Dimer-dimer zero crossing in a one-dimensional mixture

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LPTMS

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Motivation

- Bosonic mixtures of two different components with competing attractive inter and repulsive intra species interaction caught recently a great deal of attention.
- Ability to form self trapped liquid droplets.
- Finite piece of liquid in equilibrium with vacuum without any external potential !
- Many possible applications of these droplets. (cooling, ...)
- Direct manifestation of Beyond-Mean-Field (BMF) effects.

Mixture & Stability

- Mixture : 2 6= particles (", #) of equal masses and same densities.
- Interactions ($g_{"\#}, g_{""}, g_{\#\#}$) with attractive inter and repulsive intra species. What about stability (i.e. no collapse) ?

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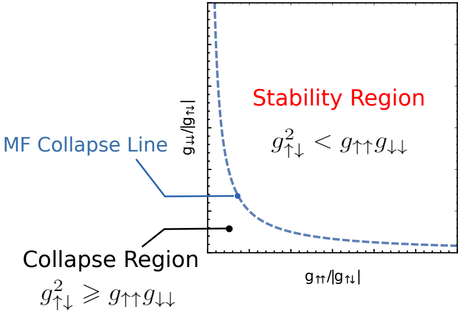
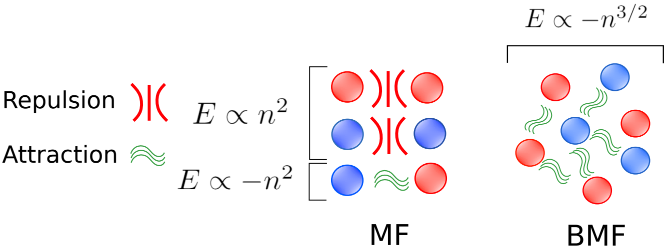
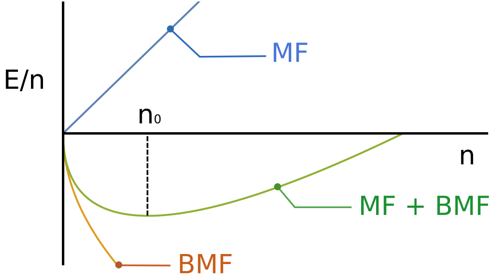


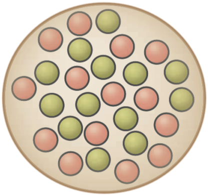
... In 1D, close to this MF collapse line, on the MF repulsive side <u>the mixture liquefy</u> !

! <u>Comes from an effectively attractive BMF term</u>.

A Brief Insight



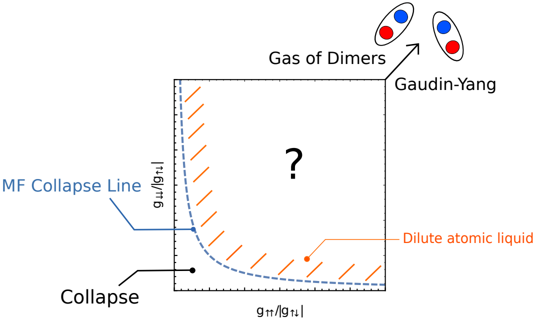




Overview of one dimensional Bose-Bose mixture at this stage

- Increasing $g_{""}g_{\#\#}/g^2_{"\#}$ makes the system more dilute which in one dimension leads to stronger correlations.

- For $g_{\sigma\sigma}$ $jg_{"\#}j$, ,the system becomes a gas of "# dimers.(Mapping with Gaudin-Yang model which has no other BS than "# dimers.)



Ψ_B, $g_{""} = g_{\#\#} = +1$, Ψ_F, $g_{""} = g_{\#\#} = 0$

Goal

By decreasing the ratio $g_{""}g_{\#\#}/g^2_{"\#}$:

! Find the line curve in the plane $fg_{""}/jg_{"\#}j, g_{\#\#}/jg_{"\#}jg$ where the dimer-dimer interaction vanishes ($a_{dd} = 1$).

