# S-wave pairing in neutron matter beyond BCS

Michael Urban (IPN Orsay)

in collaboration with

Sunethra Ramanan (IIT Madras)

# Outline

1. Introduction
2. $T_c$ within the Nozières-Schmitt-Rink approach
3. Screening corrections
4. Summary and outlook

# Reference


1. S. Ramanan and M. U., Phys. Rev. C 88, 054315 (2013)

# Neutron stars

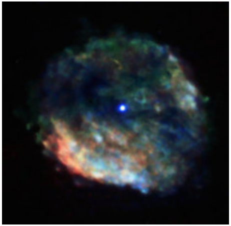
- Neutron star formed at the end of the "life" of an intermediate-mass star (supernova)
- $M \sim 1 - 2\ M_{\odot}$ in a radius of $R \sim 10 - 15$ km
  $\rightarrow$ average density $\sim 5 \times 10^{14}$ g/cm$^3$
  ($\sim 2\times$ nuclear matter saturation density)
- Cools down rapidly by neutrino emission within $\sim 1$ month: $T \lesssim 10^9$ K $\sim 100$ keV
- Internal structure of a neutron star:

  **outer crust:** Coulomb lattice of neutron rich nuclei in a degenerate electron gas

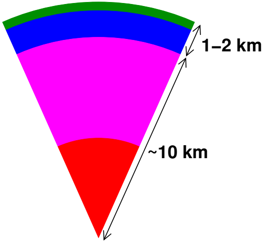
  **inner crust:** unbound neutrons form a neutron gas between the nuclei

  **outer core:** homogeneous matter ($n$, $p$, $e^-$)

  **inner core:** new degrees of freedom: hyperons? quark matter?



RCW103 [Chandra X-ray telescope]



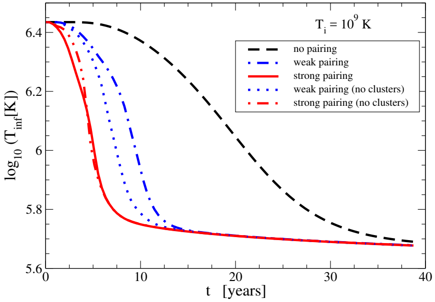
# Role of neutron pairing in neutron stars

- Outer core ($n_B \gtrsim 0.08$ fm$^{-3}$): triplet pairing ($^3P_2 - ^3F_2$ channel)
- Inner crust ($10^{-3}$ fm$^{-3} \lesssim n_B \lesssim 0.08$ fm$^{-3}$): singlet pairing ($^1S_0$ channel)

  $\rightarrow$ subject of this talk

- first approximation: treat the neutron gas in the inner crust as uniform neutron matter
- Value of the gap $\Delta$ in the inner crust strongly affects the cooling curve

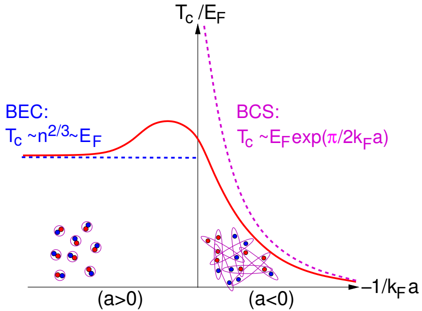
  [Fortin et al., PRC 82, 065804 (2010)]




- Superfluidity of neutron gas also responsible for 'glitches' (sudden changes in pulsar rotation frequency)

# Reminder: BEC-BCS crossover in ultracold atoms

- consider unpolarized Fermionic atoms with two spin states $\uparrow, \downarrow$
- low temperature ($\rightarrow$ low energy): contact interaction ($R = 0$)
- scattering length $a$ can be tuned (Feshbach resonance)
- on resonance: unitary limit $a \rightarrow \infty$
- molecules $\leftrightarrow$ fermionic atoms
- at zero temperature: crossover from BEC (molecules) to BCS superfluid (Cooper pairs)




- Nozières-Schmitt-Rink (NSR) theory includes non-condensed pairs above $T_c$
  $\rightarrow$ correctly interpolates between BEC and BCS limits
- At unitarity ($1/k_F a = 0$):

| | BCS | NSR | exp. |
|---|---|---|---|
| $T_c/E_F$ | 0.5 | 0.22 | 0.17 |

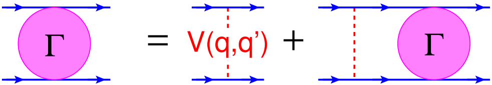
# BEC-BCS crossover in neutron matter?

