Localization and magnetic field in a quasi-one-dimensional conductor

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(Received 5 March 1992)

The semiclassical Boltzmann conductivity and the first quantum correction are calculated for a strongly two-dimensional (2D) anisotropic conductor (weakly coupled chains system) in the presence of a magnetic field. From scaling arguments, the ground state of the system at zero temperature is determined. When the coupling t between chains is much smaller than the elastic scattering rate $1/\tau$, the system behaves as a set of uncoupled 1D chains. In the other limit where $1/\tau \ll t$, the gas shows a 2D (anisotropic) behavior in zero field. A weak magnetic field leads to a negative magnetoresistance. As a consequence of the quasi-1D aspect of the Fermi surface, a strong magnetic field induces a transition from a 2D regime towards a 1D insulating state. The calculations are extended to the 3D case, where a magnetic field perpendicular to the chains can induce an Anderson localization.

I. INTRODUCTION

Among the numerous studies devoted to the physics of localization in noninteracting disordered conductors, the effect of a magnetic field has been one of the major topics.[1–3] An important aspect has been the study of the effect of a weak magnetic field. In the weakly localized regime (where pertubative calculations are possible) a negative magnetoresistance was predicted in two-dimensional (2D) and 3D systems.[4–7] However, a weak magnetic field does not destroy the localization in a 2D system.[5,8] In a strong magnetic field, the interplay between localization and Landau quantization plays a crucial role and leads to the quantization of the Hall effect.[9]

On the other hand, the 2D anisotropic electron gas that can be found experimentally in weakly coupled chains has revealed spectacular properties in a magnetic field, which have been studied extensively during the past years. For example, the quasi-1D conductors of the Bechgaard salts family present a surprising phase diagram: a cascade of spin-density wave phases appearing for increasing magnetic field. The spectrum of these field-induced spin-density wave phases is quantized, leading to a mechanism for the quantized Hall effect. More generally, the thermodynamic properties of these quasi-1D conductors result from an interplay between the 2D (or 3D) and 1D aspects of the Fermi surface.[10]

In this paper, we investigate the interplay between disorder and magnetic field in a strongly anisotropic conductor in the absence of electron-electron interaction. It is shown that the quasi-1D aspect of the Fermi surface leads to new developments in the physics of localization.[11]

We consider first a system of parallel chains arranged in one plane (x,y) with interspacing b. When the amplitude of the hopping t between the neighboring chains is small ($t \ll \mu$, μ being the Fermi energy), the Fermi surface is made of two slightly warped sheets and is well described by the dispersion law ($\hbar = 1$ throughout the paper)

$$E(\mathbf{k}) = v(|k_x| - k_F) + t\cos(k_y b) + \mu. \tag{1}$$

Here v is the Fermi velocity.[12] The linear dispersion (1) is a standard form in the field of quasi-1D conductors in a magnetic field.[13] In presence of a magnetic field H along the third direction, the semiclassical equations of motion lead to the following trajectory in real space

$$y = b\frac{t}{\omega_c}\cos(Gx), \tag{2}$$

where $G = eHb$ and $\omega_c = Gv$ is the frequency of the electronic motion. When the field is such that $\omega_c \gg t$, the electronic motion becomes 1D, confined along the direction of the chains. In presence of impurities, this 1D regime is defined by $\omega_c \gg t$ and $\omega_c\tau \gg 1$, where τ is the elastic scattering time. This point will be précised below when considering the impurity averaged one particle Green's function. This one dimensionalization should strongly modify the electronic properties of the conductor in presence of impurities. More precisely, we expect a strong magnetic field to induce a transition towards a 1D regime where the wave functions are one dimensional and localized on a length of the order of the mean free path.

