Artificial gauge potentials for neutral atoms

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The context: ultracold quantum gases

Ultracold atomic gases as many-body systems
- dilute gases but *interacting* atoms
- experimental flexibility: trapping potential, interactions, density, ...
- microscopic properties well-characterized
- well-isolated from the external world

Bose-Einstein condensates:
- Superfluid gas
  “Atom laser”
  JILA, MIT, Rice (1995)

Optical lattices:
- Superfluid-Mott insulator transition
  Munich 2002

BEC-BCS crossover:
- Condensation of fermionic pairs
  JILA, MIT, ENS (2003-2004)

Many other examples:
- gas of impenetrable bosons in 1D,
- disordered systems, ...
- non-equilibrium many-body dynamics,
Optical lattices:
interference pattern can be used to trap atoms in a periodic structure

Bosons in the Bose-Hubbard regime:
1. Quantum tunneling favor delocalization
2. Repulsive on-site interactions favor localization

Quantum phase transition from a superfluid, Bose-condensed ground state to a Mott insulator

Energy/Temperature scales: nanoKelvin
Time scales $\sim 10$ ms
Vector potential $A$ in quantum mechanics: $\hat{H} = \frac{(\hat{p} - qA)^2}{2m}$, $\nabla \times A = B$

Electrons in a magnetic field exhibit many different and fascinating effects:

- Landau diamagnetism, Shubnikov-De Haas oscillations,
- Vortices in type II superconductors,
- Coherence in mesoscopic physics, ...
- Quantum Hall effect (integer and fractional)

Fractional Quantum Hall effect:

Emergence of strongly correlated phases of matter:

- Incompressible liquids (gap)
- Exotic excitations with fractional charge and statistics (“anyons”)
- Very similar Quantum Hall states are predicted for ultracold atomic gases [Cooper, Adv. Phys. 2008].

Key elements: flat dispersion relation and interactions
Rotating Bose-Einstein condensates

What about neutral particles (atoms)?

Mathematical identity between Coriolis and Lorentz force:

\[ F_{\text{Coriolis}} = m v \times \Omega \]

\[ F_{\text{Lorentz}} = q v \times B \]

Rotation around \( z \), rotation rate \( \Omega \)  
Magnetic field along \( z \), strength \(|B|\)

Rotating superfluid atomic gases:

- Formation of quantized vortices
- Ordering into triangular vortex lattice

Rapidly rotating atomic gases (bosonic and fermionic):

- Theory: strongly correlated ground states akin to fractional quantum Hall phases  
  [Cooper, Adv. Phys. 2008]
- Experiments: so far unable to reach this regime.
Aharonov-Bohm and geometric phases

Can we explore orbital magnetism with electrically neutral atoms?

Vector potential $A$ in quantum mechanics:

$$\hat{H} = \frac{(\hat{p} - qA)^2}{2m}$$

$\mathbf{\nabla} \times \mathbf{A} = \mathbf{B}$

Aharonov-Bohm phase:

$$\phi_{AB} = \frac{q}{\hbar} \int_{C} A \cdot dl = \frac{q}{\hbar} \int_{S} \mathbf{B} \cdot d\mathbf{S}$$

What about neutral particles (atoms)?

- Orbital magnetism can be simulated by generating geometric phases

$$\phi_{geo} \equiv \frac{1}{\hbar} \int_{S} (q\mathbf{B})_{eff} \cdot d\mathbf{S}$$

- Coherent atom-light coupling in quantum optics

Review articles: J. Dalibard, F. Gerbier, P. Ohberg, G. Juzeliunas, RMP 2011

Coherent atom-light interaction

Two-level atom and monochromatic light:

- two internal states $g$ and $e$
- Hamiltonian after rotating wave approximation:

$$\hat{H}_{\text{RWA}} = \begin{pmatrix} 0 & \frac{\hbar \Omega_L}{2} e^{i\varphi} \\ \frac{\hbar \Omega_L}{2} e^{-i\varphi} & -\delta \end{pmatrix},$$

Lowest eigenstate with energy $E_- = -\frac{1}{2} \hbar \Omega = -\frac{1}{2} \sqrt{\Omega_L^2 + \delta_L^2}$:

$$|\phi_-\rangle = \sin\left(\frac{\theta}{2}\right) |g\rangle - e^{i\varphi} \cos\left(\frac{\theta}{2}\right) |e\rangle$$

where $\cos(\theta) = \frac{\delta_L}{\Omega}$.