- Neutron-neutron ${}^1S_0$ scattering length $a = -18$ fm
  much larger than range of interaction ($R \sim 1$ fm)
- At low density, one can simultaneously satisfy $k_F R \ll 1$ and $k_F |a| \gg 1$

  $\longrightarrow$ close to unitary limit: $k_F R \to 0$ and $k_F |a| \to \infty$
- At higher density: pairing gets weaker $\longrightarrow$ BCS regime
- No *nn* bound state $\longrightarrow$ BEC side of crossover cannot be realized
- NSR correction to $T_c/E_F$ should be important at low density

# T matrix with low-momentum interaction $V_{\text{low-}k}$

- $V_{\text{low-}k}$: low-momentum interaction generated from a realistic *NN* interaction by renormalization group methods (cutoff $\Lambda$)
- difficulty: numerical matrix elements $V(q,q')$, not separable
- T matrix:




$$\Gamma(K,q,q',\omega) = V(q,q') + \frac{2}{\pi}\int dq'' q''^2 V(q,q'')\bar{G}_0^{(2)}(K,q'',\omega)\Gamma(K,q'',q',\omega)$$

$$\bar{G}_0^{(2)}(K,q,\omega) = \text{angle average of } G_0^{(2)} = \frac{1 - f(\frac{\vec{K}}{2}+\vec{q}) - f(\frac{\vec{K}}{2}-\vec{q})}{\omega - \frac{K^2}{4m} - \frac{q^2}{m} + i\varepsilon}$$

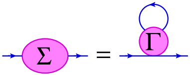
- solve this integral equation by diagonalizing $V\bar{G}_0^{(2)}$:

$$\frac{2}{\pi}\int dq' q'^2 V(q,q')\bar{G}_0^{(2)}(K,q',\omega)\phi_\nu(q',K,\omega) = \eta_\nu(K,\omega)\phi_\nu(q,K,\omega)$$

$\eta_\nu$: Weinberg eigenvalues [Weinberg (1963)]

# Contribution of non-condensed pairs to the density

- density from s.-p. Green's function: $n = \frac{2}{\beta} \sum_{\vec{k},\omega_n} \mathcal{G}(\vec{k},\omega_n)$ ($\omega_n$ =Matsubara frequency)
- BCS: $\mathcal{G} = \mathcal{G}_0 \quad \rightarrow \quad n = n_{\text{free}} = 2 \sum_{\vec{k}} f(\xi_{\vec{k}})$ (for $T \geq T_c$)
- NSR: truncate Dyson equation at 1st order in $\Sigma$:



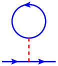

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0^2 \Sigma \quad \rightarrow \quad n = n_{\text{free}} + n_{\text{corr}}$$

$$n_{\text{corr}} = -\frac{\partial}{\partial \mu} \int \frac{K^2 dK}{2\pi^2} \int \frac{d\omega}{\pi} g(\omega)\, \text{Im} \sum_{\nu} \log(1 - \eta_\nu(K,\omega))$$ ($g$ =Bose function)

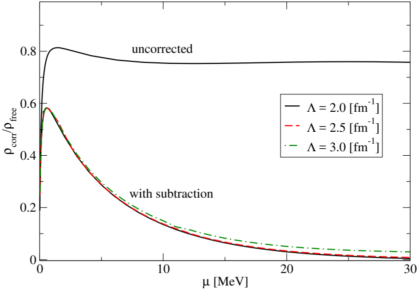
- mean-field shift $U_k = \Sigma(k, \xi_k)$ already included in s.-p. energy $\xi_k$
  [Zimmermann and Stolz (1985)]

$$\Sigma(k, i\omega_n) \rightarrow \Sigma(k, i\omega_n) - U_k$$

- approximate $U_k$ by HF self-energy

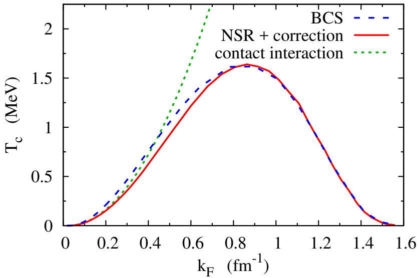


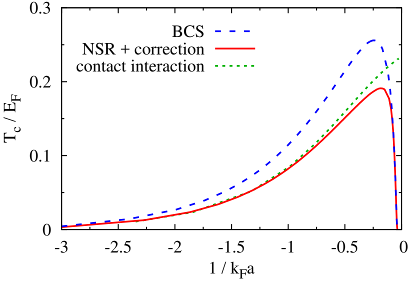
- $n_{\text{corr}}/n \rightarrow 0$ at large $n$
  but slightly cutoff dependent



# NSR critical temperature

- Thouless criterion: $\eta_\nu(K=0,\omega=0)=1$ at $T=T_c$

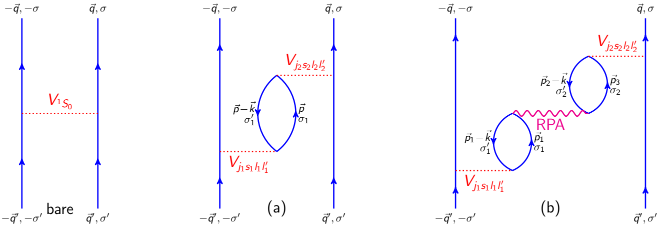



- $T_c$ up to 30% lower than $T_c^{\mathrm{BCS}}$ at low density
- $T_c \approx T_c^{\mathrm{BCS}}$ for $n \gtrsim 0.1\, n_0$ $(n_0 = 0.17\ \mathrm{fm}^{-3})$
- contact interaction is a good approximation only for $n \lesssim 0.002\, n_0$
- effects from $m^*$ and $3N$ force neglected [Hebeler and Schwenk (2010)]
- screening (particle-hole) effects?