In the weak-field limit $\omega_c\tau \ll 1$, the magnetic field can be treated semiclassically and the one dimensionalization can be neglected. However, the question arises whether the results for the isotropic 2D gas can be simply extended by introducing two anisotropic diffusion coefficients (or equivalently two anisotropic masses). A first answer can be given by comparing the elastic scattering time τ and the hopping time (of the order of $1/t$) to the neighboring chain in the clean system. When $\tau \gg 1/t$, the electronic motion in the y direction is coherent from one chain to another and becomes diffusive on a length scale much larger than b. In this case, we expect no essential difference with the isotropic case. On the other hand, when $\tau \ll 1/t$, the electronic motion along the y direction is incoherent (there is no coherent band motion). The hopping of an electron to the neighboring chain is a diffusive process. The broadening $1/\tau$ of the Fermi surface is bigger than the energy dispersion t along k_y, so that the 2D aspect of the Fermi surface is lost. In this case, the physics of localization (in a weak magnetic field) is expected to be different from that of the isotropic gas.

The magnetoresistance of a planar array of edge dislocations described by the dispersion law (1) has been studied by Nakhmedov, Pridogin, and Firsov in the weak-field limit.[14] These authors have calculated the contribution of the maximally crossed diagrams to the conductivity. Starting from a 2D anisotropic diffusion equation for the cooperon in zero field, they have taken into account the effect of the magnetic field by a Peierls substitution for a particle of charge $2e$. We show in this paper that the diffusion equation obtained in this way is correct in the weak-field limit where the semiclassical phase integral (also called eikonal) approximation for the one-particle Green's function is valid. However, this diffusion equation cannot describe the high-field limit where the electronic wave functions become one dimensional. Therefore, in order to describe the effect of a strong magnetic field, it is necessary to use the exact Green's functions as will be done in this paper.

In Sec. II, we determine the one-particle Green's functions in presence of a magnetic field and calculate the self-energy due to the elastic scattering. The lifetime of an electron at the Fermi surface does not depend on the magnetic field. We then calculate the semiclassical (Boltzmann) conductivity. Since we use the exact Green's functions, the expressions for the conductivity are valid for any value of the magnetic field. It is shown that the only field effect is to reduce the transverse (perpendicular to the chains) diffusion coefficient: this is explained by the magnetic-field-induced one dimensionalization. In Sec. III, we calculate the weak-localization correction $\delta\sigma_{xx}(H)$ to the semiclassical conductivity $\sigma_{xx}(H)$ which is given by the maximally crossed diagrams. In the weak-field limit (Sec. III A), we show that the exact Green's functions are correctly described by the eikonal approximation. The expression for $\delta\sigma_{xx}(H)$ is compared with the one obtained in the case of a closed Fermi surface. In the high-field limit (Sec. III B), the pole of the cooperon is restored because of the magnetic-field-induced one dimensionalization. The contribution of the maximally crossed diagrams is calculated. In each case (weak-field and high-field limits), we give a simple physical explanation for the expression of the first quantum correction to the conductivity. In order to determine the nature (localized or delocalized) of the ground state at zero temperature, we construct a scaling procedure based on the calculation of the weak-localization correction (Sec. IV). Our main result is that a strong magnetic field induces a transition from a 2D regime (where the localization lengths are exponentially large compared to the mean free paths) towards a 1D regime where the electronic wave functions are one dimensional with a localization length of the order of the mean free path. In Sec. V, we consider the case of a strongly anisotropic 3D conductor. In this case, it is shown that the magnetic field induces a transition from a 3D metallic regime towards a 2D or 1D insulating regime depending on the direction of the field. In conclusion, we comment on the observability of the effects described in this paper in two physical systems: the quasi-1D organic conductors and the systems of weakly coupled quantum wires.

II. CONDUCTIVITY

The Hamiltonian of the system is obtained from the dispersion law $E(\mathbf{k})$ by the Peierls substitution. Using the Landau gauge $\mathbf{A}(0, Hx, 0)$ the Hamiltonian in absence of disorder is written as

$$\mathcal{H} = v(|-i\partial_x| - k_F) + t\cos(-ib\partial_y - Gx) + \mu. \quad (3)$$

In the chosen gauge, the transverse momentum k_y is still a good quantum number so that the eigenstates can be written as $\phi_{k_y}(\mathbf{r}) = e^{ik_y y}\phi_{k_y}(x)$ where $\phi_{k_y}(x)$ is solution of the equation

$$[v(|-i\partial_x|-k_F)+t\cos(k_y b-Gx)+\mu]\phi_{k_y}(x) = \epsilon\phi_{k_y}(x). \quad (4)$$

