Effective theory for the infrared regime of QCD

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Quantum chromodynamics: energy scales

Typical QCD scale of the order of the proton mass ~ 1 GeV.

At high energies ("perturbative regime"): typical energy involved in the process is much larger than 1 GeV,
eg, collisions at LHC.
Use perturbation theory

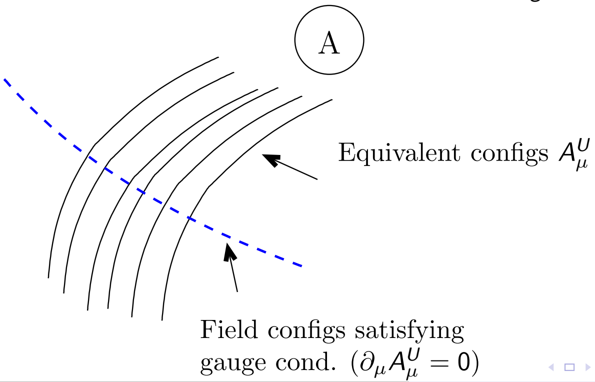
At low energies ("nonperturbative regime"): typical energy of the order of 1 GeV.
eg, Hadron spectrum, confinement criteria (Wilson loop), confinement-deconfinement phase transition, etc... As a benchmark: correlation functions.
Use Lattice simulations, Schwinger-Dyson equations, functional/nonperturbative RG, ...

Gauge fixing for QCD I

Yang-Mills theory described by Lagrangian density (in euclidean space)

$$\mathcal{L}_{\mathrm{YM}} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$
Necessary to fix the gauge ($A^U = UAU^\dagger + \frac{i}{g} U \partial U^\dagger$).



Gauge fixing for QCD II

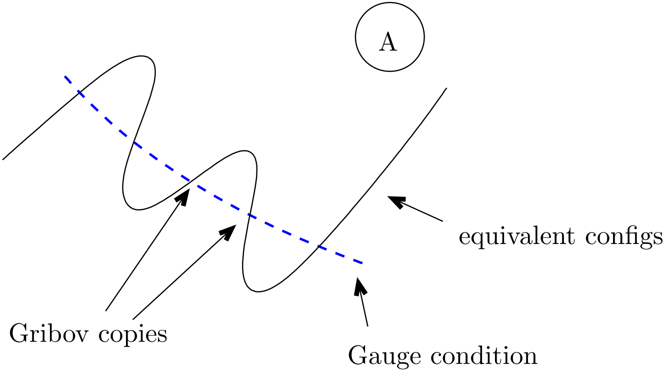
- With Faddeev-Popov construction, can be done at the expense of introducing auxiliary fields: ghost (c, $\bar{c}$) and Lagrange multiplyer (h).
- For the Landau gauge $\partial_\mu A_\mu^a = 0$,

$$\mathcal{L}_{\mathrm{FP}} = \partial_\mu \bar{c}^a (D_\mu c)^a + h^a \partial_\mu A_\mu^a$$

- The functional integral is limited to the gauge condition and the gauge group is factorized.

However... I

The situation looks more like that:



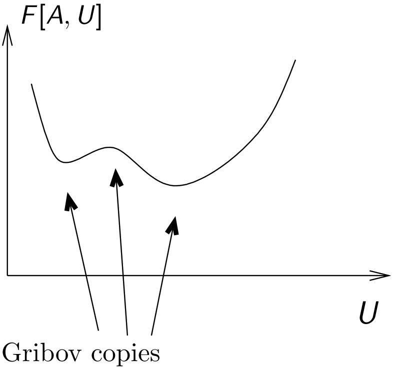
With a huge number of Gribov copies (for large lattices).

However... II

- In the presence of Gribov copies, the Faddeev-Popov construction sums over all copies, with alternating signs.
- There are as many pluses as minuses (topological constraint).
- All physical observables appear as 0/0 ratio (Neuberger's zero problem).
- Faddeev-Popov construction is not well-defined at a nonperturbative level.
- Gribov ambiguity has no influence at short distance. Up to now, we do not have a fully satisfactory starting point to describe analytically the infrared regime of QCD.
- We have to be cautious about the predictions of Faddeev-Popov in the infrared!

Input from lattice data.

