Dimensional crossover and metal-insulator transition in quasi-two-dimensional disordered conductors

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We study the metal-insulator transition in weakly coupled disordered planes on the basis of a nonlinear sigma model (NLσM). Using two different methods, a renormalization-group approach and an auxiliary-field method, we calculate the crossover length between a two-dimensional (2D) regime at small length scales and a 3D regime at larger length scales. The 3D regime is described by an anisotropic 3D NLσM with renormalized coupling constants. We obtain the critical value of the single-particle interplane hopping, which separates the metallic and insulating phases. We also show that a strong parallel magnetic field favors the localized phase and derive the phase diagram. [S0163-1829(97)09840-8]

I. INTRODUCTION

A quasi-two-dimensional (quasi-2D) weakly disordered conductor (i.e., weakly coupled planes system) exhibits a metal-insulator transition (MIT) for a critical value of the single-particle interplane hopping $t_\perp$. The existence of this quantum phase transition results from the localization of the electronic states in a 2D system by arbitrary weak disorder, while a 3D system remains metallic below a critical value of the disorder.

Although the existence of this MIT is well established, its properties are far from being understood. In particular, the critical value $t_\perp^{(c)}$ of the interplane coupling remains controversial. The self-consistent diagrammatic theory of the Anderson localization[1] predicts an exponentially small critical coupling[2] $t_\perp^{(c)} \sim \tau^{-1} e^{-\alpha k_F l}$ $(\alpha \sim 1)$ in agreement with simple scaling arguments[3] and other analytical arguments.[4] (Here τ is the elastic scattering time, k_F the 2D Fermi wave vector, and l the intraplane mean free path. $k_F l \gg 1$ for weak disorder.) A recent numerical analysis predicts a completely different result, $t_\perp^{(c)} \sim 1/\tau^2$, which is supported by analytical arguments.[5] It has also been claimed, on the basis of diagrammatic perturbative calculations, that the MIT depends on the propagating direction, in contradiction with the scaling theory of localization.[6] (Note that for a system of weakly coupled chains, it is well established, both from numerical and analytical approaches, that $t_\perp^{(c)} \sim 1/\tau$.[7])

The aim of this paper is to reconsider the MIT in quasi-2D conductors on the basis of a NLσM.[8] In the next section, we use a renormalization-group (RG) approach to show how the system crosses over from a 2D behavior at small length scales to a 3D behavior at larger length scales. The crossover length L_x is explicitly calculated. We show that the 3D regime is described by an effective NLσM with renormalized coupling constants. We obtain the critical value $t_\perp^{(c)}$ of the interplane coupling and the anisotropy of the correlation (localization) lengths in the metallic (insulating) phase. Next, we show that a parallel magnetic field tends to decouple the planes and thus can induce a MIT. In Sec. II E, we show that our expression of $t_\perp^{(c)}$ agrees with the estimate based on the weak localization correction to the Drude-Boltzmann conductivity if the latter is interpreted carefully. The results of the RG approach are recovered in Sec. III by means of an auxiliary field method. We emphasize that the latter is a general approach to study phase transitions in weakly coupled systems.

II. RENORMALIZATION-GROUP APPROACH

We consider spinless electrons propagating in a quasi-2D system with the dispersion law

$$\epsilon_{\mathbf{k}} = \mathbf{k}_\parallel^2/2m - 2t_\perp \cos(k_\perp d), \tag{2.1}$$

where $\mathbf{k}_\parallel$ and $k_\perp$ are the longitudinal (i.e., parallel to the planes) and transverse components of $\mathbf{k}$, respectively. m is the effective mass in the planes, d the interplane spacing, and $t_\perp$ the transfer integral in the transverse (z) direction. The effect of disorder is taken into account by adding a random potential with zero mean and Gaussian probability distribution:

$$\langle V_l(\mathbf{r}) V_{l'}(\mathbf{r}') \rangle = [2\pi N_2(0)\tau]^{-1} \delta_{l,l'} \delta(\mathbf{r}-\mathbf{r}'), \tag{2.2}$$

where $N_2(0) = m/2\pi$ is the 2D density of states at the Fermi level and τ the elastic scattering time. $\mathbf{r}$ is the coordinate in the plane and the integers l,l' label the different planes. We denote by k_F the 2D Fermi wave vector and by $v_F = k_F/m$ the 2D Fermi velocity.

A. A simple argument

We first determine the critical value $t_\perp^{(c)}$ by means of a simple argument whose validity will be confirmed in the next sections. Consider an electron in a given plane. If the coupling $t_\perp$ is sufficiently weak, the electron will first diffuse in the plane and then hop to the neighboring plane after a time τ_x. The corresponding diffusion length L_x is determined by the equations

$$\begin{aligned} L_x^2 &= D(L_x)\tau_x, \\ d^2 &= D_\perp \tau_x = 2t_\perp^2 d^2 \tau \tau_x. \end{aligned} \tag{2.3}$$

The length dependence of the coefficient $D(L)$ results from quantum corrections to the semiclassical 2D diffusion coefficient $D=v_F^2\tau/2$. We have assumed that the transverse diffusion is correctly described by the semiclassical diffusion coefficient $D_\perp$. As shown in the next sections, this is a consequence of the vanishing scaling dimension of the field in the NLσM approach. Equations (2.3) give $\tau_x \sim 1/t_\perp^2\tau$ and $L_x^2 \sim D(L_x)/t_\perp^2\tau$. The critical value $t_\perp^{(c)}$ is obtained from the condition $L_x \sim \xi_{2D}$ or $g(L_x)\sim 1$, where ξ_{2D} is the 2D localization length and $g(L)$ the 2D (dimensionless) conductance. These two conditions are equivalent since $g(L) = N_2(0)D(L)\sim 1$ for $L\sim \xi_{2D}$ [see Eq. (2.13) below]. We thus obtain

$$t_\perp^{(c)} \sim \frac{1}{\sqrt{\xi_{2D}^2 N_2(0)\tau}} \sim \frac{l}{\xi_{2D}\sqrt{k_F l}}\,\frac{1}{\tau}, \tag{2.4}$$

where $l=v_F\tau$ is the elastic mean free path. In the weak disorder limit ($k_F l \gg 1$), $\xi_{2D}\sim l e^{\alpha k_F l}$ ($\alpha \sim 1$) so that $t_\perp^{(c)}$ is exponentially small with respect to $1/\tau$. Apart from the factor $1/\sqrt{k_F l}$, Eq. (2.4) agrees with the result of the self-consistent diagrammatic theory of Anderson localization.[2] As we shall show in Sec. II E, it also agrees (apart from the factor $1/\sqrt{k_F l}$) with the estimate based on the weak localization correction to the Drude-Boltzmann conductivity if the latter is interpreted carefully.

B. NLσM for weakly coupled planes

The procedure to derive the NLσM describing electrons in a random potential is well established[8] and we only quote the final result in the quasi-2D case (see, however, Sec. II D). Averaging over disorder by introducing N replica of the system and using an imaginary time formalism, the effective action of the model is $S_\Lambda = S_{2D} + S_\perp$ with

$$S_{2D}[Q] = \frac{\pi}{8} N_2(0) D \sum_l \int d^2r\ \mathrm{Tr}[\boldsymbol{\nabla}_\parallel Q_l(\mathbf{r})]^2 - \frac{\pi}{2} N_2(0) \sum_l \int d^2r\ \mathrm{Tr}[\Omega Q_l(\mathbf{r})], \tag{2.5}$$

$$S_\perp[Q] = -\frac{\pi}{4} N_2(0) t_\perp^2 \tau \sum_{\langle l,l'\rangle} \int d^2r\ \mathrm{Tr}[Q_l(\mathbf{r}) Q_{l'}(\mathbf{r})],$$

where $Q_l(\mathbf{r})$ is a matrix field with elements ${}_{ij}Q_{lnm}^{\alpha\beta}(\mathbf{r})$. $\alpha,\beta = 1,\dots,N$ are replica indices, n,m refer to fermionic Matsubara frequencies ω_n,ω_m, and $i,j=1,2$ describe the particle-hole ($i=j$) and particle-particle ($i\neq j$) channels. Ω is the matrix ${}_{ij}\Omega_{lnm}^{\alpha\beta} = \delta_{i,j}\delta_{n,m}\delta_{\alpha,\beta}\omega_n$. In Eq. (2.5), Tr denotes the trace over all discrete indices. The field Q satisfies the constraints $Q_l^2(\mathbf{r}) = \underline{1}$ (with $\underline{1}$ the unit matrix), Tr $Q_l(\mathbf{r}) = 0$, and $Q^+ = C^T Q^T C = Q$, where C is the charge conjugation operator.[8] $\Lambda \sim 1/l$ is an ultraviolet cutoff for the fluctuations of the field Q in the planes. The Josephson-like interplane coupling $S_\perp$ is obtained retaining the lowest-order contribution in $t_\perp$ ($\langle l,l'\rangle$ denotes nearest neighbors).