We have now to solve a one-dimensional Schrödinger equation with a periodic potential. In first order in t, this periodic potential couples the 1D plane wave state $|k_x\rangle$ with the state $|k_x \pm G\rangle$. Since $t \ll \mu$ and $G \ll k_F$ for any value of the magnetic field, a state $|k_x\rangle$ around the Fermi level is mostly coupled with the states $|k'_x\rangle$ such that $|k_x - k'_x|/k_F \ll 1$. Consequently it is possible to neglect the coupling of the two sheets of the Fermi surface by the magnetic field. The condition $G \ll k_F$ ensures the validity of the Peierls substitution. The operator $|-i\partial_x|$ appearing in the Hamiltonian can be replaced by $-i\alpha\partial_x$ for the sheet $\alpha = +, -$ of the Fermi surface. The eigenstates and the corresponding eigenvalues are then given by

$$\phi_{\mathbf{k},\alpha}(\mathbf{r}) = \frac{1}{\sqrt{S}}e^{i\mathbf{k}\cdot\mathbf{r}+i\alpha\frac{t}{\omega_c}[\sin(k_y b-Gx)-\sin(k_y b)]}, \quad (5)$$

$$\tilde{\epsilon}_{\mathbf{k},\alpha} = v(\alpha k_x - k_F) + \mu. \quad (6)$$

Each eigenstate is labeled by the vector $\mathbf{k} = k_x, k_y$, where k_y is restricted to the first Brillouin zone $]-\pi/b, \pi/b]$ and $\alpha = \mathrm{sgn}(k_x)$. $S = L^2$ is the size of the system. Note that the eigenenergies (6) do not depend on the field. This can be understood by noting that the semiclassical orbits in the magnetic field are open: as a consequence, there is no Landau quantization, the density of states is field independent and it is possible to find another set of eigenstates with field independent eigenvalues [Eqs. (5) and (6)]. Since $\tilde{\epsilon}_{\mathbf{k},\alpha}$ does not depend on k_y, it is possible to define a set of eigenfunctions

$$\chi_{k_x,N,\alpha}(x,y=nb) = \frac{1}{L}\sum_{k_y} e^{-iNk_y b}\phi_{\mathbf{k},\alpha}(\mathbf{r})$$
$$= \frac{1}{b\sqrt{S}} e^{ik_x x - i(N-n)Gx/2 + i(N-n)\pi/2} \times J_{N-n}\left[-2\alpha\frac{t}{\omega_c}\sin\left(\frac{Gx}{2}\right)\right], \quad (7)$$

$$\tilde{\epsilon}_{k_x,N,\alpha} = \tilde{\epsilon}_{\mathbf{k},\alpha}. \quad (8)$$

Here J_n is the nth order Bessel function. The 1D behavior of the system is clearly visible on the states $\chi_{k_x,N,\alpha}$. For infinite magnetic field, the electrons are localized on chains. The crossover between the 2D and 1D regimes is reached when ω_c is of the order of t. We note $\tilde{c}^\dagger_{\mathbf{k},\alpha}$ and $\tilde{c}_{\mathbf{k},\alpha}$ the operators of creation and annihilation of a particle in the state $\phi_{\mathbf{k},\alpha}$. The corresponding Matsubara Green's function is then equal to

$$\tilde{\mathcal{G}}^{(\alpha)}(\mathbf{k},\epsilon_n) = -\int_0^\beta d\tau e^{i\epsilon_n\tau}\langle T_\tau \tilde{c}_{\mathbf{k},\alpha}(\tau)\tilde{c}^\dagger_{\mathbf{k},\alpha}(0)\rangle$$
$$= \frac{1}{i\epsilon_n - \tilde{\xi}_{\mathbf{k},\alpha}}, \quad (9)$$

where $\tilde{\xi}_{\mathbf{k},\alpha} = \tilde{\epsilon}_{\mathbf{k},\alpha} - \mu$ and $\epsilon_n = 2\pi T(n+1/2)$ is a fermion Matsubara frequency.