Outline

Reminders about 1D systems

$$[\ \frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + V(|x|)]\Psi(x) = E\Psi(x) \quad (1)$$

- $|x| \quad R_e \ \to \ \Psi(x) = A\sin(k|x| + \delta(k))$
- δ is the so-called *phase shift*
- k is the relative momentum and verify $E = \hbar^2 k^2/2\mu$.

Two-body scattering theory in 1D

$$[-\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + V(|x|)]\Psi(x) = E\Psi(x) \quad (1)$$

- $|x| \gg R_e \to \Psi(x) = A\sin(k|x| + \delta(k))$
- δ is the so-called *phase shift*
- k is the relative momentum and verify $E = \hbar^2 k^2/2\mu$.

Scattering length a

- $\lim_{x\to 0} \Psi(x) \propto |x| - a(k)$ where $a(k) = -\frac{\sin(\delta)}{k\cos(\delta)}$
- Small momenta $kR_e \ll 1$, we define the scattering length a :

  $a = \lim_{k\to 0} a(k) = \lim_{k\to 0} -\frac{\tan\delta}{k}$
- Expansion : $k\cot\delta(k) = -1/a + (r_e/2)k^2 + ...$

Two-body contact interaction

- Short range interactions between particle modeled by a δ-potential such that : $V_{ij}(x) = g_{ij}\delta(x_{ij})$.
- It's the so-called *Zero-Range Approximation*, valid when $j\lambda_{dB}j \qquad R_e$
- One can show with the logarithm derivative of Ψ_{j0} that the constant interaction g_{ij} is related to the scattering length.

$$\boxed{g_{ij} = \quad \frac{\sim^2}{\mu a_{ij}}}$$

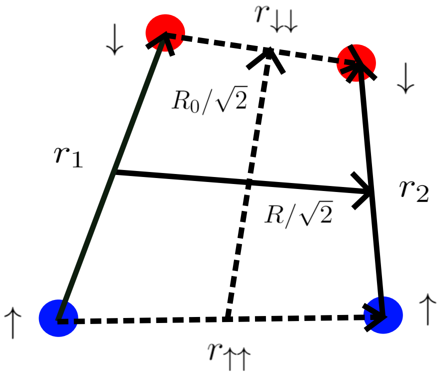
- If $g_{ij} > 0$ one deals with a repulsive potential, whereas for $g_{ij} < 0$ we deal with an attractive potential.
- If $g_{ij} < 0$ a bound state (BS) exists.

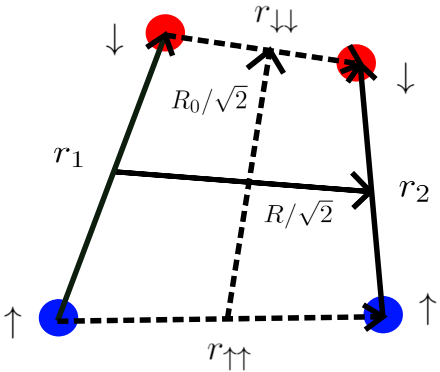
$$\boxed{\Psi_{BS} \;/\; e^{\quad jxj/a_{ij}} \qquad E_{BS}(f m_1, m_2 g, g_{ij}) = \quad \frac{\sim^2}{2\mu a_{ij}^2}}$$

! a_{ij} can be seen as the *size* of the bound state.

The Four-Body system

Schrödinger's equation





Symmetries ($e_{\sigma\sigma} = \quad 1$)

$$\Psi(r_1, r_2, R) = e_{\#\#}\Psi\left(r_+, r\;\;, \frac{r_1 \;\; r_2}{p\,\overline{2}}\right)$$

$$= e_{""}\Psi\left(r\;\;, r_+, \frac{r_2 \;\; r_1}{p\,\overline{2}}\right)$$

$$= e_{""}e_{\#\#}\Psi(r_2, r_1, \quad R)$$

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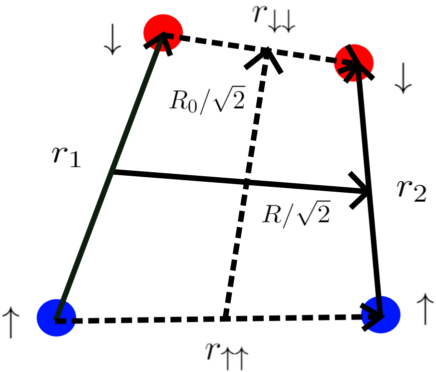

