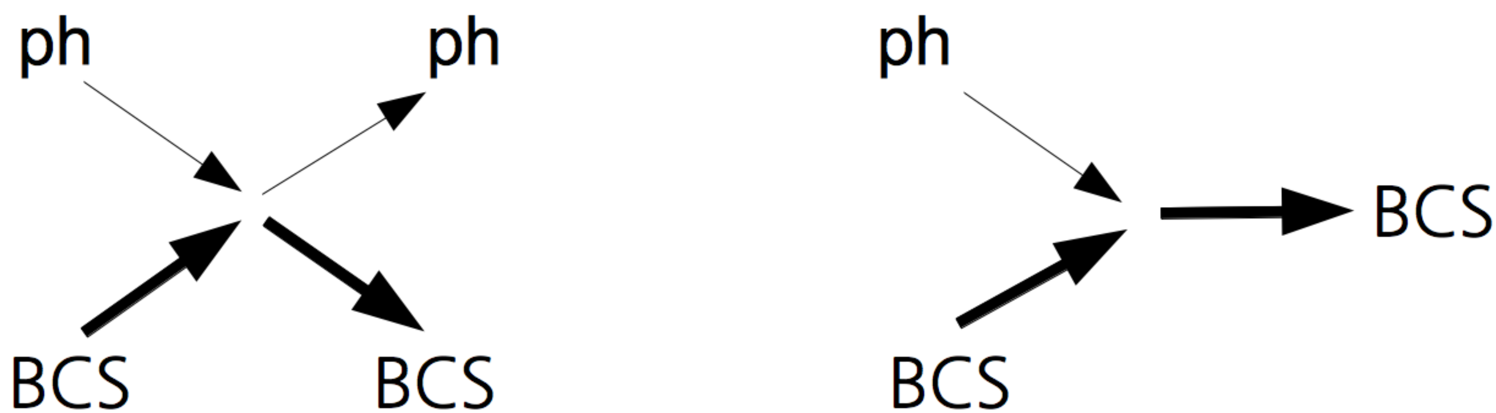


- **The T^7-law makes the observation challenging in Fermi gases (phonon lifetime ≈ second). Seems doable with liquid helium if one can excite ≈ 100 GHz sound.**

EFFET DES EXCITATIONS À BANDE INTERDITE

Hors de la limite $T \to 0$, ces excitations exponentiellemnt supprimées contribuent à l'amortissement des phonons.

- **Dans les fermions : excitations fermioniques par brisure de paire**
- **Dans l'He 4 liquide: rotons**



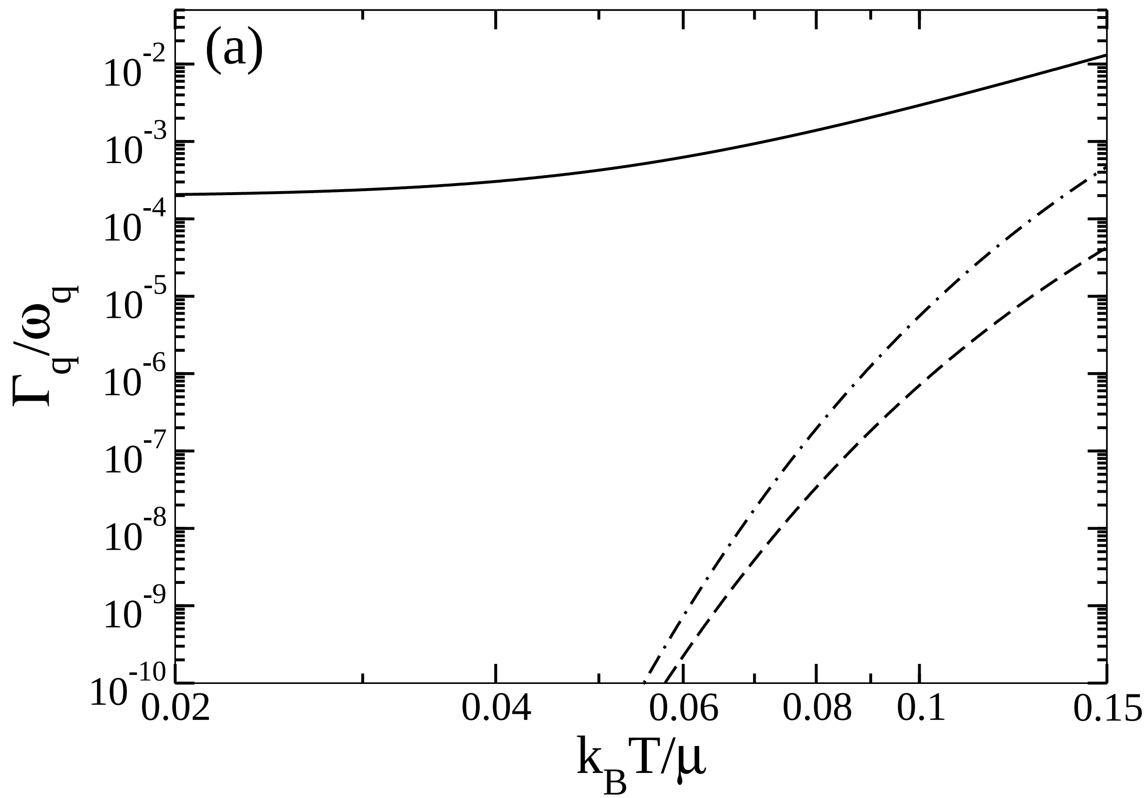
processus de diffusion **absorption-emission**