If $t_\perp^2\tau \gg D\Lambda^2 \sim 1/\tau$, the fluctuations in the transverse direction are weak. In this case, $\mathrm{Tr}[Q_l Q_{l'}]$ can be approximated by $-(d^2/2)\mathrm{Tr}[\nabla_z Q]^2$ and we obtain a NLσM with anisotropic diffusion coefficients D and $D_\perp$. For $t_\perp\tau \gg 1$, an electron in a given plane hops to the neighboring plane before being scattered. There is therefore no 2D diffusive motion in that limit so that the quasi-2D aspect does not play an essential role. The anisotropy can be simply eliminated by an appropriate length rescaling in the longitudinal and transverse directions.[9,10] We consider in the following only the limit $t_\perp\tau \ll 1$ where we expect a 2D/3D dimensional crossover at a characteristic length L_x.

C. RG approach

We analyze the action (2.5) within a RG approach following Ref. 11. The condition $t_\perp\tau \ll 1$ ensures that the transverse coupling is weak ($D\Lambda^2 \gg t_\perp^2\tau$). The initial stage of the renormalization will therefore be essentially 2D, apart from small corrections due to the interplane coupling. If we neglect the latter, we then obtain the renormalized action

$$S_{\Lambda'}[Q] = \frac{\pi}{8} N_2(0) D(\Lambda') \sum_l \int d^2r\ \mathrm{Tr}[\boldsymbol{\nabla}_\parallel Q_l(\mathbf{r})]^2 - \frac{\pi}{2} N_2(0) \sum_l \int d^2r\ \mathrm{Tr}[\Omega Q_l(\mathbf{r})] - \frac{\pi}{4} N_2(0) t_\perp^2 \tau \sum_{\langle l,l'\rangle} \int d^2r\ \mathrm{Tr}[Q_l(\mathbf{r}) Q_{l'}(\mathbf{r})], \tag{2.6}$$

where $\Lambda' < \Lambda$ is the reduced cutoff after renormalization and $D(\Lambda')$ the renormalized value of the 2D longitudinal diffusion coefficient. Since the scaling dimension of the field Q vanishes (i.e., there is no rescaling of the field),[8] there is no renormalization of the interplane coupling.[11] The 2D/3D dimensional crossover occurs when the transverse and longitudinal couplings become of the same order:

$$\tfrac{1}{2} D_x \Lambda_x^2 \sim t_\perp^2 \tau, \tag{2.7}$$

where $D_x = D(\Lambda_x)$ is the longitudinal diffusion coefficient at the crossover. In the 3D regime ($\Lambda' \leq \Lambda_x$), it is appropriate to take the continuum limit in the transverse direction.[11] Using

$$\mathrm{Tr}[Q_l Q_{l'}] = -\tfrac{1}{2}\ \mathrm{Tr}[Q_l - Q_{l'}]^2 + \mathrm{const} \to -\frac{d^2}{2}\ \mathrm{Tr}[\nabla_z Q_l]^2 \tag{2.8}$$

for l and l' nearest neighbors, and $d\Sigma_l \to \int dz$, we obtain

$$S_{\Lambda_x}[Q] = \frac{\pi}{8} N_3(0) D_x \int d^3r \{\mathrm{Tr}[\boldsymbol{\nabla}_\parallel Q(\mathbf{r})]^2 + (d\Lambda_x)^2\ \mathrm{Tr}[\nabla_z Q(\mathbf{r})]^2\} - \frac{\pi}{2} N_3(0) \int d^3r\ \mathrm{Tr}[\Omega Q(\mathbf{r})], \tag{2.9}$$

where $N_3(0) = N_2(0)/d$ is the 3D density of states at the Fermi level. The cutoffs are Λ_x and $1/d$ in the longitudinal and transverse directions, respectively. Note that $\mathbf{r}$ is now a 3D coordinate. The 3D regime is thus described by an anisotropic NLσM. However, the anisotropy is the same for the diffusion coefficients and the cutoffs and can therefore easily be suppressed by an appropriate rescaling of the lengths: $\mathbf{r}'_\parallel = \mathbf{r}_\parallel / s_1$, $z' = z/s_2$, with

$$s_1^{-2} D_x = s_2^{-2} D_x (\Lambda_x d)^2, \qquad s_1^2 s_2 = 1. \tag{2.10}$$

The last equation ensures that $\int d^3\mathbf{r}' = \int d^3\mathbf{r}$ and lets the invariant the last term of Eq. (2.9). From Eq. (2.10), we obtain $s_1 = (1/\Lambda_x d)^{1/3}$, $s_2 = (\Lambda_x d)^{2/3}$, and the effective action

$$S_{\Lambda_x}[Q] = \frac{\pi}{8} N_3(0) \bar{D} \int d^3 r \ \mathrm{Tr}[\boldsymbol{\nabla} Q(\mathbf{r})]^2 - \frac{\pi}{2} N_3(0) \int d^3 r \ \mathrm{Tr}[\Omega Q(\mathbf{r})], \tag{2.11}$$

where $\bar{D} = D_x (\Lambda_x d)^{2/3}$. The cutoff is now isotropic: $s_1 \Lambda_x \sim s_2 / d \sim \bar{\Lambda} = (\Lambda_x^2 / d)^{1/3}$. The dimensionless coupling constant of the NLσM (2.11) is

$$\lambda = \frac{4}{\pi N_3(0) \bar{D}} \bar{\Lambda} = \frac{4}{\pi N_2(0) D_x} = \frac{4}{\pi g_x}. \tag{2.12}$$

The MIT occurs for $\lambda = \lambda_c = O(1)$, i.e., when the 2D conductance $g_x = g_c = O(1)$. Using $g_x = g_c \sim 1$ for $\Lambda_x \sim \xi_{2D}^{-1}$, we recover the result (2.4) for the critical value of the interplane coupling. Notice that the factor $\sqrt{k_F l}$ appearing in Eq. (2.4) results from the 2D RG regime where the 2D conductance renormalizes from $g(l) \sim k_F l \gg 1$ to $g_x \sim 1$. In the simple argument of Sec. II A, this factor is obtained by taking into account the L dependence of the 2D diffusion coefficient.

Further information about the metallic and insulating phases can be obtained from the L dependence of the 2D dimensionless conductance. The self-consistent diagrammatic theory of Anderson localization gives[1]

$$g(L) = \frac{1}{2\pi^2} \ln\left(1 + \frac{\xi_{2D}^2}{L^2}\right)\left(1 + \frac{L}{\xi_{2D}}\right) e^{-L/\xi_{2D}}, \tag{2.13}$$

where L should be identified with Λ^{-1}. Equation (2.13) gives $g(L) \simeq (1/\pi^2)\ln(\xi_{2D}/L)$ for $L \ll \xi_{2D}$ (in agreement with perturbative RG calculation[8]) and $g(L) \simeq (1/2\pi^2)(\xi_{2D}/L) e^{-L/\xi_{2D}}$ for $L \gg \xi_{2D}$. The crossover length L_x obtained from Eqs. (2.7) and (2.13) is shown in Fig. 1. Since the crossover length is not precisely defined, we have multiplied L_x by a constant in order to have $L_x \simeq v_F / t_\perp$ deep in the metallic phase ($t_\perp \gg t_\perp^{(c)}$) (this will allow a detailed comparison with the results of Sec. III).