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$$= e_{""}e_{\#\#}\Psi(r_2, r_1, \quad R)$$

$$(\quad r^{2}_{X} \quad E)\Psi(r_1, r_2, R) = [\quad g_{"\#}(\delta(r_1) + \delta(r_2) + \delta(r_+) + \delta(r\quad)) \quad g_{""}\delta(r_{""}) \quad g_{\#\#}\delta(r_{\#\#})]\Psi(r_1, r_2, R) \qquad (2)$$

Skorniakov-Ter-Martirosian (STM)

We introduce the function $f_{"\#}$ which corresponds by definition to the wavefunction Ψ when one pair $f"\#g$ coincide.

$$\lim_{r_1 \ ! \ 0} \Psi(r_1, r_2, R) = f_{"\#}(r_2, R) \tag{3}$$

We do the same for $r_{""} \ \ ! \ \ 0$ and then $r_{\#\#} \ \ ! \ \ 0$:

$$\lim_{r_{""} \ ! \ 0} \Psi(r_1, r_2, R) = f_{""}(r_{\#\#}, R_0) \tag{4}$$

$$\lim_{r_{\#\#} \ ! \ 0} \Psi(r_1, r_2, R) = f_{\#\#}(r_{""}, R_0) \tag{5}$$

STM Equation

$$(\mathbf{P}^2 - E)\tilde{\Psi}(p_1,p_2,p) = -g_{\uparrow\downarrow}\tilde{f}_{\uparrow\downarrow}(p_2,p) - e_{\uparrow\uparrow}e_{\downarrow\downarrow}g_{\uparrow\downarrow}\tilde{f}_{\uparrow\downarrow}(p_1,-p)$$
$$- e_{\downarrow\downarrow}g_{\uparrow\downarrow}\tilde{f}_{\uparrow\downarrow}\left(\frac{p_1+p_2-\sqrt{2}p}{2}, \frac{p_1-p_2}{\sqrt{2}}\right)$$
$$- e_{\uparrow\uparrow}g_{\uparrow\downarrow}\tilde{f}_{\uparrow\downarrow}\left(\frac{p_1+p_2+\sqrt{2}p}{2}, \frac{p_2-p_1}{\sqrt{2}}\right) \quad (6)$$
$$- g_{\uparrow\uparrow}\tilde{f}_{\uparrow\uparrow}\left(\frac{p_2-p_1+\sqrt{2}p}{2}, \frac{p_1+p_2}{\sqrt{2}}\right)$$
$$- g_{\downarrow\downarrow}\tilde{f}_{\downarrow\downarrow}\left(\frac{p_1-p_2+\sqrt{2}p}{2}, \frac{p_1+p_2}{\sqrt{2}}\right)$$

Where $\mathbf{P} = \{p_1, p_2, p\}$ corresponds to a 3D vector in momentum space. Idea is then to end up with a system of three coupled integral equations for $\{\tilde{f}_{\uparrow\downarrow}, \tilde{f}_{\uparrow\uparrow}, \tilde{f}_{\downarrow\downarrow}\}$ since we can show that :

$$\begin{cases} \tilde{f}_{\uparrow\downarrow}(k,q) = \int \frac{du}{2\pi}\tilde{\Psi}(u,k,q) \\ \tilde{f}_{\uparrow\uparrow}(k,q) = \int \frac{du}{\pi}\tilde{\Psi}(u, \sqrt{2}q - u, \sqrt{2}(k+u) - q) \\ \tilde{f}_{\downarrow\downarrow}(k,q) = \int \frac{du}{\pi}\tilde{\Psi}(u, \sqrt{2}q - u, \sqrt{2}(k-u) + q) \end{cases} \quad (7)$$

Dimer-dimer Scattering

- We fix $g_{"\#} < 0$ (attractive interspecies).
- The total energy is $E = \quad 2j\epsilon_{"\#}j + \epsilon_0$, where $\epsilon_{"\#} = \quad \sim^2/ma^2_{"\#}$ and ϵ_0 is the dimer-dimer collisional energy.
- Starting from gas of dimers "# (Yang Gaudin), we decrease the ratio $g_{""}g_{\#\#}/g^2_{"\#}$ and look at zero-collision d-d energy to extract $a_{dd}/a_{"\#}$.

