Renormalization-group approach to Fermi-liquid theory

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We show that the renormalization-group (RG) approach to interacting fermions at one-loop order recovers Fermi-liquid-theory results when the forward scattering zero sound and exchange channels are both taken into account. The Landau parameters are related to the fixed point value of the Ω limit of the forward scattering vertex. We specify the conditions under which the results obtained at one-loop order hold at all orders in a loop expansion. We also emphasize the similarities between our RG approach and the diagrammatic derivation of Fermi-liquid theory. [S0163-1829(96)10526-9]

Much of our understanding of interacting fermions is based on Fermi-liquid theory (FLT).[1,2] Although the latter was first formulated as a phenomenological theory, its microscopic foundation was rapidly established using field theoretical methods.[2,3] The discovery of new materials showing non-Fermi-liquid behavior, like high-T_c superconductors, has motivated a lot of theoretical work in order to clarify the validity and the limitations of FLT.

Several authors have recently applied renormalization-group (RG) methods to interacting fermions (see Refs. 4–7 and references therein). While these methods are well known in the context of one- and quasi-one dimensional interacting fermion systems where they have been very successful,[8] their application to isotropic systems of dimension d greater than one is more recent. In his study of interacting fermions in $d=2,3$, Skankar used both RG methods and a standard perturbative calculation.[5] While RG arguments were used to identify the relevant couplings, the low-energy degrees of freedom were explicitly integrated out in the Landau interaction channel by means of standard diagrammatic calculations. Extending Shankar's approach to finite temperature, Chitov and Sénéchal have recently shown how this interaction channel can be treated by the RG method without use of any additional perturbative calculation.[7] Moreover, the finite-temperature formalism clearly establishes the difference between the Q and Ω limits of the forward scattering vertex and therefore differentiates the Landau function (i.e., the Landau parameters F_l^s and F_l^a) from the (physical) forward scattering amplitude. It is clear that both approaches[5,7] amount to summing the series of bubble diagrams in the forward scattering zero sound channel. Since it is well known that such a random-phase-approximation- (RPA) type calculation reproduces the results of FLT,[9] the agreement between the FLT and RG approaches is *a posteriori* not surprising. Although the selection of Feynman diagrams appearing in the RG procedure was justified on the basis of an expansion in the small parameter Λ_0/K_F (K_F is the Fermi wave vector and Λ_0 a low-energy cutoff), it is nevertheless rather unexpected that the RG approach reduces to a RPA calculation while the diagrammatic microscopic derivation of FLT (Refs. 2,3) is obviously more than a simple RPA calculation.

The aim of this paper is to reconsider the RG approach to interacting fermions along the lines developed in Refs. 5,7 in order to clarify its connection with FLT. First, we derive the RG equation for the Q (Ω) limit of the forward scattering vertex Γ^Q (Γ^Ω) at one-loop order. In order to respect the Fermi statistics, the forward scattering zero sound (ZS) and exchange (ZS$'$) channels are both taken into account. As a result, we find that both flows of Γ^Q and Γ^Ω are nonzero. We show that the antisymmetry of Γ^Q under exchange of the two incoming or outgoing particles is conserved under the RG equation, while the antisymmetry of Γ^Ω is lost. We then solve (approximately) the RG equations to obtain a relation between the fixed point (FP) values Γ^{Q*} and $\Gamma^{\Omega*}$. The standard relation between Γ^{Q*} and the Landau parameters F_l^s, F_l^a (which is one of the key results of the microscopic diagrammatic derivation of FLT) is recovered if one identifies these latter with $\Gamma^{\Omega*}$. This result differs from previous RG approaches[5,7] where the Landau parameters were identified with the bare interaction of the low-energy effective action (which is the starting point of the RG analysis). We show that the relation between $\Gamma^{\Omega*}$ and the Landau parameters obtained at one-loop order holds at all orders if one assumes that the only singular contribution to the RG flow is due to the one-loop ZS graph.