In the metallic phase, $t_\perp \gg t_\perp^{(c)}$, we have

$$g_x \simeq 2N_2(0) t_\perp^2 \tau \xi_{2D}^2 e^{-2\pi^2 g_x} \gtrsim 1, \qquad L_x \simeq \xi_{2D} e^{-\pi^2 g_x} \lesssim \xi_{2D}. \tag{2.14}$$

The isotropic NLσM (2.11) defines an isotropic correlation length $\bar{L}$.[12] Using Eq. (2.10), we then obtain the anisotropy of the correlation lengths (in the original length units):

$$\frac{L_\perp}{L_\parallel} = \frac{s_2}{s_1} = \Lambda_x d. \tag{2.15}$$

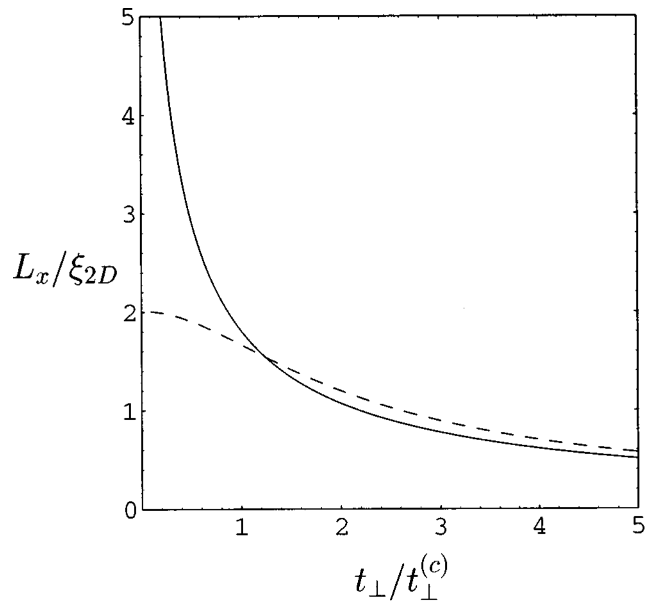

FIG. 1. Crossover length L_x vs interplane coupling $t_\perp$. Solid line: RG approach; dashed line: auxiliary field method. We have used $k_F l = 50$ and $\xi_{2D}/l = 5\times 10^4$ (l and ξ_{2D} were considered as independent parameters).

Deep in the metallic phase ($t_\perp \gg t_\perp^{(c)}$), we have $L_\perp / L_\parallel \sim d t_\perp / v_F \sim t_\perp / t_\parallel$ (where $t_\parallel \sim v_F k_F \sim v_F / d$ is the transfer integral in the planes). Close to the transition ($t_\perp \gtrsim t_\perp^{(c)}$) we have $L_\perp / L_\parallel \sim d/\xi_{2D} \sim \sqrt{k_F l}(t_\perp / t_\parallel)$. This differs from the result $L_\perp / L_\parallel \sim t_\perp / t_\parallel$ predicted by a 3D NLσM.[9,13] (Notice again that the factor $k_F l$ originates in the 2D stage of the RG procedure. This stage is not considered if one uses an anisotropic 3D NLσM from the very beginning of the analysis.)

In the insulating phase ($t_\perp \ll t_\perp^{(c)}$) we have

$$g_x = \frac{1}{2\pi^2} \xi_{2D} \sqrt{\frac{2N_2(0) t_\perp^2 \tau}{g_x}} e^{-\sqrt{[g_x / 2N_2(0) t_\perp^2 \tau]}/\xi_{2D}} \lesssim 1, \qquad L_x \simeq \sqrt{\frac{g_x}{2N_2(0) t_\perp^2 \tau}} \gtrsim \xi_{2D}. \tag{2.16}$$

The anisotropy of the localization lengths $\xi_\perp / \xi_\parallel$ is the same as that of the correlation lengths and is therefore given by Eq. (2.15) (see Fig. 2). Close to the transition, $\xi_\perp / \xi_\parallel \sim \sqrt{k_F l}(t_\perp / t_\parallel)$. Again this differs from the result $\xi_\perp / \xi_\parallel \sim t_\perp / t_\parallel$ predicted by a 3D anisotropic NLσM.[9]

D. Effect of a parallel magnetic field

We consider in this section the effect of an external magnetic field $\mathbf{H} = (0, H, 0)$ parallel to the planes. In second quantized form, the interplane hopping Hamiltonian is

$$\mathcal{H}_\perp = -t_\perp \sum_{\langle l,l'\rangle} \int d^2\mathbf{r} e^{ie\int_{(\mathbf{r},ld)}^{(\mathbf{r},l'd)} \mathbf{A}(\mathbf{s})\cdot d\mathbf{s}} \psi^\dagger(\mathbf{r},l) \psi(\mathbf{r},l') = -t_\perp \sum_{\langle l,l'\rangle, \mathbf{k}_\parallel} \psi^\dagger[\mathbf{k}_\parallel + (l-l')\mathbf{G}, l'] \psi(\mathbf{k}_\parallel, l) \tag{2.17}$$

in the gauge $\mathbf{A}(0,0,-Hx)$. $\mathbf{G} = (G,0,0)$ with $G = -eHd$. In the second line of Eq. (2.17), we have used a mixed repre-

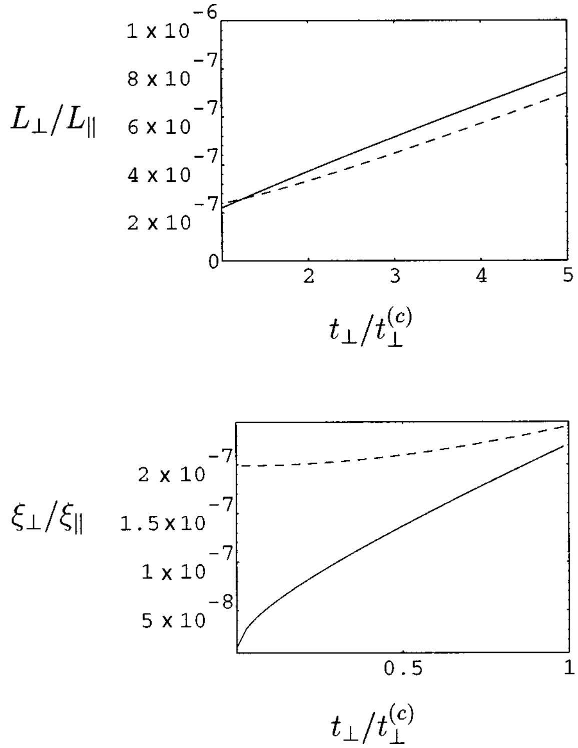

FIG. 2. Anisotropy of the correlation (localization) lengths for $t_\perp > t_\perp^{(c)}$ ($t_\perp < t_\perp^{(c)}$). Solid line: RG approach, dashed line: auxiliary field method. The parameters are the same as in Fig. 1 with $k_F d \sim 1$.