- By substituting $\tilde{f}_{"\#}$ by an appropriate expression, homogeneous STM equation becomes an inhomogeneous equation $MX = Y$
- Leads to a linear problem that we put on the grid to extract $a_{dd}/a_{"\#}$.

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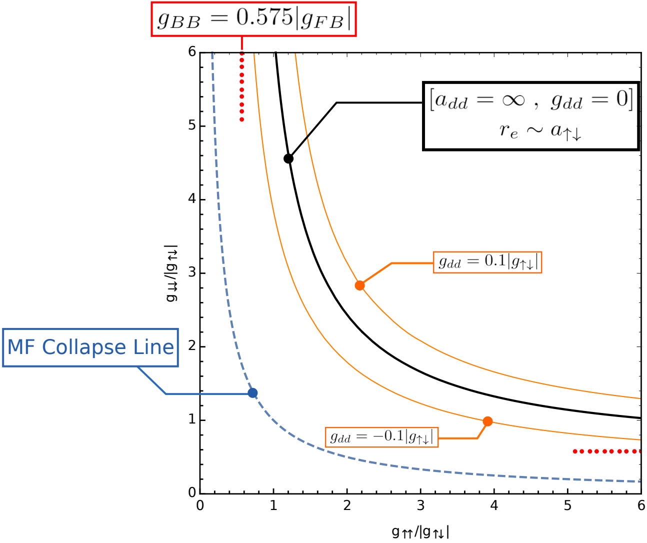
Goal reminder

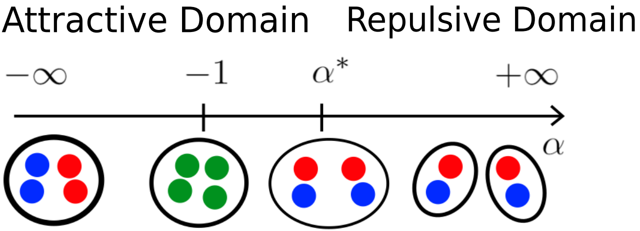
Find the line curve in the plane $f g_{""}/jg_{"\#}j, g_{\#\#}/jg_{"\#}jg$ where the dimer-dimer interaction vanishes.

Results

Overview of the Bose-Bose mixture in the plane $fg_{""}/jg_{"\#}j, g_{\#\#}/jg_{"\#}jg$

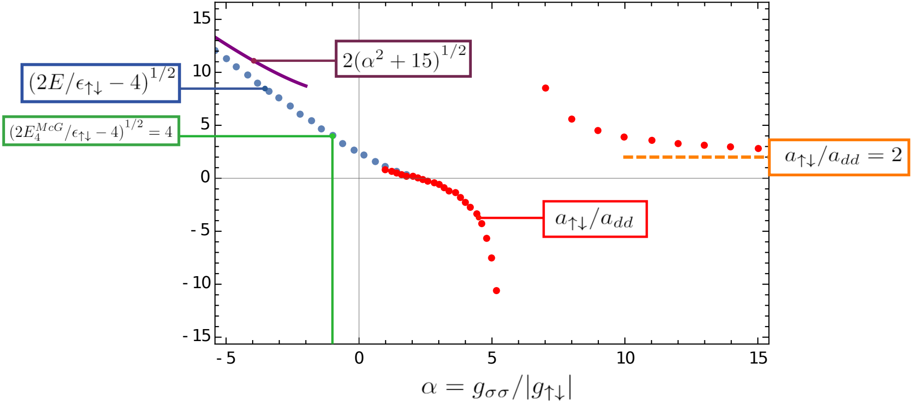


Overview in symmetric case ($g_{\uparrow\uparrow} = g_{\downarrow\downarrow}$) in function of $\alpha = g_{\uparrow\uparrow}/|g_{\uparrow\downarrow}|$