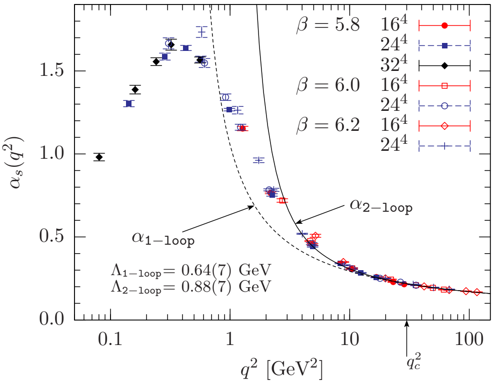
- In lattice simulations, no need to fix the gauge, but can be implemented.
- The extrema of $F[A,U]=\int \mathrm{Tr}\, A^U_\mu A^U_\mu$ satisfy $\partial_\mu A^U_\mu=0$.



- *Bona fide* gauge fixing. But it is not the Faddeev-Popov construction.

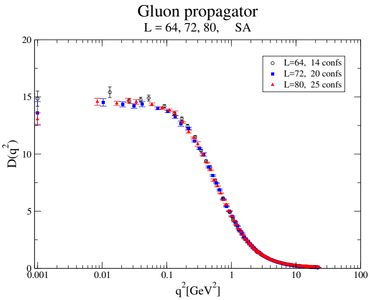
However... IV

Lattice data for the coupling (Sternbeck et al '05), extracted from ghost-gluon vertex (Beware that the coupling constant is not universal at low energies.):



The expansion parameter is $N\alpha/(4\pi)$. Not so large.

Gluon propagator is massive (Sternbeck et al '07)!



Hardly compatible with the Faddeev-Popov action (BRST symmetry + analyticity of correlation functions prevent this mass term).
Possible interpretation: Lattice data are indeed not described by the Faddeev-Popov action (beware, everybody would not agree with this interpretation).

Curci-Ferrari model I

Idea: Put the gluon mass by hand in the gauge-fixed bare action.

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \partial_\mu \bar{c}^a c(D_\mu c)^a + h^a \partial_\mu A^a_\mu + \frac{1}{2} m^2 (A^a_\mu)$$

We think of the mass term as an effective way of taking into account the Gribov copies.

Cons:

- We do not have a clean procedure to generate this mass.
- As a consequence, one more parameter in the theory...
- BRST symmetry is explicitely broken.
- Therefore the usual construction of the physical space does not apply (Ojima '82).

Curci-Ferrari model II

Pros:

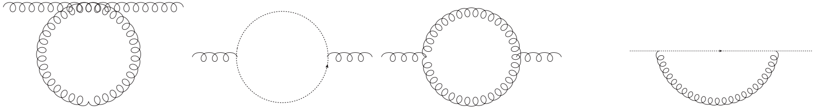
- BRST symmetry is softly broken. The theory is renormalizable (De Boer et al, '95). (There is a modified BRST symmetry, which is however not nilpotent.)
- Note that the mass term is added to the gauge-fixed action.
- Feynman rules are identical to usual ones, except for the massive gluon propagator:

$$\langle A_\mu A_\nu \rangle_0(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{1}{p^2 + m^2}$$

- Mass term regularizes the IR behavior of the theory (all diagrams are IR finite for non-exceptional momenta).
- Mass term does not modify the UV behavior. All UV properties of Yang-Mills theory are recovered.
- Ghosts remain massless. The compensation between gluon and ghost loops is only partial in the IR.

One-loop gluon and ghost propagators

Need to compute 4 Feynman diagrams



Define $\langle A_\mu A_\nu \rangle(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) G(p)$ $\qquad \langle c\bar{c}\rangle(p) = \frac{1}{p^2} F(p).$

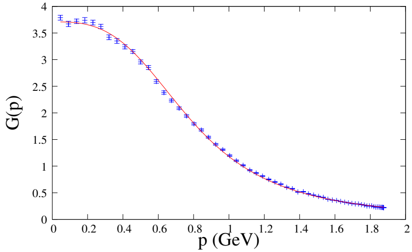
Introduce 4 renormaization parameters and you get $(s = p^2/m^2.)$:

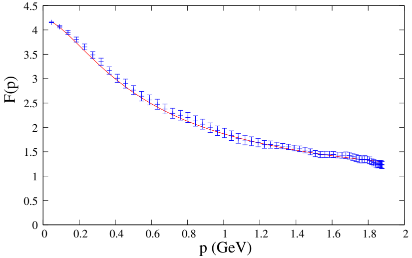
$$G^{-1}(p)/m^2 = s + 1 + \frac{g^2 N}{384\pi^2} s \Big\{ 111 s^{-1} - 2s^{-2} + (2 - s^2)\log s$$
$$+ (4s^{-1} + 1)^{3/2} \left(s^2 - 20s + 12\right) \log\left(\frac{\sqrt{4+s} - \sqrt{s}}{\sqrt{4+s} + \sqrt{s}}\right)$$
$$+ 2(s^{-1} + 1)^3 \left(s^2 - 10s + 1\right) \log(1+s) - (s \to \mu^2/m^2) \Big\},$$

$$F^{-1}(p) = 1 + \frac{g^2 N}{64\pi^2} \Big\{ - s \log s + (s+1)^3 s^{-2} \log(s+1) - s^{-1} - (s \to \mu^2/m^2) \Big\}.$$

Comparison with lattice data

For SU(2) (Cucchieri et al '08)





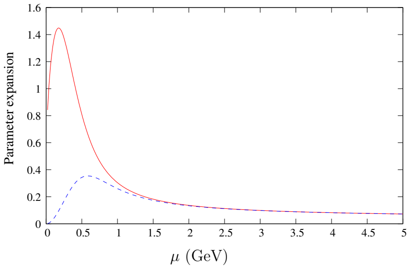
Renormalization-group flow

From renormalization factors, deduce a set of coupled β functions for g and m:

In the UV ($\mu \gg m$) $\beta_g \simeq -\frac{g^3 N}{16\pi^2}\frac{11}{3}$

In the IR ($\mu \ll m$) $\beta_g \simeq +\frac{g^3 N}{16\pi^2}\frac{1}{6}$

Moreover, there is an IR supression due to the coupling of the ghosts through massive gluons.



Similarly, gluon mass tends to 0 at high energy.

The phase diagram of QCD I

In heavy ion collisions, and core of neutron stars, matter reaches extreme conditions, with temperatures of the order of $\sim 10^{12}$ K, densities of $\sim 10^{18}$ kg/m^3.

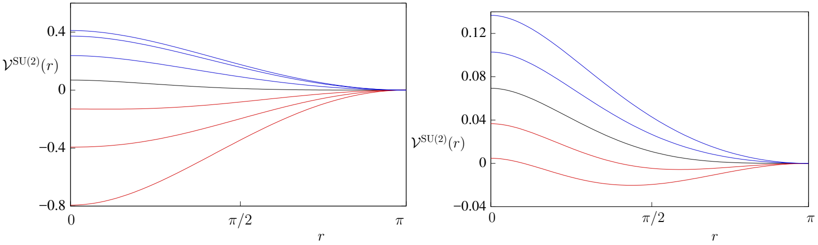
Typical values for strong interactions. In strong interactions units: $T \sim 1$ GeV, $\rho \simeq 1$ GeV/fm^3.

In the quenched approximation (no dynamic quarks), lattice simulations clearly show a phase transition at a temperature ~ 250 MeV, which is in the nonperturbative regime.

Extension to finite chemical potential is intricate. Lattice simulation are hard!

Phase diagram of QCD II

Describe the confinment/deconfinement transition in terms of a potential. If the minimum lies at π: confining phase (Polyakov loop vanishes).
At high temperatures (red), $V \to F_0(\beta g \bar{A})$.
At low temperatures (blue), $V \to -\frac{1}{2} F_0(\beta g \bar{A})$.



The leading order approximation captures the good physics!
Was extended to take into account the chemical potential, to next order at $\mu = 0$ ...

Conclusions

- Curci-Ferrari seems to capture many "nonperturbative" properties of QCD within perturbation theory.
- This would mean that the major nonperturbative ingredient is the gluon mass.
- We have a nice model to study low-energy properties of QCD. Tested in several situations.
- Can control chiral symmetry breaking along similar lines.
  - Wilson loop?
  - Two-loop calculations for the propagators?
  - Transport coefficients?
  - ...
- Can we generate the mass from first principles (relation with problems with disorder in stat. phys.)?
- Can we build a physical subspace?