The effect of disorder is taken into account by adding the term $V_{\text{imp}} = V\sum_I \delta(\mathbf{r}-\mathbf{R}_I)$ in the Hamiltonian (3). We assume that there is no correlation between the positions $\mathbf{R}_I$ of the different impurities. In second quantization, V_{imp} can be written as a function of the operators $\tilde{c}_{\mathbf{k},\alpha}$ and $\tilde{c}^\dagger_{\mathbf{k},\alpha}$

$$V_{\text{imp}} = \frac{V}{S}\sum_{\mathbf{k},\mathbf{q}}\sum_N A_N^{(\alpha,\alpha')}(k_y,k_y+q_y) \times \rho_i(\mathbf{q}-N\mathbf{G})\tilde{c}^\dagger_{\mathbf{k}+\mathbf{q},\alpha'}\tilde{c}_{\mathbf{k},\alpha}. \quad (10)$$

Here $\rho_i(\mathbf{q}) = \sum_I \exp(-i\mathbf{q}\cdot\mathbf{R}_I)$ is the Fourier transform of the density of impurities. The coefficients $A_N^{(\alpha,\alpha')}$ are defined by

$$A_N^{(\alpha,\alpha')}(k_y,k_y+q_y) = \sum_n \gamma_n^{(\alpha)}(k_y)\gamma_{n+N}^{(\alpha')}(k_y+q_y)^*, \quad (11)$$

$$\gamma_n^{(\alpha)}(k_y) = \int_0^{2\pi}\frac{du}{2\pi}e^{inu+i\alpha\frac{t}{\omega_c}[\sin(k_y b-u)-\sin(k_y b)]}$$
$$= e^{ink_y b - i\alpha\frac{t}{\omega_c}\sin(k_y b)}J_n\left(\alpha\frac{t}{\omega_c}\right). \quad (12)$$

The self-energy $\tilde{\Sigma}^{(\alpha)}(\mathbf{k},\epsilon_n)$ associated with the impurity averaged single-particle Green's function is calculated in lowest order in n_i (the impurity density) and V (Born approximation) as shown in Fig. 1. The impurity average is carried out by standard methods.[15] The contributions of the diagrams of Figs. 1(a) and 1(b) are given by

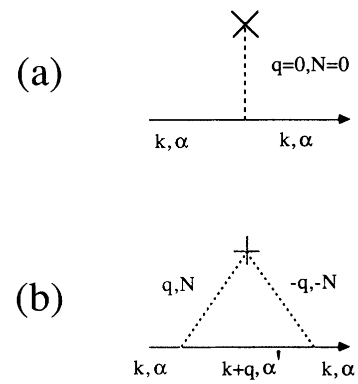

FIG. 1. Diagrams for the self-energy in the Born approximation. Each interaction line with an impurity is labeled by a vector **q** and an integer N [see Eq. (10)].

$$\tilde{\Sigma}_a^{(\alpha)}(\mathbf{k},\epsilon_n) = n_i V A_0^{(\alpha,\alpha)}(k_y,k_y), \quad (13)$$

$$\tilde{\Sigma}_b^{(\alpha)}(\mathbf{k},\epsilon_n) = \frac{n_i V^2}{S}\sum_N\sum_{q_y}|A_N^{(\alpha,\alpha')}(k_y,k_y+q_y)|^2 \times \sum_{q_x}\tilde{\mathcal{G}}^{(\alpha')}(k_x+q_x,\epsilon_n). \quad (14)$$

In the preceding equation, we have used the fact that the Green's function $\tilde{\mathcal{G}}^{(\alpha)}(\mathbf{k},\epsilon_n)$ depends only on k_x. $\tilde{\Sigma}_a^{(\alpha)}(\mathbf{k},\epsilon_n)$ and $\tilde{\Sigma}_b^{(\alpha)}(\mathbf{k},\epsilon_n)$ are calculated using the properties

$$A_0^{(\alpha,\alpha)}(k_y,k_y) = 1, \quad (15)$$

$$\frac{1}{L}\sum_{q_y,N}|A_N^{(\alpha,\alpha')}(k_y,k_y+q_y)|^2 = \frac{1}{b}. \quad (16)$$