We consider a two-dimensional system of interacting spin-$\frac{1}{2}$ fermions with a circular Fermi surface (the results obtained in this paper can be straightforwardly extended to the three-dimensional case). Following Refs. 5,7, we write the partition function as a functional integral over Grassmann variables, $Z=\int \mathcal{D}\psi^*\mathcal{D}\psi e^{-S}$, where S is a low-energy effective action (we set $\hbar=k_B=1$):

$$S=-\sum_{\widetilde{K},\sigma}\psi^*_\sigma(\widetilde{K})(i\omega-\epsilon(\mathbf{K})+\mu)\psi_\sigma(\widetilde{K})$$
$$+\frac{1}{4\beta\nu}\sum_{\widetilde{K}_1,\ldots,\widetilde{K}_4}\sum_{\sigma_1,\ldots,\sigma_4}U_{\sigma_1\sigma_2,\sigma_3\sigma_4}(\mathbf{K}_1,\mathbf{K}_2,\mathbf{K}_3,\mathbf{K}_4)$$
$$\times\psi^*_{\sigma_4}(\widetilde{K}_4)\psi^*_{\sigma_3}(\widetilde{K}_3)$$
$$\times\psi_{\sigma_2}(\widetilde{K}_2)\psi_{\sigma_1}(\widetilde{K}_1)\delta_{\mathbf{K}_1+\mathbf{K}_2,\mathbf{K}_3+\mathbf{K}_4}\delta_{\omega_1+\omega_2,\omega_3+\omega_4}+\cdots,\tag{1}$$

where the dots denote terms which are irrelevant at the tree level.[5] Here $\mathbf{K}$ is a two-dimensional vector with $|K-K_F|<\Lambda_0\ll K_F$. μ is the chemical potential, K_F the Fermi wave vector, and the cutoff Λ_0 fixes the energy scale of the effective action. $\widetilde{K}=(\mathbf{K},\omega)$ and ω is a fermionic Matsubara frequency. $\beta=1/T$ is the inverse temperature and ν the size of the system. $\sigma=\uparrow,\downarrow$ refers to the electron spins. The antisymmetrized coupling function $U_{\sigma_1\sigma_2,\sigma_3\sigma_4}(\mathbf{K}_1,\mathbf{K}_2,\mathbf{K}_3,\mathbf{K}_4)$ is assumed to be a nonsingular function of its arguments. Ignoring irrelevant terms, we write the single-particle energy as $\epsilon(\mathbf{K})=\mu+v_Fk$ where $K=K_F+k$ and v_F is the Fermi velocity. The summation over the wave vectors is defined by

$$\frac{1}{\nu}\sum_{\mathbf{K}}=\int\frac{d^2\mathbf{K}}{(2\pi)^2}\equiv K_F\int_{-\Lambda_0}^{\Lambda_0}\frac{dk}{2\pi}\int_0^{2\pi}\frac{d\theta}{2\pi},\tag{2}$$

keeping only the relevant term in the integration measure at the tree level. Shankar's analysis of the coupling functions of the quartic interaction shows that only two such functions survive under the RG flow for $\Lambda_0\ll K_F$: the forward scattering coupling function and the BCS coupling function. In the following, we neglect this latter by assuming it is irrelevant at one-loop order so that no BCS instability occurs. The forward scattering coupling function is denoted by $\Gamma_{\sigma_i}(\widetilde{K}_1,\widetilde{K}_2,\widetilde{K}_2-\widetilde{Q},\widetilde{K}_1+\widetilde{Q})$ where $\widetilde{Q}=(\mathbf{Q},\Omega)$ with $Q\ll K_F$ and Ω is a bosonic Matsubara frequency (we use the notation $\Gamma_{\sigma_i}\equiv\Gamma_{\sigma_1\sigma_2,\sigma_3\sigma_4}$). Since the dependence on $k_{1,2}$ and $\omega_{1,2}$ is irrelevant, we introduce the coupling function $\Gamma_{\sigma_i}(\theta_1,\theta_2,\widetilde{Q})=\Gamma_{\sigma_i}(\mathbf{K}_1^F,\mathbf{K}_2^F,\mathbf{K}_2^F-\widetilde{Q},\mathbf{K}_1^F+\widetilde{Q})$ where $\mathbf{K}^F=K_F\mathbf{K}/K=K_F(\cos\theta,\sin\theta)$ is a wave vector on the Fermi surface. The forward scattering coupling function can be decomposed into a spin-triplet amplitude Γ_t and a spin-singlet amplitude Γ_s:[10,11]