- Interaction between dimers become attractive when $\alpha < \alpha$
- 3 known integrable cases : $\alpha \to +\infty$, $\alpha = -1$, $\alpha \to -\infty$

Overview of the dimerized symmetric Bose-Bose mixture in function of α

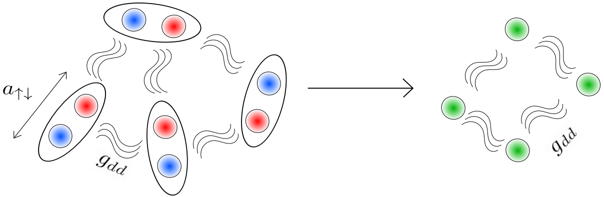


$g_{dd} = 0$ for $\alpha = 2.2$

Discussions

Soliton ?

- Consider $N_d > 2$ dimers close to the dimer-dimer zero crossing line in the attractive regime where $a_{dd} \quad a_{"\#} \quad r_e$.



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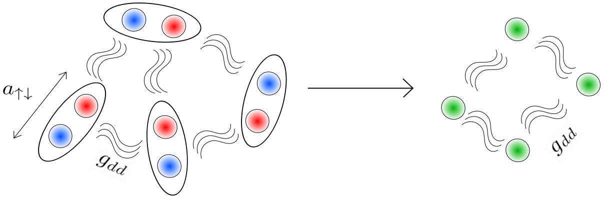
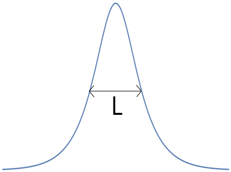


Soliton

$$\mathrm{E}_{N_d} = \frac{g_{dd}^2 N_d (N_d^2 \quad 1)}{12}$$

! Ground State ?



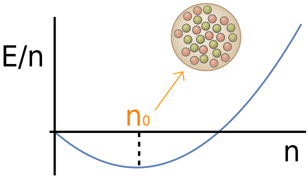
L a_{dd}/N_d

Breaks down for N_d ! $+1$ (cf. above the collapse line)

MF & 3-Body Repulsive Interaction

- Idea : A liquid state which is a result of a competition between two- and three- dimer forces ? ($g_{dd} < 0$ and assume $g_3 > 0$)
- MF (for dimers) treatment (cf. Bulgac) :

$$\epsilon := E_{N_d}/N_d = g_{dd} n_d/2 + g_3 n_d^2/6 \quad (8)$$



Minimum : $n_d^0 = \quad 3g_{dd}/2g_3$

- Applicability : Interaction energy shift much smaller than the energy scale $E \quad n_d{}^2$! $f a_{dd} n_d \quad 1$ and $g_3 \quad 1g$
- Both of these conditions (at n_d^0) lead to $g_3 \quad 1$

About 3-Body Interaction

- What is this g_3 ?

$$\boxed{\text{3-Body in 1D } ! \quad \text{2-Body in 2D} \quad , \quad \Psi_3 \ / \ \ln(\rho/a_3) \ , \ a_3 > 0}$$

- 3-dimer effective potential taken as :

$$g_3 = \frac{\sqrt{3}\pi}{2\ln(2e \quad \gamma/a_3\kappa)} \tag{9}$$

- κ is the typical momentum of the system
- In the leading order of $g_3 \quad 1$, by assuming that $a_3 \quad a_{"\#}$, we have in the leading order of g_3 :