sentation by taking the Fourier transform with respect to the intraplane coordinate $\mathbf{r}$. The effective action $S[Q]$ of the NLσM can be simply obtained by calculating the particle-hole bubble $\Pi(\mathbf{q},\omega_\nu)$ in the semiclassical (diffusive) approximation. To lowest order in $t_\perp$, we have $\Pi = \Pi^{(0)} + \Pi^{(1)} + \Pi^{(2)}$ where $\Pi^{(0)} \simeq 2\pi N_2(0)\tau(1-|\omega_\nu|\tau - D\tau q_\parallel^2)$ (for $|\omega_\nu|\tau, D\tau q_\parallel^2 \ll 1$) is the 2D result. Here ω_ν is a bosonic Matsubara frequency. $\Pi^{(1)}$ and $\Pi^{(2)}$ are given by (see Fig. 3)

$$\Pi^{(1)} = \frac{2t_\perp^2}{L^2} \sum_{\mathbf{k}_\parallel, \delta=\pm 1} G^2(\mathbf{k}_\parallel,\omega_n) G(\mathbf{k}_\parallel - \delta\mathbf{G},\omega_n) G(\mathbf{k}_\parallel,\omega_{n+\nu})$$
$$= -8\pi N_2(0) \frac{t_\perp^2 \tau^3}{(1+\omega_c^2\tau^2)^{1/2}},$$
$$\Pi^{(2)} = \frac{t_\perp^2}{L^2} \sum_{\mathbf{k}_\parallel, \delta=\pm 1} e^{iq_\perp \delta d} G(\mathbf{k}_\parallel,\omega_n) G(\mathbf{k}_\parallel - \delta\mathbf{G},\omega_n) \times G(\mathbf{k}_\parallel,\omega_{n+\nu}) G(\mathbf{k}_\parallel - \delta\mathbf{G},\omega_{n+\nu})$$
$$= 8\pi N_2(0) \frac{t_\perp^2 \tau^3}{(1+\omega_c^2\tau^2)^{1/2}} \cos(q_\perp d) \tag{2.18}$$

for $\omega_n \omega_{n+\nu} < 0$ and $q_\parallel, \omega_\nu \to 0$. L^2 is the area of the planes and

$$G(\mathbf{k}_\parallel,\omega_n) = [i\omega_n + (i/2\tau)\operatorname{sgn}(\omega_n) - \mathbf{k}_\parallel^2/2m + \mu]^{-1} \tag{2.19}$$

is the 2D one-particle Green's function (μ is the Fermi energy). We have introduced the characteristic magnetic energy $\omega_c = v_F G$. The diffusion modes are determined by

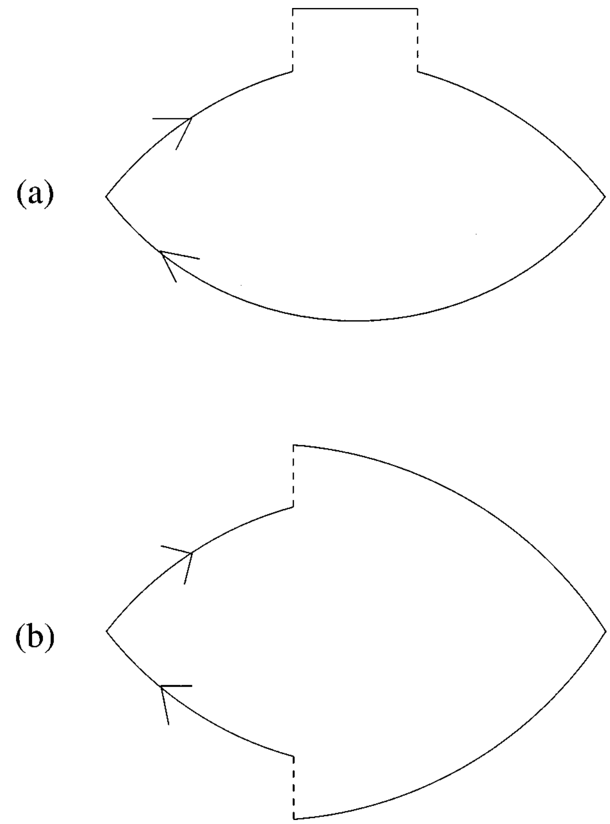

FIG. 3. Diagrammatic representation of $\Pi^{(1)}$ (the symmetric diagram is not shown) (a) and $\Pi^{(2)}$ (b). The solid lines represent the 2D Green's functions while the dashed lines denote single-particle interplane hopping.

$$1 - [2\pi N_2(0)\tau]^{-1}\Pi(\mathbf{q},\omega_\nu) = |\omega_\nu|\tau + D\tau q_\parallel^2 + \frac{8t_\perp^2\tau^2}{(1+\omega_c^2\tau^2)^{1/2}} \sin^2(q_\perp d/2), \tag{2.20}$$

which yields the following interplane coupling in the NLσM:

$$S_\perp[Q] = -\frac{\pi}{4} N_2(0) \frac{t_\perp^2 \tau}{(1+\omega_c^2\tau^2)^{1/2}} \sum_{\langle l,l'\rangle} \times \int d^2r \sum_{inm\alpha\beta} {}_{ii}Q_{lnm}^{\alpha\beta}(\mathbf{r})\, {}_{ii}Q_{l'mn}^{\beta\alpha}(\mathbf{r}). \tag{2.21}$$

Notice that we have retained only the diagonal part of the field Q since the magnetic field suppresses the interplane diffusion modes in the particle-particle channel. This is strictly correct only above a characteristic field corresponding to a "complete" breakdown of time reversal symmetry (this point is further discussed below). Equation (2.21) shows that the magnetic field not only breaks down time reversal symmetry but also reduces the amplitude of the interplane hopping. This quantum effect (important only when $\omega_c\tau \gg 1$) can be understood from the consideration of the semiclassical electronic orbits.[14] Notice that such an effect cannot be described in the semiclassical phase integral (or eikonal) approximation for the magnetic field. The action S_{2D} of the independent planes is not modified by the magnetic field since the latter is parallel to the planes.

We can now apply the same RG procedure as in the preceding section. The initial (2D) stage of the renormalization is not modified by the parallel magnetic field, and the 2D/3D dimensional crossover is determined by

$$\frac{1}{2}D_x\Lambda_x^2 \sim \frac{t_\perp^2\tau}{(1+\omega_c^2\tau^2)^{1/2}}. \tag{2.22}$$

The crossover length $L_x(t_\perp, \omega_c)$ therefore satisfies the scaling law

$$L_x(t_\perp,\omega_c)=L_x\left(\frac{t_\perp}{(1+\omega_c^2\tau^2)^{1/4}},0\right)\equiv L_x\left(\frac{t_\perp}{(1+\omega_c^2\tau^2)^{1/4}}\right), \tag{2.23}$$

where $L_x(t_\perp)$ is the zero field crossover length obtained in the preceding section.

In the 3D regime, the diffusion modes are 3D. The smallest volume corresponding to a diffusive motion is of order $L_x^2 d$. This corresponds to a magnetic flux HL_xd for a parallel field. Time reversal symmetry is ''completely'' broken when this magnetic flux is at least of the order of the flux quantum, i.e., when $L_x \gtrsim 1/G$. It is easy to verify that this defines a characteristic field H_0 such that $\omega_c\tau \ll 1$. Above H_0, the diffusion modes are completely suppressed in the particle-particle channel. The effective action is then given by Eq. (2.9) where only the diagonal part $(i=j)$ of the field Q should be considered. D_x and Λ_x depend on H according to Eq. (2.22).

The critical value of the interplane coupling for $H>H_0$ is obtained from $g_x=g_c'$ where g_c' is the critical value of the dimensionless conductance in the unitary case (no time-reversal symmetry). Thus, we have

$$t_\perp^{(c)}(H)=t_\perp^{(c)\prime}(1+\omega_c^2\tau^2)^{1/4}, \tag{2.24}$$

where $t_\perp^{(c)\prime} \sim [g_c'/(\tau N_2(0)\xi_{2D}^2)]^{1/2} < t_\perp^{(c)}$ is the critical coupling for $\omega_c=0$ in the unitary case. Close to the MIT, we have $L_x \sim \xi_{2D}$ (since $g_c' \sim 1$) so that H_0 is defined by $\omega_c \sim v_F/\xi_{2D}$, which corresponds to an exponentially small value of the field: $\omega_c \sim \tau^{-1}e^{-\alpha k_F l}$. The phase diagram in the $t_\perp - \omega_c$ plane is shown in Fig. 4. For $H \lesssim H_0$, the curve is not correct and should reach $t_\perp^{(c)}$ at $H=0$. Therefore, we obtain that a weak magnetic field favors the metallic phase $[t_\perp^{(c)}(H) < t_\perp^{(c)}$ for $0<H \ll H_0]$ while a strong magnetic field favors the insulating phase $[t_\perp^{(c)}(H) > t_\perp^{(c)}$ for $H \gg H_0]$. This agrees with the results of Ref. 3 obtained from scaling arguments based on the weak localization correction to the Drude-Boltzmann conductivity.[15]

In the metallic phase, the anisotropy of the correlation lengths is given by

$$\frac{L_\perp}{L_\parallel}=\frac{d}{L_x[t_\perp/(1+\omega_c^2\tau^2)^{1/4}]}. \tag{2.25}$$

The same result holds for the anisotropy of the localization lengths $\xi_\perp/\xi_\parallel$ in the insulating phase.