$$\Gamma_{\sigma_i}(\theta_1,\theta_2,\widetilde{Q})=\frac{\Gamma_t(\theta_1,\theta_2,\widetilde{Q})}{2}(\delta_{\sigma_1,\sigma_4}\delta_{\sigma_2,\sigma_3}+\delta_{\sigma_1,\sigma_3}\delta_{\sigma_2,\sigma_4})$$
$$+\frac{\Gamma_s(\theta_1,\theta_2,\widetilde{Q})}{2}(\delta_{\sigma_1,\sigma_4}\delta_{\sigma_2,\sigma_3}$$
$$-\delta_{\sigma_1,\sigma_3}\delta_{\sigma_2,\sigma_4}).\tag{3}$$

We now introduce the Q limit (Γ^Q) and Ω limit (Γ^Ω) of the forward scattering vertex:

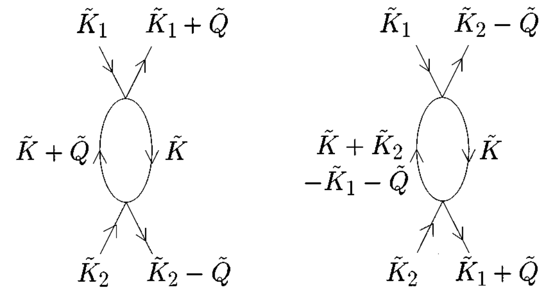

FIG. 1. One-loop diagrams for the renormalization of the vertex Γ_{σ_i} in the ZS and ZS′ channels (the spin indices are not shown).

$$\Gamma^Q_{\sigma_i}(\theta_1-\theta_2)=\lim_{Q\to0}[\Gamma_{\sigma_i}(\theta_1,\theta_2,\widetilde{Q})|_{\Omega=0}],$$
$$\Gamma^\Omega_{\sigma_i}(\theta_1-\theta_2)=\lim_{\Omega\to0}[\Gamma_{\sigma_i}(\theta_1,\theta_2,\widetilde{Q})|_{Q=0}].\tag{4}$$

$\Gamma^Q_{\sigma_i}$ and $\Gamma^\Omega_{\sigma_i}$ can be decomposed into singlet and triplet amplitudes according to (3). The only remnant of the antisymmetry of Γ_{σ_i} is then the condition $\Gamma_t^{Q,\Omega}(\theta=0)=0$ for the bare vertices.[7]

We now derive the RG equation (using the Kadanoff-Wilson approach[5]) for the coupling Γ_{σ_i} when the cutoff Λ_0 is reduced according to $\Lambda(t)=\Lambda_0e^{-t}$. Three diagrams have to be considered at one-loop order, corresponding to the ZS, ZS′, and BCS channels. As pointed out in Ref. 7, the ZS graph alone does not respect the Fermi statistics. Indeed, if one exchanges the two incoming or outgoing lines, the ZS graph transforms into the ZS′ graph and vice versa (Fig. 1). It is therefore necessary to consider the ZS and ZS′ graphs on the same footing. We ignore momentarily the symmetry-preserving contribution of the BCS diagram which will be discussed later.