Adiabatic preparation:

$$\theta = \pi \longrightarrow \theta = 0$$

$$|\phi_-\rangle = |g\rangle \longrightarrow |\phi_-\rangle = -e^{i\varphi} |e\rangle$$
Harper Hamiltonian for a charged particle on a tight-binding lattice

**Bulk:**

\[ \hat{H} = \frac{(\hat{p} - qA)^2}{2m} \]
\[ \nabla \times A = B \]

**Tight-binding lattice:**

\[ H = - \sum_{\langle r_i, r_j \rangle} J e^{i\phi_{AB}(r_i \rightarrow r_j)} \hat{a}_i^{\dagger} \hat{a}_j + \text{h.c.} \]

\( J \): single-particle tunnel energy

**Complex tunnel coefficients:**

\[ \phi_{AB}(r_i \rightarrow r_j) = \frac{q}{\hbar} \int_{r_i}^{r_j} A \cdot dl \]

\[ \alpha = \frac{|q| B d^2}{\hbar} = \frac{\text{Magnetic flux/unit cell}}{\text{Magnetic flux quantum}} \]

Landau gauge: \( A = -By e_x \)

\[ \alpha = \begin{cases} 
\sim 10^{-4} \text{ in usual solids with } \sim 50 \text{ T} \\
\sim 2\pi \text{ in solid-state superlattices or cold atoms.} 
\end{cases} \]
Hofstadter butterfly

Energy spectrum vs flux:
Flux per unit cell: $2\pi\alpha$

- Fragmentation of the Bloch bands
- Wide gaps, flat bands

Rational flux $\alpha = p/q$:

Magnetic unit cell $(1 \times q)$: $q$ topological bands with Chern number $C \neq 0$: 

\[
E(k_x) = J \left( k_x - \frac{1}{q} \right)^2
\]
Floquet approach

- Fast modulation in the Hamiltonian at $\Omega$, induced “micromotion” at the same frequency
- Slow (“secular”) motion governed by an effective Hamiltonian $\hat{H}_{\text{eff}}$

$$\hat{H}_{\text{eff}} \approx \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} \hat{H}(t) dt$$

- Shaken optical lattices: Pisa, Hamburg, Zürich, Chicago, ...
- Sliding optical lattices: Munich, MIT

“Quantum optics” approach

- internal states coupled by one or two-photon transitions
- affects the external state as well due to the recoil effect
  - Adiabatic dressing (bulk systems): NIST 2008
  - Synthetic dimensions: NIST, Florence 2015

- Most of the schemes (proposed and realized) rely on optical lattices,
- All involve breaking some symmetry of the “bare” lattice Hamiltonian and projection onto a low-energy subspace.
Topological phases of matter

Phase of matter characterized by one (or more) integer-valued *Topological invariants* linked to certain physical properties.

- gapped phase (band gap in fermionic insulators)
- robustness with respect to microscopic changes (as long as the gap do not close)
- **Bulk-edge correspondence**:
  - Edge-state channels in a bounded geometry, which carry current (or heat or ...)
  - Protected against scattering
  - Number of channels determined by topological properties of the bulk

Materials/Phases that are insulating in the bulk, but with a perfect (or very good...) conducting surface

Note that these phases usually escape the standard classification of phases in condensed matter using the concept of order parameter and Landau theory.
Snapshots of experiments using the Floquet approach

Lattice shaking:

\[ V_{\text{lat}}(x, t) = -V_0 \cos [k_L (x - x_0(t))] \]

Staggered flux in hexagonal lattices:

Figures from J. Struck et al., Hamburg
Also Pisa, Chicago, Zürich, Munich, ...

