Zoo of Quantum Halos

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Plan of this talk

1. Introduction to quantum halos
   • Efimov effect
   • Super Efimov effect

2. Semi-super Efimov effect
   • Model analysis
   • RG analysis and universality

3. Classification of quantum halos
   • Even more universality classes?

4. Summary

Introduction
Quantum halos

Structure and reactions of quantum halos

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(Published 5 February 2004)

This article provides an overview of the basic principles of the physics of quantum halo systems, defined as bound states of clusters of particles with a radius extending well into classically forbidden regions. Exploiting the consequences of this definition, the authors derive the conditions for occurrence in terms of the number of clusters, binding energy, angular momentum, cluster charges, and excitation energy. Transitions between different cluster divisions and the importance of thresholds for cluster or particle decay are discussed, with particular attention to the Efimov effect and the related exotic states. The pertinent properties can be described by the use of dimensionless variables. Then universal and specific properties can be distinguished, as shown in a series of examples selected from nuclear, atomic, and molecular systems. The neutron dripline is especially interesting for nuclei and negative ions for atoms. For molecules, in which the cluster division comes naturally, a wider range of possibilities exists. Halos in two dimensions have very different properties, and their states are easily spatially extended, whereas Borromean systems are unlikely and spatially confined. The Efimov effect and the Thomas collapse occur only for dimensions between 2.3 and 3.8 and thus not for 2. High-energy reactions directly probe the halo structure. The authors discuss the reaction mechanisms for high-energy nuclear few-body halo breakup on light, intermediate, and heavy nuclear targets. For light targets, the strong interaction dominates, while for heavy targets, the Coulomb interaction dominates. For intermediate targets these processes are of comparable magnitude. As in atomic and molecular physics, a geometric impact-parameter picture is very appropriate. Finally, the authors briefly consider the complementary processes involving electroweak probes available through beta decay, electromagnetic transitions, and capture reactions.

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Quantum halos

Efimov effect

✓ 3 bosons
✓ 3 dimensions
✓ s-wave resonance

Infinite bound states with universal scaling

\[ E_n \sim e^{-2\pi n} \]

Quantum halos can be arbitrarily large for \( n \gg 1 \)!
Quantum halos

Super Efimov effect

✓ 3 fermions
✓ 2 dimensions
✓ p-wave resonance

Infinite bound states with universal scaling

\[ E_n \sim e^{-2e^{3\pi n/4}} \]


Quantum halos can be arbitrarily large for \( n \gg 1 \)!
Quantum halos

Semi-super Efimov effect

✓ 4 bosons
✓ 2 dimensions
✓ 3-body resonance

Infinite bound states with universal scaling

\[ E_n \sim e^{-2(\pi n)^2/27} \]

\[ R_n \sim e^{(\pi n)^2/27} r_0 \]

Quantum halos can be arbitrarily large for \( n \gg 1 \)!

Y. Nishida, PRL (2017)
Semi-super Efimov effect
RG analysis

3-body scattering T-matrix in effective range expansion

\[ T_3(\varepsilon) = \frac{\pi^2}{1 + \varepsilon \ln\left(\frac{\Lambda}{\sqrt{-\varepsilon}}\right) + O(\varepsilon^2)} \rightarrow \frac{\pi^2}{\ln\left(\frac{\Lambda}{\sqrt{-\varepsilon}}\right)} \times \frac{1}{E - \frac{k^2}{6m} + i0^+} \]

( collision energy \( \varepsilon = E - \frac{k^2}{6m} + i0^+ \) )

\[ \Rightarrow \text{Propagation of a point-like trimer} \]

Singular wave function at origin: \( \Psi(r_1, r_2, r_3) \rightarrow \frac{C}{R^2} \)

Cf. p-wave in 2D, s-wave in 4D
RG analysis

Low-energy effective field theory

\[ \mathcal{L} = \phi^\dagger \left( i\partial_t + \frac{\nabla^2}{2m} \right) \phi + v_2 \phi^\dagger \phi^\dagger \phi \phi \]
\[ + \Phi^\dagger \left( i\partial_t + \frac{\nabla^2}{6m} \right) \Phi + g \Phi^\dagger \phi \phi \phi + g \phi^\dagger \phi^\dagger \phi^\dagger \Phi \]

2-body interaction

3-body int.