The contribution of the ZS graph is[7]

$$\left.\frac{d\Gamma^Q_{\sigma_i}(\theta_1-\theta_2)}{dt}\right|_{ZS}=-\frac{N(0)\beta_R}{\cosh^2(\beta_R)}$$
$$\times\int\frac{d\theta}{2\pi}\sum_{\sigma,\sigma'}\Gamma^Q_{\sigma_1\sigma',\sigma\sigma_4}(\theta_1-\theta)\Gamma^Q_{\sigma\sigma_2,\sigma_3\sigma'}(\theta-\theta_2),$$
$$\left.\frac{d\Gamma^\Omega_{\sigma_i}(\theta_1-\theta_2)}{dt}\right|_{ZS}=0,\tag{5}$$

where $\beta_R=v_F\beta\Lambda(t)/2$ is a dimensionless inverse temperature and $N(0)=K_F/2\pi v_F$ the density of states per spin. Since $\lim_{\beta\to\infty}(\beta/4)\cosh^{-2}(\beta x/2)=\delta(x)$, the ZS graph gives a singular contribution to the RG flow of Γ^Q when $T\to0$. Consider now the contribution of the ZS′ graph. For a given $\widetilde{Q}$, this graph involves the quantity (Fig. 1)

$$T\sum_\omega G(\widetilde{K})G(\widetilde{K}+\mathbf{K}^F_{21}-\widetilde{Q})$$
$$=\frac{1}{2}\frac{\tanh[(\beta/2)\epsilon(\mathbf{K}+\mathbf{K}^F_{21}-\mathbf{Q})]-\tanh[(\beta/2)\epsilon(\mathbf{K})]}{-i\Omega+\epsilon(\mathbf{K})-\epsilon(\mathbf{K}+\mathbf{K}^F_{21}-\mathbf{Q})}.\tag{6}$$

Here $G(\widetilde{K})=(i\omega-v_Fk)^{-1}$ is the one-particle Green's function and $\mathbf{K}^F_{21}=\mathbf{K}^F_2-\mathbf{K}^F_1$. In general, the limit $\widetilde{Q}\to0$ can be taken without any problem (and is independent of the order in which the limits $Q,\Omega\to 0$ are taken) and (6) gives a smooth contribution to the flow of Γ^Q and Γ^Ω. As pointed out by Mermin,[12] problems arise when $\mathbf{K}_{21}^F$ is small since the limits $\tilde{Q}\to 0$ and $\mathbf{K}_{21}^F\to 0$ do not commute. For small $\mathbf{K}_2^F-\mathbf{K}_1^F$, i.e., for $|\theta_1-\theta_2|\ll T/E_F$, (6) becomes

$$\frac{v_F\hat{\mathbf{K}}\cdot(\mathbf{K}_{21}^F-\mathbf{Q})}{-i\Omega-v_F\hat{\mathbf{K}}\cdot(\mathbf{K}_{21}^F-\mathbf{Q})}\,\frac{\beta/4}{\cosh^2(\beta v_F k/2)},\tag{7}$$

where $\hat{\mathbf{K}}=\mathbf{K}/K$ is a unit vector. This quantity [apart from the thermal factor $\beta\cosh^{-2}(\beta v_F k/2)$] has been analyzed in detail by Mermin who showed that the antisymmetry of the vertex is strongly related to the order in which the different limits are taken. Following Ref. 12, we first take the limit $\tilde{Q}\to 0$ (which is well defined for $\mathbf{K}_1^F\neq\mathbf{K}_2^F$) and then $\theta_1-\theta_2\to 0$. This ensures that $\Gamma_{\sigma_i}^{Q,\Omega}(\theta)$ is a continuous function at $\theta=0$. We then have ($\tilde{Q}=0$)

$$\lim_{\theta_1\to\theta_2}\left[T\sum_\omega G(\tilde{K})G(\tilde{K}+\mathbf{K}_{21}^F)\right]=-\frac{\beta}{4\cosh^2(\beta v_F k/2)}.\tag{8}$$