# Screening

- diagrams (analogous to screening of Coulomb interaction)




($V$ = anti-symmetrized matrix element)

- contact interaction at weak coupling (diagram (a) only):
  repulsive exchange of spin fluctuations ($S = 1$) reduces $T_c$ by $\sim 50\%$
  [Gor'kov and Melik-Barkhudarov (1961)]

- away from weak-coupling limit: necessary to include RPA (diagram (b))

- previous work mostly uses drastic approximations:
  $V$ replaced with average matrix element to factorize loop integrals
  [e.g., Cao, Lombardo, and Schuck, PRC 74, 064301 (2006)]

# RPA effect on the $S = 0$ and $S = 1$ contributions

- diagram (a): $S = 0$ contribution attractive, $S = 1$ repulsive and about $3\times$ stronger than $S = 0$

- RPA in Landau approximation: ($\Pi_0$ = Lindhard function)

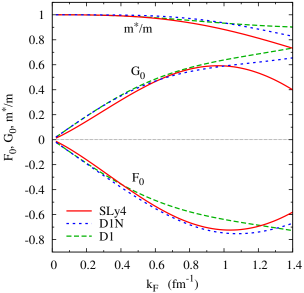
$$V_{ph}^{RPA} = \frac{f_0}{1 - f_0 \Pi_0} + \frac{g_0}{1 - g_0 \Pi_0} \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

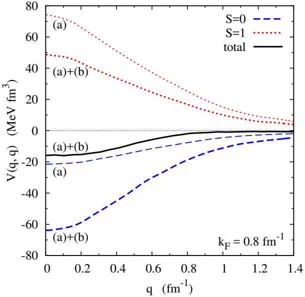
- generally $f_0 < 0$, $g_0 > 0$ (at least at low density) (contact interaction: $g_0 = -f_0$)

- $f_0 < 0 \;\rightarrow\;$ RPA enhances $S = 0$ contribution
  $g_0 > 0 \;\rightarrow\;$ RPA reduces $S = 1$ contribution

- RPA effect (diagram (b)) gets more important with increasing density

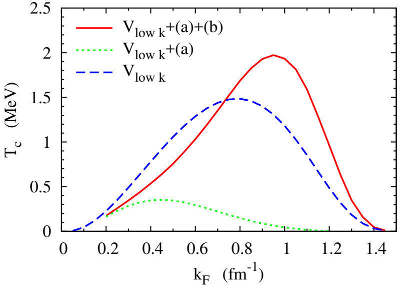
- example: at $k_F = 0.8\ \text{fm}^{-1}$, net result is attractive $\rightarrow$ antiscreening instead of screening!

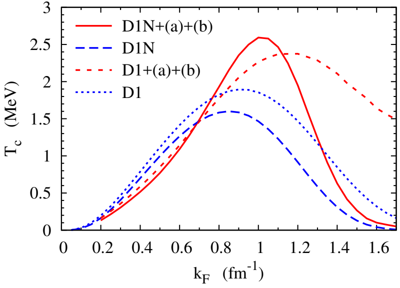


# Critical temperature

- $V_{\text{low-}k}+$ Landau parameters from SLy4:
- diagram (a) results in dramatic screening
- from $k_F \sim 0.7\ \text{fm}^{-1}$ ($n \sim 0.01\ \text{fm}^{-3}$), screening turns into antiscreening
  $\rightarrow$ $T_c$ is increased, not reduced!
- repeat calculation with Gogny D1 and D1N:
- $T_c$ depends on the choice of the interaction
- again, screening turns into antiscreening at $k_F \approx 0.7 - 0.8\ \text{fm}^{-1}$
- NSR effect not included here
- additional reduction of $T_c$ from quasiparticle residue ($Z$ factor $< 1$)?
  [Cao, Lombardo, and Schuck]



# Summary

- superfluid transition temperature $T_c$ of dilute neutron matter relevant for neutron stars (cooling, glitches)
- large theoretical uncertainties
- non-condensed pairs (NSR theory) reduce $T_c$ at low density ($\lesssim 0.01$ fm$^{-3}$) by up to 30%
- screening corrections: single bubble exchange diagram insufficient
- RPA bubble exchange: calculation without the usual approximations suggests that screening turns into antiscreening beyond $0.01 - 0.02$ fm$^{-3}$

# Outlook

- use screened interaction in NSR calculation
- reduction of $T_c$ due to quasiparticle residue $Z < 1$
- derive Fermi-liquid parameters and pairing from one interaction: in-medium similarity renormalization group (IMSRG) instead of $V_{\text{low-}k}$