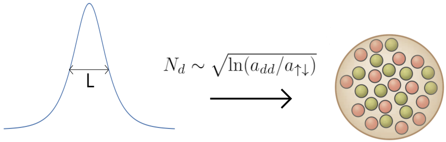
$$g_3 = \sqrt{3}\pi/2\ln(a_{dd}/a_{"\#}) \quad 1$$

$$\boxed{n_d^0 = (\sqrt{3}/\pi a_{dd})\ln(a_{dd}/a_{"\#}) \quad , \quad \mu = \epsilon = (\sqrt{3}/4\pi a_{dd}^2)\ln(a_{dd}/a_{"\#})}$$

Different regimes

- In the region $a_3 \quad a_{\uparrow\downarrow} \quad n_d{}^{1}$, precisely $1/\ln(a_{\uparrow\downarrow} n_d) \quad 1/\ln(a_3 n_d) \quad 1$

<u>Crossover : Soliton to Liquid Droplet when increasing N_d</u>



- Dimer-dimer effective range correction (per dimer) ?
  - ! Scales as $r_e \epsilon n_d \quad \epsilon g_3{}^{1} e^{\quad \sqrt{3}\pi/2g_3}$ smaller than any powers of g_3
- Case $a_{\uparrow\downarrow} \quad 1/n_d \quad a_3$?
  - ! Weak 3-body attraction leads to high density phase (cf. Nishida)
  - ! Solution breaks down for same reasons than soliton.

Conclusion

Summary

1. We derived STM equations for the 4 body-problem in the case of a mixture with intercomponent dimers.

2. We implemented these equations numerically and verify our numerical method in known integrable cases.

3. We calculated the line where the dimer-dimer interaction vanishes (particularly in the Bose symmetric case $\alpha \ = 2.2$ and in the BF case $g_{BB} = 0.575 j g_{FB} j$)

4. For a weak dimer-dimer interaction, we predict a dilute dimerized liquid phase stabilized against collapse by a repulsive three-dimer force.

Open questions

Solve the three-dimer problem / Three dimer zero crossing point ? / Liquid density imbalanced / Pentamer ...

Bose-Fermi Mapping

- In 1D, one can map the case of N impenetrable bosons with an ideal Fermi gas of N particles.
- For fermions, thanks to Pauli principle, the wavefunction vanishes with contact of intraspecies.
- For bosons, if we impose an infinite contact repulsion (impenetrable bosons), we reproduce artificially the Pauli principle.

$$\begin{cases} \Psi_B(x_1, x_2, ..., x_n) = A(x_1, ..., x_n)\Psi_F(x_1, x_2, ..., x_n) \\ A(x_1, ..., x_n) = \prod_{i>j} \mathrm{sgn}(x_i \quad x_j) \end{cases} \tag{10}$$

! Same characteristic such as energy.

- This mapping has been at center of investigations in 1-dimension, in our case, we will resume this by :

$$\boxed{\Psi_B, \;\; g_{""} = g_{\#\#} = +1 \qquad , \qquad \Psi_F, \;\; g_{""} = g_{\#\#} = 0} \tag{11}$$

Trimer Threshold

- Let us consider the ""# combination (or equivalently, ##") and apply STM.
- In the case $g_{"\#} < 0$, ""# is always bound except in the limit $g_{""} = +1$ where $(\epsilon_{""\#} \quad \epsilon_{"\#}) = 0$ and a_{ad} diverges.
- The trimer ""# can be formed if $\epsilon_{""\#} < E = \quad 2j\epsilon_{"\#}j$ for zero dimer-dimer collision energy.

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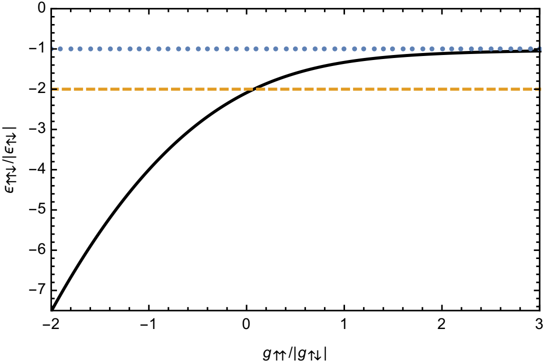