Equation (8) shows that when $T\to 0$ the ZS$'$ graph gives a singular contribution to the RG flow of $\Gamma^{Q,\Omega}(\theta)$ for $|\theta|\ll T/E_F$.[13] Taking into account the spin dependence of the coupling, we obtain that the contributions of the ZS and ZS$'$ graphs to the RG flow of $\Gamma_t^Q(\theta=0)$ cancel each other. Consequently, $\Gamma_t^Q(\theta=0)=0$ for any value of the flow parameter t. The antisymmetry of Γ^Q is therefore conserved under the RG equation. Since the contribution of the ZS graph to the RG flow of $\Gamma^\Omega(\theta=0)$ vanishes, while the contribution of the ZS$'$ graph does not, the antisymmetry of Γ^Ω is not conserved under RG. This result agrees with standard diagrammatic calculations.[12]

Taking into account both the contributions of the ZS and ZS$'$ graphs, the RG equations of $\Gamma^{Q,\Omega}$ can be written

$$\frac{d\Gamma_{\sigma_i}^Q}{dt}=\frac{d\Gamma_{\sigma_i}^Q}{dt}\Big|_{\mathrm{ZS}}+\frac{d\Gamma_{\sigma_i}^Q}{dt}\Big|_{\mathrm{ZS}'},$$
$$\frac{d\Gamma_{\sigma_i}^\Omega}{dt}=\frac{d\Gamma_{\sigma_i}^\Omega}{dt}\Big|_{\mathrm{ZS}'}=\frac{d\Gamma_{\sigma_i}^Q}{dt}\Big|_{\mathrm{ZS}'}.\tag{9}$$

The two preceding equations can be combined [using also (5)] to obtain

$$\frac{d\Gamma_{\sigma_i}^Q(\theta_1-\theta_2)}{dt}=\frac{d\Gamma_{\sigma_i}^\Omega(\theta_1-\theta_2)}{dt}-\frac{N(0)\beta_R}{\cosh^2(\beta_R)}\int\frac{d\theta}{2\pi}\sum_{\sigma,\sigma'}\Gamma^Q_{\sigma_1\sigma',\sigma\sigma_4}(\theta_1-\theta)\times\Gamma^Q_{\sigma\sigma_2,\sigma_3\sigma'}(\theta-\theta_2).\tag{10}$$

In order to solve this RG equation, we Fourier transform $\Gamma_{\sigma_i}^{Q,\Omega}(\theta)$ and introduce the spin symmetric ($A^{Q,\Omega}$) and antisymmetric ($B^{Q,\Omega}$) parts:

$$\Gamma_{\sigma_i}^{Q,\Omega}(l)=\int\frac{d\theta}{2\pi}e^{-il\theta}\Gamma_{\sigma_i}^{Q,\Omega}(\theta),$$

$$2N(0)\Gamma_{\sigma_i}^{Q,\Omega}(l)=A_l^{Q,\Omega}\delta_{\sigma_1,\sigma_4}\delta_{\sigma_2,\sigma_3}+B_l^{Q,\Omega}\boldsymbol{\tau}_{\sigma_1\sigma_4}\cdot\boldsymbol{\tau}_{\sigma_2\sigma_3},\tag{11}$$

where $\boldsymbol{\tau}$ denote the Pauli matrices. Equation (11) holds when the spin-dependent part of the particles interaction is due purely to exchange. Equation (10) then takes the simple form

$$\frac{dA_l^Q}{dt}=\frac{dA_l^\Omega}{dt}-\frac{\beta_R}{\cosh^2(\beta_R)}A_l^{Q2},\tag{12}$$

and the same equation relating B_l^Q and B_l^Ω. Integrating these equations between 0 and t, we obtain (writing explicitly the t dependence)

$$A_l^Q(t)=A_l^\Omega(t)-\int_0^t dt'\,\frac{\beta_R}{\cosh^2(\beta_R)}A_l^Q(t')^2\tag{13}$$