E. Perturbative estimate of $t_\perp^{(c)}$

The first quantum correction to the Drude-Boltzmann conductivity was calculated in Ref. 6. On the basis of this diagrammatic calculation, it was concluded that the MIT depends on the propagating direction. The aim of this section is to show that there is no disagreement between the NLσM approach and the diagrammatic calculation if the latter is interpreted carefully.

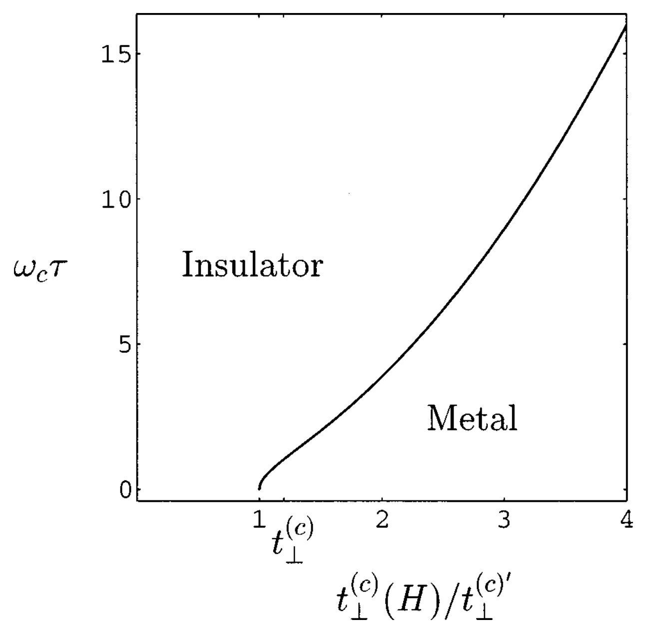

FIG. 4. Phase diagram in a parallel magnetic field. The position of $t_\perp^{(c)}$ $(>t_\perp^{(c)\prime})$ is arbitrary. At very low field, $\omega_c\tau \sim l/\xi_{2D}$ the curve is not correct since one should have $t_\perp^{(c)}(H=0)=t_\perp^{(c)}$ (see text).

We first make a few comments on the Drude-Boltzmann conductivities. They are given by the Einstein relation:

$$\sigma_\mu=e^2N_3(0)D_\mu, \quad \mu=\parallel,\perp, \tag{2.26}$$

where the semiclassical diffusion coefficients are $D_\parallel = v_F^2\tau/2$ and $D_\perp = 2t_\perp^2 d^2\tau$. $\sigma_\parallel$ is well defined only in the diffusive regime, i.e., at length scales larger than the intraplane mean free path $l=v_F\tau$. $\sigma_\perp$ is meaningful only at length scales (in the planes) larger than the (semiclassical) crossover length $L_x^0 \sim v_F/t_\perp$. [L_x^0 is obtained from Eq. (2.3) neglecting any quantum correction to the semiclassical 2D diffusive coefficient.] At length scales smaller than L_x^0, the system behaves as a set of decoupled planes and a transverse conductivity cannot be defined.

We now consider the first quantum correction to the conductivities obtained by summing the maximally crossed diagrams. It is given by[6]

$$\delta\sigma_\mu = -\frac{e^2}{\pi}D_\mu\int_{-\pi/d}^{\pi/d}\frac{dq_z}{2\pi}\int_{q_\parallel \lesssim l^{-1}} \times\frac{d^2q_\parallel}{(2\pi)^2}\,\frac{\delta_{\mu,\parallel}+\delta_{\mu,\perp}\cos(q_zd)}{D_\parallel q_\parallel^2+8t_\perp^2\tau\sin^2(q_zd/2)}. \tag{2.27}$$

Following Ref. 6, we could obtain directly the expression of $\delta\sigma_\mu$ from Eq. (2.27). In order to clarify the underlying physics, we shall proceed in a slightly different way. We write $\delta\sigma_\mu$ as a sum of a 2D contribution and a 3D contribution, i.e., $\delta\sigma_\mu=\delta\sigma_\mu^{2D}+\delta\sigma_\mu^{3D}$, by splitting the sum over $q_\parallel$ into two parts:

$$\int_{q_\parallel \lesssim l^{-1}} \frac{d^2 q_\parallel}{(2\pi)^2} = \int_{L_x^{0-1} \lesssim q_\parallel \lesssim l^{-1}} \frac{d^2 q_\parallel}{(2\pi)^2} + \int_{q_\parallel \lesssim L_x^{0-1}} \frac{d^2 q_\parallel}{(2\pi)^2}. \tag{2.28}$$

Since $D_\parallel / L_x^{0^2} \sim 8 t_\perp^2 \tau$, we can neglect $8 t_\perp^2 \tau \sin^2(q_z d/2)$ in the denominator of Eq. (2.27) in the 2D regime ($q_\parallel \gtrsim L_x^{0-1}$). In the 3D regime ($q_\parallel \lesssim L_x^{0-1}$), only the small values of q_z contribute significantly so that we can expand both the numerator and the denominator for $q_z d \ll 1$. We then obtain

$$\delta\sigma_\mu^{2\mathrm{D}} = -\frac{e^2}{\pi} D_\mu \int_{-\pi/d}^{\pi/d} \frac{dq_z}{2\pi} \times \int_{L_x^{0-1} \lesssim q_\parallel \lesssim l^{-1}} \frac{d^2 q_\parallel}{(2\pi)^2} \frac{\delta_{\mu,\parallel} + \delta_{\mu,\perp} \cos(q_z d)}{D_\parallel q_\parallel^2}, \tag{2.29}$$

$$\delta\sigma_\mu^{3\mathrm{D}} = -\frac{e^2}{\pi} D_\mu \int_{-\Lambda_\perp}^{\Lambda_\perp} \frac{dq_z}{2\pi} \int_{q_\parallel \lesssim L_x^{0-1}} \frac{d^2 q_\parallel}{(2\pi)^2} \frac{\delta_{\mu,\parallel} + \delta_{\mu,\perp}}{D_\parallel q_\parallel^2 + D_\perp q_z^2},$$

where $\Lambda_\perp \sim 1/d$. The preceding equations lead to

$$\delta\sigma_\parallel^{2\mathrm{D}} = -\frac{e^2}{2\pi^2 d} \ln\left(\frac{L_x^0}{l}\right), \quad \delta\sigma_\perp^{2\mathrm{D}} = 0,$$
$$\delta\sigma_\mu^{3\mathrm{D}} = -\frac{e^2}{2\pi^3} \frac{D_\mu}{(D_\parallel D_\perp)^{1/2}} \frac{1}{L_x^0}. \tag{2.30}$$

Since $|\delta\sigma_\parallel^{3\mathrm{D}}| \ll |\delta\sigma_\parallel^{2\mathrm{D}}|$, we have

$$\frac{\delta\sigma_\parallel}{\sigma_\parallel} \simeq \frac{\sigma_\parallel^{2\mathrm{D}}}{\sigma_\parallel} = -\frac{1}{2\pi^2 d N_3(0) D_\parallel} \ln\left(\frac{L_x^0}{l}\right) \sim -\frac{1}{\pi E_F \tau} \ln\left(\frac{1}{t_\perp \tau}\right), \tag{2.31}$$

$$\frac{\delta\sigma_\perp}{\sigma_\perp} = \frac{\delta\sigma_\perp^{3\mathrm{D}}}{\sigma_\perp} = -\frac{1}{2\pi^3 N_3(0) (D_\parallel D_\perp)^{1/2} L_x^0} \sim -\frac{1}{2\pi^2 E_F \tau},$$

where $E_F = m v_F^2/2$ is the Fermi energy. Equations (2.31) agree with Ref. 6. $\delta\sigma_\parallel^{2\mathrm{D}}$ is nothing else but the first quantum correction for a 2D system of area $L_x^{0^2}$. The fact that $\delta\sigma_\perp^{2\mathrm{D}}$ vanishes is not surprising since the transverse conductivity is not defined in the 2D regime. [In the 2D regime, all the electronic paths with closed loops (which are responsible for the first quantum correction to the conductivity) are strictly two dimensional and therefore cannot contribute to the transverse conductivity.] We find for $\delta\sigma_\mu^{3\mathrm{D}}$ the usual expression for an anisotropic three-dimensional system except that the short-distance cutoff is not the mean free path but the crossover length L_x^0.