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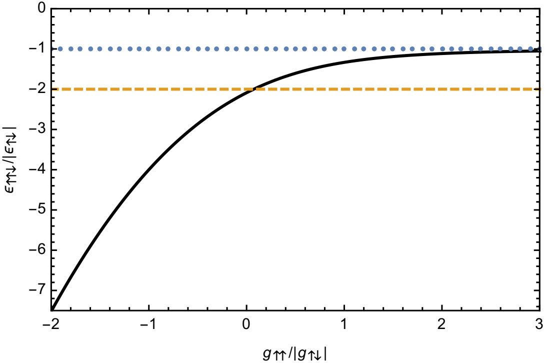


$$\epsilon_{""\#} = \quad 2j\epsilon_{"\#}j \quad , \quad g_{""} = 0.0738jg_{"\#}j$$

Case α ! $+1$

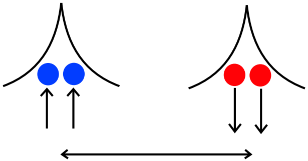
- Thanks to the BFM, the case of infinite repulsion between intracomponents lead to study interacting two species Fermi gas.
- Corresponds equivalently in this study to the fermionic case where $g_{""} = g_{\#\#} = 0$! We end up with 1 Integral equation.
- Four attractively interacting fermions in 1D ! Integrable case (solved by C. Mora) :
- Scattering properties of the two dimers ("#) system are described by the dimer-dimer scattering length a_{dd}.

$$\boxed{a_{dd} = 0.5 a_{"\#}} \tag{12}$$

Case α ! 1

Intraspecies are infinitely attractive : $g_{""} = g_{\#\#} = \quad 1$

! Four-body bound state composed of two intracomponent dimers.



$$[\quad \frac{\sim^2}{2\mu}\frac{d^2}{dx^2} + 4g_{"\#}\delta(x)]\chi_r = E_{BS}\chi_r \tag{13}$$

$$E = \quad \frac{2}{ma_{""}{}^2} \quad E_{BS}(f2m, 2mg, g = 4g_{"\#}) \tag{14}$$

$$\boxed{E/j\epsilon_{"\#}j = \quad 2\alpha^2 \quad 32} \tag{15}$$

Case $\alpha = \quad 1$

- Known as Lieb-Liniger / Mc Guire model
- Take N as the arbitrary number of particles of equal masses M all interacting via one another via equal strength δ-function potentials.

$$[\quad \frac{\sim^2}{2M} \sum_{i=1}^{N} \frac{d^2}{d{x_i}^2} + C \sum_{i>j} \sum_{j=1}^{N} \delta(x_i \quad x_j)]\Psi = E\Psi \tag{16}$$

- We put $\sim = M = 1$ and $g = \quad p\overline{\ } 2C$ and consider the case of a δ-attractive potential between particles. We end up with the energy of the N-body bound state :

$$E = \quad \frac{g^2}{48} N(N^2 \quad 1) \tag{17}$$

In our units for our four-body problem :

$$\boxed{E(N=4) = \quad 10 j \epsilon_{"\#} j} \tag{18}$$

Case α ' α

- Appearance of a weakly bound four-body bound state :

$$E = \frac{2}{m a_{"\#}{}^2} \quad E_{BS}(f2m, 2mg, g_{dd})$$

- Where g_{dd} is the strength of interaction between the two intercomponent dimers and which verify $g_{dd} = \; 1/a_{dd}$.

$$E/j\epsilon_{"\#}j = \; 2 \quad \frac{a_{"\#}{}^2}{a_{dd}{}^2} \qquad (19)$$

- One can interest to the function A defined by :

$$A(\alpha) = p\overline{2}\sqrt{\frac{E}{\epsilon_{"\#}} \quad 2} \underset{\alpha \, ' \, \alpha}{=} \frac{a_{"\#}}{a_{dd}} \qquad (20)$$

- $A(\alpha \;)$ passes through zero when a_{dd} diverge, namely for the ratio α of the gas-liquid transition :

$$A(\alpha \;) = 0 \;\; , \;\; a_{dd} = \mathit{1} \;\; , \;\; g_{dd} = 0 \qquad (21)$$