⇒ Propagation of a point-like trimer

Singular wave function at origin: \( \Psi(r_1, r_2, r_3) \xrightarrow{R \to 0} \frac{C}{R^2} \)

Cf. p-wave in 2D, s-wave in 4D
RG analysis

Low-energy effective field theory

\[ \mathcal{L} = \phi^\dagger \left( i \partial_t + \frac{\nabla^2}{2m} \right) \phi + v_2 \phi^\dagger \phi^\dagger \phi \phi \\
+ \Phi^\dagger \left( i \partial_t + \frac{\nabla^2}{6m} \right) \Phi + g \Phi^\dagger \phi \phi \phi + g \phi^\dagger \phi^\dagger \phi^\dagger \Phi \\
+ v_4 \phi^\dagger \Phi^\dagger \Phi \phi + v_6 \Phi^\dagger \Phi^\dagger \Phi \Phi \]

- **2-body interaction**
- **3-body int.**
- **4-body int.**

**coupling constants**

\[ \Rightarrow \text{running couplings} \]
RG analysis

Low-energy effective field theory

\[ \mathcal{L} = \phi^\dagger \left( i \partial_t + \frac{\nabla^2}{2m} \right) \phi + v_2 \phi^\dagger \phi^\dagger \phi \phi \]

2-body interaction

\[ + \Phi^\dagger \left( i \partial_t + \frac{\nabla^2}{6m} \right) \Phi + g \Phi^\dagger \phi \phi \phi + g \phi^\dagger \phi^\dagger \phi^\dagger \Phi \]

3-body int.

\[ + v_4 \phi^\dagger \Phi^\dagger \Phi \phi + v_6 \Phi^\dagger \Phi^\dagger \Phi \Phi \]

4-body int.

\[ \Rightarrow \quad \text{RG equations} \]

\[ (s \equiv \ln \Lambda / \kappa) \]

\[ \frac{dv_2}{ds} = \frac{v_2^2}{\pi} \]

\[ \frac{dg}{ds} = -\frac{g^3}{2\pi^2} + \frac{3gv_2}{\pi} \]

\[ = \frac{9g^2}{\pi} + \frac{3v_4^2}{4\pi} - \frac{g^2v_4}{\pi^2} + O(g^2v_2) \]
RG analysis

Solution for $v_2=0$ in the low-energy limit $s \equiv \ln \Lambda / \kappa \to \infty$

2-body

$$\frac{dv_2}{ds} = \frac{v_2^2}{\pi} \quad \Rightarrow \quad v_2(s) = 0$$

3-body

$$\frac{dg}{ds} = -\frac{g^3}{2\pi^2} + \frac{3gv_2}{\pi} \quad \Rightarrow \quad g^2(s) \to \frac{\pi^2}{s}$$

4-body

$$\frac{dv_4}{ds} = \frac{9g^2}{\pi} + \frac{3v_4^2}{4\pi} - \frac{g^2v_4}{\pi^2} + O(g^2v_2)$$

$$\Rightarrow \quad v_4(s) \to -\frac{6\pi}{\sqrt{3s}} \cot (\sqrt{27s} - \theta)$$

$$\quad \text{diverges at} \quad \sqrt{27s} = \pi n + \theta$$

$$\Rightarrow \quad 4\text{-body binding energies at} \quad \kappa_n \to e^{-(\pi n + \theta)^2 / 27 \Lambda}$$

Semi-super Efimov effect!
RG analysis

Solution for $v_2 \neq 0$ in the low-energy limit $s \equiv \ln \Lambda / \kappa \to \infty$