We now turn to the main question. How can we deduce from $\delta\sigma_\mu$ an estimate for $t_\perp^{(c)}$? From both the perturbative calculation of this section and the NLσM approach, the mechanism of the MIT is clear. At small length scales ($L \ll L_x^0$), the system behaves as a 2D system and the localization effects are strong. At larger length scales ($L \gg L_x^0$), the behavior is 3D and the localization effects are much weaker. Thus the MIT is primarily driven by the 2D localization effects. It should occur when the latter are strong, i.e., when

$$|\delta\sigma_\parallel^{2\mathrm{D}}| \sim \sigma_\parallel. \tag{2.32}$$

This leads to $t_\perp^{(c)} \tau \sim l/\xi_{2\mathrm{D}}$, in agreement with Eq. (2.4) apart from the factor $1/\sqrt{k_F l}$. This factor originates in the L dependence of the 2D diffusive coefficient [see Eq. (2.3)] and cannot be obtained from the simple criterion (2.32). Since $\delta\sigma_\perp = \delta\sigma_\perp^{3\mathrm{D}}$, the transverse conductivity does not give any information about the 2D localization effects. Consequently, $\delta\sigma_\perp$ *cannot* be used to estimate $t_\perp^{(c)}$.

III. AUXILIARY FIELD METHOD

The aim of this section is to recover the results of the RG approach by means of a completely different method. In order to study how the ''correlations'' are able to propagate in the transverse direction, we will study an effective field theory which is generated by a Hubbard-Stratonovich transformation of the interplane coupling $S_\perp[Q]$. This methods bears some obvious similarities with standard approaches in critical phenomena.[16] It is particularly useful when the low-dimensional problem (here the 2D Anderson localization) can be solved (at least approximately) by one or another method. This kind of approach has already been used for weakly coupled 1D Ginzburg-Landau models[17] or weakly coupled Luttinger liquids.[18] While these studies were done in the high-temperature (or disordered) phase, we start in this paper from the ''ordered'' (i.e., metallic) phase where the low-energy modes are Goldstone modes.

Introducing an auxiliary matrix field ζ to decouple $S_\perp$, we rewrite the partition function as

$$Z = \int \mathcal{D}\zeta \, e^{-\sum_{l,l'} \int d^2 r \, \mathrm{Tr}[\zeta_l(\mathbf{r}) J_{\perp l,l'}^{-1} \zeta_{l'}(\mathbf{r})]} \times \int \mathcal{D}Q \, e^{-S_{2\mathrm{D}}[Q] + 2\sum_l \int d^2 r \, \mathrm{Tr}[\zeta_l(\mathbf{r}) Q_l(\mathbf{r})]}. \tag{3.1}$$

The field ζ should have the same structure as the field Q and therefore satisfies the condition $\zeta^+ = C^T \zeta^T C = \zeta$. $J_{\perp l,l'}^{-1}$ is the inverse of the matrix $J_{\perp l,l'} = J_\perp(\delta_{l,l'+1} + \delta_{l,l'-1})$ where $J_\perp = (\pi/4) N_2(0) t_\perp^2 \tau$ (the matrix $J_\perp$ is diagonal in the indices α,n,i).

We first determine the value of the auxiliary field in the saddle-point approximation. Assuming a solution of the form ${}_{ij}(\zeta^{\mathrm{SP}})_{lnm}^{\alpha\beta}(\mathbf{r}) = \zeta_0 \delta_{i,j} \delta_{\alpha,\beta} \delta_{n,m}$, we obtain the saddle-point equation

$$\zeta_0 = J_\perp(q_\perp = 0) \langle {}_{ii}Q_{lnn}^{\alpha\alpha}(\mathbf{r}) \rangle_{\mathrm{SP}}, \tag{3.2}$$

where $J_\perp(q_\perp) = 2 J_\perp \cos(q_\perp d)$ is the Fourier transform of $J_{\perp l,l'}$. The average $\langle \cdots \rangle_{\mathrm{SP}}$ should be taken with the saddle-point action

$$S_{\mathrm{SP}}[Q] = S_{2\mathrm{D}}[Q] - 2\sum_l \int d^2 r \, \mathrm{Tr}[\zeta^{\mathrm{SP}} Q_l(\mathbf{r})]. \tag{3.3}$$

Note that the saddle-point value ζ^{SP} acts as a finite external frequency. The mean value of the field Q is related to the density of states and is a nonsingular quantity. We have[8] $\langle {}_{ii}Q_{lnn}^{\alpha\alpha}(\mathbf{r}) \rangle_{\mathrm{SP}} = \mathrm{sgn}(\omega_n)$, which yields

$$\zeta_0 = J_\perp(q_\perp = 0) \mathrm{sgn}(\omega_n) = 2 J_\perp \, \mathrm{sgn}(\omega_n). \tag{3.4}$$

This defines a characteristic frequency for the 2D/3D dimensional crossover

$$\omega_x = \frac{8}{\pi N_2(0)} |\zeta_0| = 4 t_\perp^2 \tau. \tag{3.5}$$

We now consider the fluctuations around the saddle-point solution. As in the standard localization problem, the Sp($2N$) symmetry of the Lagrangian is spontaneously broken by the ''frequency'' Ω to Sp(N)$\times$Sp(N).[8] The term Tr$[\Omega Q]$ breaks the symmetry of $S_{2\mathrm{D}}[Q]$. Via the coupling Tr$[\zeta Q]$ between the fields ζ and Q, it also breaks the symmetry of the effective action $S[\zeta]$ of the field ζ. Our aim is now to obtain the effective action of the (diffusive) Goldstone modes associated with this spontaneous symmetry breaking. Following Ref. 8, we shift the field according to $\zeta \rightarrow \zeta + \zeta^{\mathrm{SP}} - \pi N_2(0)\Omega/4$ and expand the action to lowest order in Ω and ζ. The partition function becomes

$$Z = \int \mathcal{D}\zeta e^{-\Sigma_{l,l'} \int d^2r \{ \mathrm{Tr}[\zeta_l(\mathbf{r}) J^{-1}_{\perp l,l'} \zeta_{l'}(\mathbf{r})] + 2\,\mathrm{Tr}[\zeta^{\mathrm{SP}} J^{-1}_{l,l'} \zeta_{l'}(\mathbf{r})] - (\pi/2) N_2(0) \mathrm{Tr}[\Omega J^{-1}_{l,l'} \zeta_{l'}(\mathbf{r})]\}} \int \mathcal{D}Q e^{-\widetilde{S}_{2\mathrm{D}}[Q] + 2\Sigma_l \int d^2r\ \mathrm{Tr}[\zeta_l(\mathbf{r}) Q_l(\mathbf{r})]}, \tag{3.6}$$

where we have introduced the 2D action

$$\widetilde{S}_{2\mathrm{D}}[Q] = S_{2\mathrm{D}}[Q] + \frac{\pi}{2} N_2(0) \sum_l \int d^2r\ \mathrm{Tr}[\Omega Q_l(\mathbf{r})] - 2 \sum_l \int d^2r\ \mathrm{Tr}[\zeta^{\mathrm{SP}} Q_l(\mathbf{r})]$$

$$= \frac{\pi}{8} N_2(0) D \sum_l \int d^2r\ \mathrm{Tr}[\boldsymbol{\nabla}_\parallel Q_l(\mathbf{r})]^2 - 2 \sum_l \int d^2r\ \mathrm{Tr}[\zeta^{\mathrm{SP}} Q_l(\mathbf{r})]. \tag{3.7}$$