M. Aidelsburger et al. (Munich), 2012

Uniform flux in square lattices:

Figures from J. Struck et al., Hamburg
M. Aidelsburger et al. (Munich), 2015
Similar experiments at MIT (Ketterle group)

Sliding lattice:

\[ W(x, t) = W_0 \cos [\delta k \cdot r - \Delta \omega t] \]

Staggered flux in square lattices:

Figures from J. Struck et al., Hamburg
M. Aidelsburger et al. (Munich), 2012

Also Pisa, Chicago, Zürich, Munich, ...

M. Aidelsburger et al. (Munich), 2015
Similar experiments at MIT (Ketterle group)
Quantized Hall Conductivity and Chern Number

**Hall conductivity**: current along $x$ flowing in response to an applied electric field $E_y$ along $y$, $j_x = \sigma_H E_y$

Linear response (Kubo formalism) for fermionic insulators (Fermi energy inside a gap):

$$\sigma_H = \frac{e^2}{h} \times \sum_{\epsilon_n(k) < E_F} C_n$$

$C_n$: Chern number of the $n$th band
Necessarily an integer!
Topological invariant that do not change by smooth deformation of the Hamiltonian

**Topological classification of surfaces in 3D space**:
Surfaces in ordinary 3D space can be classified by their *genus* ($\equiv$ number of handles):

Mapping $(x, y) \rightarrow S(x, y)$

Gauss-Bonnet formula: $\int_S \mathcal{G} = 4\pi(1 - g)$

$\mathcal{G}$: Gaussian curvature

Mapping $(k_x, k_y) \rightarrow \hat{H}(k)$

Chern formula: $\int_{BZ} \mathcal{B}(k) = C$

$\mathcal{B}$: Berry curvature
• apply additional “electric field” $V = \mathbf{F} \cdot \mathbf{r}$ [$\mathbf{F}$: constant force]

• $\mathcal{B} \neq 0$: additional deflection of the c.o.m. transverse to $\mathbf{F}$, $\bar{x} \propto FCt$

Transverse displacement after one period $T_B = C \times$ lattice spacing

Munich experiment: Aidelsburger et al., Nat. Phys. 2014

• Hofstadter lattice with $\alpha = 1/4$

• bandwidth $\ll k_B T \ll$ band gap

$P_0 \approx 1 \implies C_0 \approx 0.9(1)$

Direct measurements of Berry curvature: Fläschner et al., Science 2016
see also: Duca et al., Science 2014.
Synthetic dimensions

Internal degree of freedom (Zeeman states) $\equiv$ sites of a fictitious lattice

Two-photon Raman transition $\equiv$ hopping in a tight-binding model

References: M. Mancini et al.; B. Stuhl et al., Science 2015
Figure taken from the LENS experiment (Mancini et al.)

- Allows to study four-dimensional physics!
- System size necessarily small in the synthetic dimension
- Interactions are local in real space, of “infinite range” in the synthetic dimension
Bosons in topological bands

Generally we expect bosons at low $T$ to condense into the single-particle minima.

$\alpha = 0, \ 1/2$ : BEC observed

2D: Struck et al., Nature Physics 2012

3D : Kennedy et al., Nature Physics 2015

Experiments with $\alpha \neq 0, \ 1/2$ : BEC does not survive, lowest band (almost) uniformly filled ($T \gg$ bandwidth)

- Heating generally observed in shaking experiments (timescale $\sim 50$ ms)
- Redistribution of the “micromotion” energy by collisions
- Possibly off-resonant transfer from the ground to higher bands?
- Currently under active investigation in several experimental groups
• Realization of topological band structures with cold atoms

• Two broad classes of approaches:
  - Floquet methods with rapidly modulated potentials,
  - “Quantum optics” methods using coherent manipulation of the atom internal degrees of freedom.