**2-body**
\[
\frac{dv_2}{ds} = \frac{v_2^2}{\pi} \Rightarrow v_2(s) \to -\frac{\pi}{s}
\]

**3-body**
\[
\frac{dg}{ds} = -\frac{g^3}{2\pi^2} + \frac{3gv_2}{\pi} \Rightarrow g^2(s) \to \frac{C}{s^6}
\]

**4-body**
\[
\frac{dv_4}{ds} = \frac{9g^2}{\pi} + \frac{3v_4^2}{4\pi} - \frac{g^2v_4}{\pi^2} + O(g^2v_2)
\]
\[
\Rightarrow v_4(s) \to -\frac{4\pi}{3s}
\]

Semi-super Efimov effect disappears

$\Rightarrow$ Double fine-tunings of 2-body and 3-body int. are needed but possible with ultracold atoms!

D.S.Petrov, PRL (2014); A.J.Daley, J. Simon, PRA (2014);
D.S.Petrov, PRA (2014); S.Paul, P.R.Johnson, E.Tiesinga, PRA (2016)
Quantum Halo Zoo
Universal scaling laws

Spinless bosons

- **3D + 2-body res.** ⇒ \( \kappa_n \sim e^{-\pi n/1.006} \)
  - V.Efimov, PLB (1970)
- **2D + 3-body res.** ⇒ \( \kappa_n \sim e^{-(\pi n)^2/27} \)
  - Y.Nishida, PRL (2017)
- **1D + 4-body res.** ⇒ \( \kappa_n \sim e^{-\pi n/1.247} \)

Spinless fermions

- **2D + 2-body res.** ⇒ \( \kappa_n \sim e^{-e^{3\pi n/4}} \)
- unknown in 3D & 1D

Interesting hierarchy & interplay among statistics, dimensionality, required interaction, and emergent universal scaling laws
Universal scaling laws

Spinless bosons

• $3D + 2$-body res. $\Rightarrow$ Efimov effect

• $2D + 3$-body res. $\Rightarrow$ Semi-super Efimov effect

• $1D + 4$-body res. $\Rightarrow$ Efimov effect

Spinless fermions

• $2D + 2$-body res. $\Rightarrow$ Super Efimov effect

• unknown in $3D & 1D$

Interesting hierarchy & interplay among statistics, dimensionality, required interaction, and emergent universal scaling laws
Known universal scaling laws are classified into

**(Normal) Efimov class**

\[ \kappa_n \sim e^{-\pi n / \gamma} \]

- 3 bosons in 3D (Efimov, 1970)
- 4 anyons in 2D (Nishida, 2008)
- 5 bosons in 1D (Nishida, Son, 2010)
- mass-imbalanced 3, 4, 5 fermions in 3D (Efimov, 1973; Castin et al., 2010; Bazak, Petrov, 2017)
- mixed dimensions (Nishida, Tan, 2008; 2011)

**(Semi-super Efimov class)**

\[ \kappa_n \sim e^{-\left(\frac{\pi n}{\gamma}\right)^2} \]

- 4 bosons in 2D (Nishida, 2017)
- mixed dimensions (Zhang, Yu, 2017)

**(Super Efimov class)**

\[ \kappa_n \sim e^{-e^{\pi n / \gamma}} \]

- 3 fermions in 2D (Nishida et al., 2013)
- mass-imbalanced bosons / fermions in 2D (Moroz, Nishida, 2014)
- mixed dimensions (Zhang, Yu, 2017)

next talk!
Known universal scaling laws are classified into

- **(Normal) Efimov class**
  \[ \kappa_n \sim e^{-\frac{\pi n}{\gamma}} \]

- **Semi-super Efimov class**
  \[ \kappa_n \sim e^{-\left(\frac{\pi n}{\gamma}\right)^2} \]

- **Super Efimov class**
  \[ \kappa_n \sim e^{-e^{\frac{\pi n}{\gamma}}} \]

TRiO of few-body universality classes

Q3. Even more universality classes?