$\widetilde{S}_{2\mathrm{D}}$ is the action of the decoupled planes ($t_\perp = 0$) at the finite frequency ω_x. To proceed further, we note that

$$\int \mathcal{D}Q e^{-\widetilde{S}_{2\mathrm{D}}[Q] + 2\Sigma_l \int d^2r\ \mathrm{Tr}[\zeta_l(\mathbf{r}) Q_l(\mathbf{r})]} = \widetilde{Z}_{2\mathrm{D}} e^{W[\zeta]}, \tag{3.8}$$

where $W[\zeta]$ is the generating functional of connected Green's functions calculated with the action $\widetilde{S}_{2\mathrm{D}}$.[19] $\widetilde{Z}_{2\mathrm{D}}$ is the partition function corresponding to the action $\widetilde{S}_{2\mathrm{D}}$. We have

$$W[\zeta] = 2 \sum_l \int d^2r\ \mathrm{Tr}[\zeta_l(\mathbf{r}) \langle Q_l(\mathbf{r}) \rangle_{\widetilde{S}_{2\mathrm{D}}}] + 2 \sum_l \int d^2r_1 d^2r_2 \sum_{ijnm\alpha\beta} {}_{ij}\zeta^{\alpha\beta}_{lnm}(\mathbf{r}_1)\, {}_{ij}\widetilde{R}^{\alpha\beta}_{lnm}(\mathbf{r}_1,\mathbf{r}_2)\, {}_{ji}\zeta^{\beta\alpha}_{lmn}(\mathbf{r}_2) + \cdots, \tag{3.9}$$

where[20]

$${}_{ij}\widetilde{R}^{\alpha\beta}_{lnm}(\mathbf{r}_1,\mathbf{r}_2) = \langle {}_{ji}Q^{\beta\alpha}_{lmn}(\mathbf{r}_1)\, {}_{ij}Q^{\alpha\beta}_{lnm}(\mathbf{r}_2) \rangle_{\widetilde{S}_{2\mathrm{D}}}. \tag{3.10}$$

Using the saddle-point equation (3.2), we obtain to quadratic order in the field ζ and lowest order in Ω the effective action

$$S[\zeta] = \sum_{l,l'} \int d^2r_1 d^2r_2 \sum_{ijnm\alpha\beta} {}_{ij}\zeta^{\alpha\beta}_{lnm}(\mathbf{r}_1) [J^{-1}_{\perp l,l'} \delta(\mathbf{r}_1 - \mathbf{r}_2) - 2\, {}_{ij}\widetilde{R}^{\alpha\beta}_{lnm}(\mathbf{r}_1,\mathbf{r}_2) \delta_{l,l'}]_{ji} \zeta^{\beta\alpha}_{lmn}(\mathbf{r}_2)$$

$$- \frac{\pi}{2} N_2(0) \sum_{l,l'} \int d^2r\ \mathrm{Tr}[\Omega J^{-1}_{\perp l,l'} \zeta_l(\mathbf{r})]. \tag{3.11}$$

For $\omega_n \omega_m < 0$, ${}_{ij}\widetilde{R}^{\alpha\beta}_{lnm}(\mathbf{r}_1,\mathbf{r}_2)$ is the propagator of the Goldstone modes of the action $\widetilde{S}_{2\mathrm{D}}[Q]$. Its Fourier transform is given by

$${}_{ij}\widetilde{R}^{\alpha\beta}_{lnm}(\mathbf{q}_\parallel) \equiv \widetilde{R}(\mathbf{q}_\parallel) = \frac{4}{\pi N_2(0)(D_x q_\parallel^2 + \omega_x)}, \tag{3.12}$$

where D_x is the exact 2D diffusion coefficient at the finite frequency ω_x. Notice that the finite frequency ω_x gives a mass to the Goldstone modes. The preceding equation defines the crossover length $L_x = (D_x/\omega_x)^{1/2}$. Since $\widetilde{R}(\mathbf{q}_\parallel = 0) = 1/2J_\perp(q_\perp = 0)$, the fluctuations of ζ around its saddle point value are massless for $\omega_n \omega_m < 0$. On the other hand, it is clear that the fluctuations are massive for $\omega_n \omega_m > 0$. Having identified the Goldstone modes resulting from the spontaneous symmetry breaking, we now follow the conventional NLσM approach.[8] We suppress the massive fluctuations imposing on the field ζ the constraints $\zeta^2 = \zeta^{\mathrm{SP}2}$ and Tr$\zeta = 0$. Rescaling the field ζ in order to have $\zeta^2 = \underline{1}$ and introducing the Fourier transformed fields $\zeta(\mathbf{q})$, we obtain

$$S[\zeta] = J_\perp^2 \sum_{\mathbf{q}} [J_\perp^{-1}(q_\perp) - 2\widetilde{R}(\mathbf{q}_\parallel)] \mathrm{Tr}[\zeta(\mathbf{q}) \zeta(-\mathbf{q})]$$

$$- \frac{\pi}{2} N_2(0) \mathrm{Tr}[\Omega \zeta(\mathbf{q} = 0)]. \tag{3.13}$$

In the 3D regime, $q_\parallel \lesssim 1/L_x$ and $q_\perp \lesssim 1/d$, we can expand $J_\perp^{-1}(q_\perp) - 2\widetilde{R}(\mathbf{q}_\parallel)$ in lowest order in $q_\parallel$ and $q_\perp$ to obtain

$$S[\zeta]=\frac{\pi}{8}N_2(0)\sum_{\mathbf{q}}\ (D_x q_\parallel^2+2t_\perp^2 d^2\tau q_\perp^2)\mathrm{Tr}[\zeta(\mathbf{q})\zeta(-\mathbf{q})]$$

$$-\frac{\pi}{2}N_2(0)\mathrm{Tr}[\Omega\zeta(\mathbf{q}=0)]. \qquad (3.14)$$

Going back to real space and taking the continuum limit in the z direction (which introduces a factor $1/d$), we eventually come to

$$S[\zeta]=\frac{\pi}{8}N_3(0)\int d^3r\{D_x\ \mathrm{Tr}[\boldsymbol{\nabla}_\parallel\zeta]^2+2t_\perp^2 d^2\tau\ \mathrm{Tr}[\nabla_z\zeta]^2\}$$

$$-\frac{\pi}{2}N_3(0)\int d^3r\ \mathrm{Tr}[\Omega\zeta]. \qquad (3.15)$$

The cutoffs are $\Lambda_x=L_x^{-1}$ in the longitudinal directions and $1/d$ in the transverse direction.

Equation (3.15) is similar to the action (2.9) we have obtained in the RG approach. The only difference is that the crossover length is not defined in the same way. In the RG approach, $L_x\sim D(L_x)/\omega_x$ is defined via the length-dependent 2D diffusion coefficient while the auxiliary field method involves the frequency-dependent 2D diffusion coefficient. We approximate the latter by[1]

$$D(\omega_\nu)=\frac{D}{1+l^2/\xi_{2\mathrm{D}}^2|\omega_\nu|\tau}, \qquad (3.16)$$

where ω_ν is a bosonic Matsubara frequency. This yields

$$\Lambda_x^2=L_x^{-2}=\frac{\omega_x}{D}\left(1+\frac{l^2}{\xi_{2\mathrm{D}}^2\omega_x\tau}\right). \qquad (3.17)$$

The critical interplane coupling $t_\perp^{(c)}$ is determined by $N_2(0)D_x\equiv N_2(0)D(\omega_x)\sim 1$, which leads again to Eq. (2.4). The crossover length L_x is shown in Fig. 1. For $t_\perp\gtrsim t_\perp^{(c)}$, there is an agreement with the RG approach. The reason is that in the weak coupling limit ($g\gtrsim 1$), $D(L)$ and $D(\omega_\nu)$ approximately coincide.[1] Deep in the insulating phase, this is not the case and the two approaches give different results: $L_x\gg\xi_{2\mathrm{D}}$ in the RG approach while $L_x\sim\xi_{2\mathrm{D}}$ in the auxiliary field method. This disagreement should not be surprising since neither one of the two methods is exact. In the RG approach, the dimensional crossover is treated very crudely since all the effects due to $t_\perp$ are neglected in the first (2D) stage of the renormalization procedure. However, this method should give qualitatively correct results. In particular, we expect the result $L_x\gg\xi_{2\mathrm{D}}$ for $t_\perp\ll t_\perp^{(c)}$ to be correct. In the auxiliary field method, the effective action (3.15) was obtained by completely neglecting the massive modes (more precisely sending their mass to infinity). In principle, these latter should be integrated out, which would lead to a renormalization of the diffusion coefficients appearing in Eq. (3.15). The comparison with the RG result suggests that this renormalization is important in the insulating phase.