• Experiments have demonstrated single-particle effects tied to the topological band structure.

• The goal of studying strongly interacting topological phases is still elusive: heating issues encountered in experiments must be resolved.

• Non-Abelian gauge potentials can also be realized using similar techniques:
  - Spin-orbit coupling, 2D or 3D Kane and Hasan, RMP 2010
  - Topological superfluids: Pairing interaction required
    • $p$−wave order parameter
    • zero-energy modes behaving as Majorana fermions
• fermionic band insulator with the Fermi energy inside a gap
• topological band structure

**Chern insulators**: topological invariant = Chern number

• integer Quantum Hall states (yet to be demonstrated with cold atoms)
• Haldane insulator (realized by the ETH Zürich group)

Many more possibilities with **non-Abelian gauge potentials**:

• **Spin-orbit coupling**, 2D or 3D Kane and Hasan, RMP 2010

• **Topological superfluids**: Pairing interaction required
  • $p-$wave order parameter
  • zero-energy modes behaving as Majorana fermions
Towards atomic fractional Quantum Hall states?

Relevant parameter:

$$\nu = \frac{\text{atomic density}}{\text{flux per unit cell}} = \frac{n}{\alpha}$$

- Analogue of continuum (≡ Lowest Landau level) states exist.

Example: Laughlin states

- fermions: $\nu = \frac{1}{3}, \ldots$
- bosons: $\nu = \frac{1}{2}, \ldots$

- Many possible states without continuum counterparts [Möller and Cooper, PRL 2009].

Example for $\alpha = \frac{1}{5}$:

- Laughlin state for particles at $n = \frac{1}{10}$
- Laughlin state for holes at $n = 1 - \frac{1}{10}$

Gaps are small:

at most $\sim 0.1J$ for the $\nu = \frac{1}{2}$ bosonic Laughlin state [Hafezi et al., PRA 2007]

Narrow slices in the global phase diagram

Sorensen et al., PRL 2005
Hafezi et al., PRA 2007, EPL 2008
Palmer, Klein, Jaksch, PRL 2006; PRA 2008
Möller, Cooper, PRL 2009 ...
Some recent works:

- **Revealing the Topology of Quasicrystals with a Diffraction Experiment**
  

- **Clock spectroscopy of interacting bosons in deep optical lattices**
  
Rabi spectroscopy on the clock transition: time domain

Strong driving: Rabi oscillations of a BEC in the optical domain

Total atom number: \( N \approx 8 \times 10^4 \)

For much lower atom numbers: \( N \approx 8 \times 10^3 \)
Optical atomic clock technology to study many-body phenomena

Level structure of Ytterbium

- "clock" transition \( J = 0 \rightarrow J' = 0 \)
- virtually no spontaneous emission
  \( \rightarrow \) coherent manipulation

State-dependent 2D optical lattice

- \( y \) lattice at "magic" wavelength: \( V_e(y) = V_g(y) \)
- \( x \) lattice at "anti-magic" wavelength: \( V_e(x) = -V_g(x) \)

- regular tunneling along \( y \)
- supressed tunneling along \( x \)

\( \lambda_x = 610 \text{ nm} \)
\( \lambda_y = 759.5 \text{ nm} \)
Proposal for alkali atoms in [Jaksch and Zoller, NJP 2003]

- two internal states $g$ and $e$
- state-dependent potential confining the atoms at distinct places depending on their internal state

Coupling laser $|g; R_g\rangle \rightarrow |e; R_e\rangle$:

$\langle e; R_e | \hat{V}_{AL} | g; R_g \rangle \propto e^{i k_L \cdot \frac{R_g + R_e}{2}}$

Not enough to get $\oint A \cdot dl \neq 0$, but good starting point!

Coherent driving of the clock transition: Bougaanne et al., NJP 2017