and a similar equation for $B_l^Q(t)$. The RG equations in their symmetry-preserving form (12), (13) relate two FP's $\Gamma_{\sigma_i}^{Q*}$ and $\Gamma_{\sigma_i}^{\Omega*}$ in a fashion more general than the standard RPA-like form [see Eq. (15) below] with all harmonics decoupled. Deferring study of such a fixed point, which is beyond the scope of the present paper,[14] we concentrate now on the approximation leading to the standard FLT results. Because of the thermal factor $\beta_R/\cosh^2(\beta_R)$, the second term of the right-hand side (RHS) of (13) becomes different from zero only when $\Lambda(t)\lesssim T/v_F$. On the other hand, we have shown above that $\Gamma_{\sigma_i}^\Omega(\theta)$ is a smooth function of $\Lambda(t)$ except for $|\theta|\ll T/E_F$. The Fourier transform $\Gamma_{\sigma_i}^\Omega(l)$ is also a smooth function of $\Lambda(t)$. At low temperature, we can therefore make the approximation $A_l^\Omega|_{\Lambda(t)\lesssim T/v_F}\simeq A_l^{\Omega*}$, where $A_l^{\Omega*}=A_l^\Omega|_{\Lambda(t)=0}$ is the FP value of A_l^Ω. This allows us to rewrite (13) for $\Lambda(t)\lesssim T/v_F$ as

$$A_l^Q(t)=A_l^{\Omega*}-\int_0^t dt'\,\frac{\beta_R}{\cosh^2(\beta_R)}A_l^Q(t')^2.\tag{14}$$

Equation (14) is solved by introducing the parameter $\tau=\tanh\beta_R$. The FP values of A_l^Q and B_l^Q are given in the zero-temperature limit by

$$A_l^{Q*}=\frac{A_l^{\Omega*}}{1+A_l^{\Omega*}},\quad B_l^{Q*}=\frac{B_l^{\Omega*}}{1+B_l^{\Omega*}}.\tag{15}$$

Equation (15) shows that the standard results of the microscopic FLT are recovered if one identifies the Landau parameters with the FP values of A_l^Ω and B_l^Ω:

$$F_l^s=A_l^{\Omega*},\quad F_l^a=B_l^{\Omega*}.\tag{16}$$

Alternatively, (16) can be written as $f_{\sigma_i}(\theta)=\Gamma_{\sigma_i}^{\Omega*}(\theta)$ where $f_{\sigma_i}(\theta)$ is Landau's quasiparticle interaction function. Since the singular contribution (8) of the ZS$'$ graph to $\Gamma_{\sigma_i}^\Omega$ was neglected when approximating (13) by (14), $\Gamma_{\sigma_i}^{Q*}(\theta)$ obtained from (15) is correct only for $|\theta|\gtrsim T/E_F$. The determination of $\Gamma_{\sigma_i}^{Q*}(\theta)$ for $|\theta|\lesssim T/E_F$ would require the consideration of the singular contribution of the ZS$'$ graph. It should be noted that the diagrammatic derivation of FLT (Refs. 2,3) also neglects the zero-angle singularity in the ZS$'$ channel and therefore does not respect the antisymmetry of the Q limit of the forward scattering vertex. The condition $\Gamma_t^Q(\theta=0)=0$ is usually enforced, giving the ‘‘amplitude sum rule'' of FLT.[10] For physical quantities (like collective modes or response functions) which probe all values of the angle θ, it is nevertheless justified to neglect the singularity of the ZS$'$ channel.