The generalization of the preceding results in order to include a parallel magnetic field is straightforward. The interplane coupling is now given by Eq. (2.21). The auxiliary field ζ has therefore only a diagonal part ($i=j$). The saddle-point equation yields

$${}_{ii}(\zeta^{\mathrm{SP}})^{\alpha\beta}_{lnm}(\mathbf{r})=\delta_{\alpha,\beta}\delta_{n,m}\ \mathrm{sgn}(\omega_n)\ \frac{\pi}{2}N_2(0)\ \frac{t_\perp^2\tau}{(1+\omega_c^2\tau^2)^{1/2}}. \qquad (3.18)$$

This defines the crossover frequency

$$\omega_x=\frac{4t_\perp^2\tau}{(1+\omega_c^2\tau^2)^{1/2}}. \qquad (3.19)$$

In the 3D regime, the massless fluctuations around the saddle point value are described by the action

$$S[\zeta]=\frac{\pi}{8}N_3(0)\int d^3r\Bigg[D_x\ \mathrm{Tr}[\boldsymbol{\nabla}_\parallel\zeta]^2+\frac{2t_\perp^2 d^2\tau}{(1+\omega_c^2\tau^2)^{1/2}}\ \mathrm{Tr}[\nabla_z\zeta]^2\Bigg]-\frac{\pi}{2}N_3(0)\int d^3r\ \mathrm{Tr}[\Omega\zeta], \qquad (3.20)$$

with the usual constraints on the field ζ. Here D_x is the coefficient diffusion calculated at the magnetic-field-dependent frequency ω_x. Again we recover the result of the RG approach, the only difference coming from the definitions of L_x and Λ_x.

IV. CONCLUSION

Using two different methods, we have studied the Anderson MIT in quasi-2D systems. We have found that the critical value of the single-particle interplane coupling is given by Eq. (2.4). Apart from the factor $1/\sqrt{k_F l}$, this result agrees with the diagrammatic self-consistent theory of Anderson localization and, in contradiction with Abrikosov's interpretation,[6] with estimates based on the weak localization correction to the Drude-Boltzmann conductivity (see Sec. II E). Nevertheless, it differs from recent numerical calculations according to which $t_\perp^{(c)}\sim 1/\tau^2$.[5] In the weak disorder limit ($k_F l\gg 1$), this latter result seems to us in contradiction with the scaling theory of Anderson localization since the 2D localization length $\xi_{2\mathrm{D}}$ is exponentially large with respect to the mean free path l. Because of the latter property, we indeed expect an exponentially small value of the critical coupling $t_\perp^{(c)}$ with respect to the elastic scattering rate $1/\tau$.[11] For a very large 2D localization length $\xi_{2\mathrm{D}}$, it seems unlikely that the dimensional crossover and the MIT can be studied with numerical calculations on finite systems. The numerical calculations of Ref. 5 are done in a strong disorder regime ($k_F l\sim 1$). In this regime, the exponential dependence of $t_\perp^{(c)}$ on $k_F l$ may be easily overlooked.

We have also studied the anisotropy of the correlation (localization) lengths in the metallic (insulating) phase and shown that it differs from the result predicted by a 3D anisotropic NLσM. The phase diagram in the presence of a magnetic field was also derived: our approach formalizes and extends previous results obtained for weakly coupled chains.[3]

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[1] D. Vollhardt and P. Wölfle, Phys. Rev. B **22**, 4666 (1980); in *Electronic Phase Transitions*, edited by W. Hanke and Yu. V. Kopaev (Elsevier, New York, 1992).

[2] V. N. Prigodin and Yu. A. Firsov, J. Phys. C **17**, L979 (1984).

[3] N. Dupuis and G. Montambaux, Phys. Rev. B **46**, 9603 (1992).

[4] Qiming Li, C. M. Soukoulis, E. N. Economou, and G. S. Grest, Phys. Rev. B **40**, 2825 (1989).

[5] I. Zambetaki, Qiming Li, E. N. Economou, and C. M. Soukoulis, Phys. Rev. Lett. **76**, 3614 (1996).

[6] A. A. Abrikosov, Phys. Rev. B **50**, 1415 (1994).

[7] O. N. Dorokhov, Solid State Commun. **46**, 605 (1983); **51**, 381 (1984); N. A. Panagiotides, S. N. Evangelou, and G. Theodorou, Phys. Rev. B **49**, 14 122 (1980); V. N. Prigodin and Yu. A. Firsov, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 241 (1983) [Sov. Phys. JETP **38**, 284 (1983)].

[8] For a review on the NLσM approach to Anderson localization, see D. Belitz and T. R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).

[9] P. Wölfle and R. N. Bhatt, Phys. Rev. B **30**, 3542 (1984).

[10] This is possible because the anisotropy is the same for the diffusion coefficients (D and $D_\perp$) and the ultraviolet cutoffs ($1/\sqrt{D\tau}$ and $1/\sqrt{D_\perp \tau}$) (Ref. 9).

[11] I. Affleck and B. I. Halperin, J. Phys. A **29**, 2627 (1996).

[12] The correlation length corresponds to the length at which the Ohmic regime is reached. In this regime, we have $\beta(g)\sim d-2$ where d is the dimension and $\beta(g)$ the beta function expressed as a function of the dimensionless conductance. See, for instance, P. A Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[13] In Ref. 9, an isotropic NLσM is obtained from the very beginning of the analysis (i.e., the 2D stage of the RG procedure is absent) by an appropriate rescaling of the lengths (see the equation following Eq. 15 of Ref. 9). This rescaling implies that all the characteristic lengths have an isotropy given by $t_\perp / t_\parallel$.

[14] N. Dupuis and G. Montambaux, Phys. Rev. B **49**, 8993 (1994). See in particular Sec. VI.

[15] Note that in Ref. 3, the planes themselves are highly anisotropic with an open Fermi surface. In that case, $(1+\omega_c^2\tau^2)^{1/2}$ should be replaced by $1+\omega_c^2\tau^2$ in Eq. (2.21).

[16] For example, in the Ising model, one usually obtains an effective ϕ^4 theory from a decoupling of the spin-spin interaction by means of a Hubbard-Stratonovich transformation. See, for instance, R. J. Creswick, H. A. Farach, and C. P. Poole, *Introduction to Renormalization Group methods in Physics* (John Wiley, New York, 1991).

[17] R. H. McKenzie, Phys. Rev. B **51**, 6249 (1995).

[18] D. Boies, C. Bourbonnais, and A.-M. S. Tremblay, Phys. Rev. Lett. **74**, 968 (1995); Proceedings of the XXXI Rencontres de Moriond, series: Les Arcs, Savoie, France, January 20–27, 1996, *Correlated Fermions and Transport in Mesoscopic systems*, edited by T. Martin, G. Montambaux, and J. Trân Thanh Vân (Éditions Frontières, Gif-sur-Yvette, 1996).

[19] See, for instance, J. W. Negele and H. Orland, *Quantum Many Particle Systems* (Addison-Wesley, New York, 1988).

[20] Since we consider only the two-point Green's function, we do not need to distinguish between connected and disconnected Green's functions.