Nous calculâmes les taux d'amortissement correspondants, en étendant/corrigeant Landau !

DANS UN GAZ UNITAIRE DE FERMIONS

Phonons $q = mc/2\hbar$ à l'unitarité $a^{-1} = 0$, $\gamma > 0$, la plupart des paramètres des phonons et des quasi-particules fermioniques mesurés ou déduits de l'invariance d'échelle.

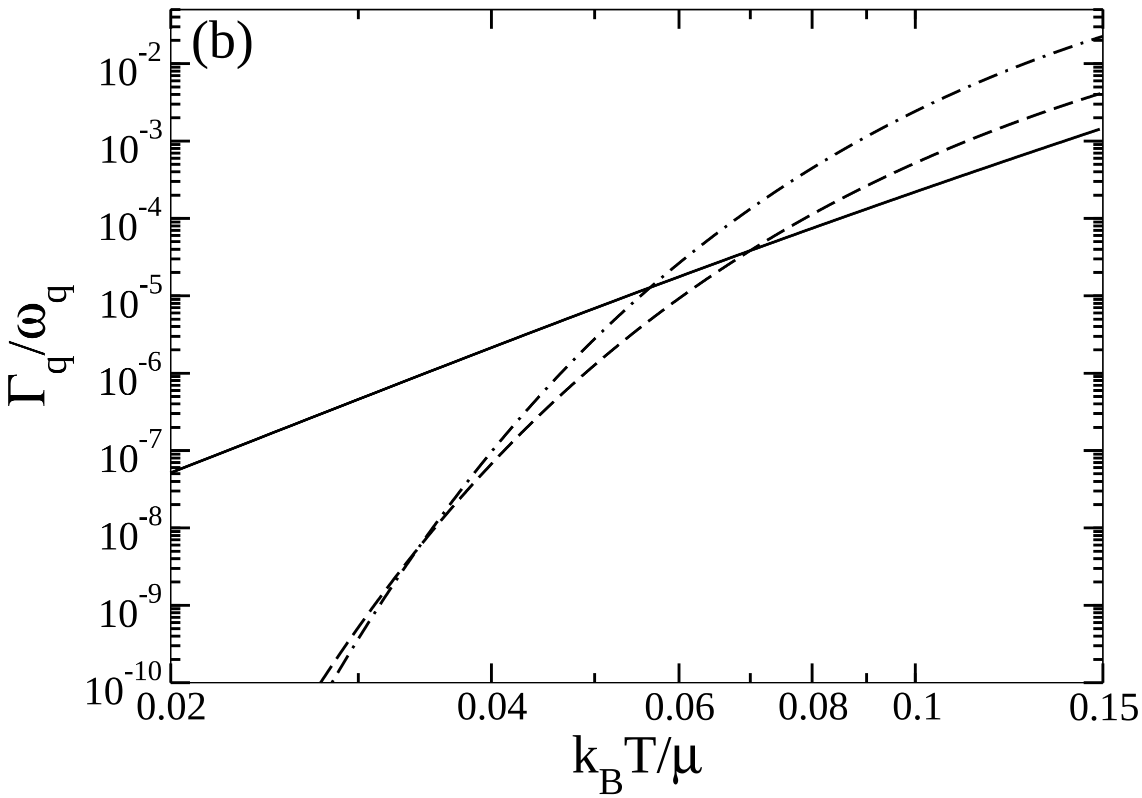
$\mu = \xi\epsilon_F$ with $\xi \simeq 0.376$



Plein: amor. Beliaev-Landau 3-phonon; Tireté: diffusion ph-BCS; Tireté-pointillé: absorption-emission

DU CÔTÉ BCS

Phonons $q = mc/2\hbar$ dans des fermions du côté BCS $1/k_F a^{-1} = -0.389$, $\gamma \simeq -0.30 < 0$. Paramètres des phonons et des quasi-particules fermioniques estimés par la théorie BCS $\mu/\epsilon_F \simeq 0.809$



Plein: amor. Landau-Khalatnikov 4-phonon; Tireté: diffusion ph-BCS; Tireté-pointillé : absorption-émission