The relation (15) between Γ^{Q*} and the Landau parameters has been obtained at one-loop order. It appears therefore as an approximate relation whose validity is restricted to the weak-coupling limit. However, it turns out that (15) holds at all orders in a loop expansion if we assume that the only singular contribution to the RG flow comes from the one-loop ZS graph (again we neglect the singular contribution of the ZS$'$ graph). Note that the same kind of assumption is at the basis of the diagrammatic derivation of FLT.[2,3] In this case, the RG flows of Γ^Q and Γ^Ω are determined by

$$\frac{d\Gamma^Q_{\sigma_i}}{dt}=\left.\frac{d\Gamma^Q_{\sigma_i}}{dt}\right|_{\rm ZS}+\left.\frac{d\Gamma^Q_{\sigma_i}}{dt}\right|_{{\rm ZS}',{\rm BCS},2\ {\rm loops}\ldots}, \tag{17}$$

$$\frac{d\Gamma^\Omega_{\sigma_i}}{dt}=\left.\frac{d\Gamma^\Omega_{\sigma_i}}{dt}\right|_{{\rm ZS}',{\rm BCS},2\ {\rm loops}\ldots}=\left.\frac{d\Gamma^Q_{\sigma_i}}{dt}\right|_{{\rm ZS}',{\rm BCS},2\ {\rm loops}\ldots}. \tag{18}$$

The contribution of the one-loop ZS graph [first term of the RHS of (17)] has been separated from the nonsingular contributions. Note that we have included in this latter the one-loop BCS graph which had been neglected up to now. Equations (17), (18) can be combined to obtain

$$\frac{d\Gamma^Q_{\sigma_i}}{dt}=\frac{d\Gamma^\Omega_{\sigma_i}}{dt}+\left.\frac{d\Gamma^Q_{\sigma_i}}{dt}\right|_{\rm ZS}, \tag{19}$$

where the second term of the RHS of (19) is given by (5). Since, according to our assumption, $\Gamma^\Omega_{\sigma_i}$ is a nonsingular function of $\Lambda(t)$, Eq. (19) is similar to Eq. (10) and can be solved in the same way, yielding again the result (15). Thus, higher-order contributions change the FP value $\Gamma^{\Omega*}_{\sigma_i}$ obtained at one-loop order, but not the relation (15) between $\Gamma^{Q*}_{\sigma_i}$ and $\Gamma^{\Omega*}_{\sigma_i}$.

It should be pointed out that the Landau parameters are not determined by the bare coupling function $U_{\sigma_i}(\mathbf{K}_1,\ldots,\mathbf{K}_4)$ of the effective action (1), but are related to the FP value $\Gamma^{\Omega*}_{\sigma_i}$. Usually, the FP values of physical quantities are related to some bare effective values, so that their calculation by means of the RG (within the framework of a loop expansion) is approximative and valid only in the weak-coupling regime. FLT, which assumes that the only singular contribution to the RG flow is due to the one-loop ZS graph, does not rely on any kind of weak-coupling condition.

The approximation (14) is equivalent to the Bethe-Salpeter equation in the ZS channel for the vertex $\Gamma^Q_{\sigma_i}$, $\Gamma^{\Omega*}_{\sigma_i}$ being the two-particle-irreducible (2PI) vertex. The rearrangement of diagram summations leading to the Bethe-Salpeter equation in FLT (Refs. 2,3) is based on the assumption that this 2PI vertex is a regular function of its variables, neglecting the zero-angle singularity in the ZS$'$ channel. As a consequence, the antisymmetry of the Q limit of the forward scattering vertex is not guaranteed in the final result, and ''the amplitude sum rule'' must be imposed. From this point of view, there is a strict equivalence between the present RG approach and the diagrammatic microscopic derivation of FLT.

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[13] Note that $\mathbf{K}$ and $\mathbf{K}+\mathbf{K}^F_{21}-\mathbf{Q}$ can both be within an infinitesimal shell near the cutoff $\Lambda(t)$ only if $\mathbf{Q},\mathbf{K}^F_{21}\to 0$. When this latter condition is not realized, the ZS$'$ diagram is generated from the six-point vertex function. The importance of m-point ($m\geqslant 6$) vertex functions, which was pointed out by Shankar in a preliminary version of Ref. 5, will be discussed in detail elsewhere.

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