⇒ My speculation is …
Known universal scaling laws are classified into:

1. **(Normal) Efimov class**
   - $\kappa_n \sim e^{-\pi n/\gamma}$
   - $V(R) \sim \frac{#}{R^2}$
   - Hyperspherical potential
   - No (arbitrarily large) quantum halos if power > 2

2. **Semi-super Efimov class**
   - $\kappa_n \sim e^{-(\pi n/\gamma)^2}$
   - $V(R) \sim \frac{#}{R^2(\ln R)}$

3. **Super Efimov class**
   - $\kappa_n \sim e^{-e^{\pi n/\gamma}}$
   - $V(R) \sim \frac{#}{R^2(\ln R)^2}$

References:
- V.Efimov, PLB (1970)
- Volosniev et al., JPB (2014); C.Gao et al., PRA (2015)
Known universal scaling laws are classified into:

- **(Normal) Efimov class**
  \[ \kappa_n \sim e^{-\pi n/\gamma} \]
  \[ V(R) \sim \frac{\#}{R^2} \]

- **Semi-super Efimov class**
  \[ \kappa_n \sim e^{-e^{\pi n/\gamma}} \]
  \[ V(R) \sim \frac{\#}{R^2(\ln R)^2} \]

- **Super Efimov class**
  \[ \kappa_n \sim e^{-e^{e^{\pi n/\gamma}}} \]
  \[ V(R) \sim \frac{\#}{R^2(\ln R)^2(\ln \ln R)^2} \]

If \( V(R) \sim \frac{\#}{R^2(\ln R)^2(\ln \ln R)} \), then
  \[ \Rightarrow \kappa_n \sim e^{-e^{\pi n/\gamma}} \]

- **Semi-hyper Efimov class**
- **Hyper Efimov class**
Q4. Do semi-hyper and hyper Efimov effects emerge in quantum few-body systems with short-range interactions?

⇒ I don’t know (at this moment).

\[
\begin{align*}
\text{If } & V(R) \sim \frac{\#}{R^2 (\ln R)^2 (\ln \ln R)} \\
\Rightarrow & \quad \kappa_n \sim e^{-e^{(\pi n/\gamma)^2}} \\
\text{Semi-hyper Efimov class} \\
\end{align*}
\]

\[
\begin{align*}
\text{If } & V(R) \sim \frac{\#}{R^2 (\ln R)^2 (\ln \ln R)^2} \\
\Rightarrow & \quad \kappa_n \sim e^{-e^{\pi n/\gamma}} \\
\text{Hyper Efimov class} \\
\end{align*}
\]
“Phase diagram”

Fates of N bosons with few-body attraction

<table>
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<tr>
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<tr>
<td>1D</td>
<td>$N^3$</td>
<td>$e^{8N^2}/\sqrt{3}\pi$</td>
<td>Efimov</td>
</tr>
<tr>
<td>2D</td>
<td>$e^{2.148 N}$</td>
<td>semi-super Efimov</td>
<td>—</td>
</tr>
<tr>
<td>3D</td>
<td>Efimov</td>
<td>—</td>
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Semi-universal
- low-lying states are non-universal
- higher excited states show universal scaling law

McGuire, JMP (1964); Sekino, Nishida, PRA (2018); Nishida, Son, PRA (2010); Hammer, Son, PRL (2004); Nishida, PRL (2917); Efimov, PLB (1970)
Known (arbitrarily large) quantum halos are classified into TRiO of few-body universality classes

✓ **(Normal) Efimov class**
\[ \kappa_n \sim e^{-\frac{\pi n}{\gamma}} \]

✓ **Semi-super Efimov class**
\[ \kappa_n \sim e^{-\left(\frac{\pi n}{\gamma}\right)^2} \]

✓ **Super Efimov class**
\[ \kappa_n \sim e^{-e^{\frac{\pi n}{\gamma}}} \]

Q. Even more universality classes such as

✓ **Semi-hyper Efimov class**
\[ \kappa_n \sim e^{-e^{\left(\frac{\pi n}{\gamma}\right)^2}} \]

✓ **Hyper Efimov class**
\[ \kappa_n \sim e^{-e^{e^{\frac{\pi n}{\gamma}}}} \]

✓ ... 

emerge in quantum few-body systems with short-range interactions ??