$\tilde{\Sigma}_a^{(\alpha)}(\mathbf{k},\epsilon_n)$ is real and can be absorbed in a renormalization of the chemical potential. Performing the sum over q_x as in the isotropic case,[16] we come to the following expression for the self-energy:

$$\tilde{\Sigma}^{(\alpha)}(\mathbf{k},\epsilon_n) = -\frac{i}{2\tau}\text{sgn}(\epsilon_n), \quad (17)$$

$$\frac{1}{\tau} = 2\pi N(0)n_i V^2, \quad (18)$$

where $N(0) = 1/\pi b v$ is the density per spin of the states $\phi_{\mathbf{k},\alpha}$ at the Fermi level. We recover the usual expression for the lifetime of an electron at the Fermi surface. τ is not affected by the magnetic field: this is due to the fact that the density of states does not depend on the field.

In order to calculate the conductivity, we need an expression for the current operator associated with the Hamiltonian (3). To take advantage of the conservation of the transverse momentum in the Landau gauge, we define the fermion operators $\psi^{(\alpha)}(x,k_y)$ and their conjugated operators $\psi^{(\alpha)\dagger}(x,k_y)$ by

$$\psi^{(\alpha)}(x,k_y)=\frac{1}{\sqrt{L}}\int_0^L dy\, e^{-ik_y y}\psi^{(\alpha)}(\mathbf{r}), \tag{19}$$

$$\psi^{(\alpha)}(\mathbf{r})=\sum_{\alpha k_x>0}\sum_{k_y}\phi_{\mathbf{k},\alpha}(\mathbf{r})\tilde{c}_{\mathbf{k},\alpha}. \tag{20}$$

The current operators are obtained from the velocity operators $v_x=i[\mathcal{H},x]$ and $v_y=i[\mathcal{H},y]$. The Fourier transforms with respect to y of the current operators $j_x(\mathbf{r})$ and $j_y(\mathbf{r})$ at point $\mathbf{r}$ are given by (Appendix A)

$$j_x(x,q_y)=ev\sum_{\alpha,k_y}\alpha\psi^{(\alpha)\dagger}(x,k_y-q_y)\psi^{(\alpha)}(x,k_y), \tag{21}$$

$$\begin{aligned}j_y(x,q_y)=-\frac{ebt}{2}\sum_{k_y}\sum_{\alpha,\alpha'}&\left[\sin(k_yb-Gx)\right.\\&\left.+\sin(k_yb-q_yb-Gx)\right]\\&\times\psi^{(\alpha)\dagger}(x,k_y-q_y)\psi^{(\alpha')}(x,k_y).\end{aligned} \tag{22}$$

In Eqs. (21) and (22) and in the following, it is assumed that the size L of the system is normalized to unity. The conductivity at $\mathbf{q}=0$ and $T=0$ is given by the Kubo formula[15]

$$\sigma_{\mu\nu}(\omega,H)=\frac{e^2}{2\pi}\sum_{\alpha,\alpha'}\sum_{\beta,\beta'}\sum_{k_y,k_y'}\int dx\,dx' v_\mu^{(\alpha,\alpha')}(x,k_y)v_\nu^{(\beta,\beta')}(x',k_{y'})\langle G^{R^{(\alpha',\beta')}}_{E_F+\omega}(x,x',k_y,k_y')G^{A^{(\beta,\alpha)}}_{E_F}(x',x,k_y',k_y)\rangle_{\rm imp}, \tag{23}$$

$$v_x^{(\alpha,\alpha')}(x,k_y)=\alpha v\delta_{\alpha,\alpha'}, \tag{24}$$

$$v_y^{(\alpha,\alpha')}(x,k_y)=-bt\sin(k_yb-Gx). \tag{25}$$

Here $G_\epsilon^{R,A^{(\alpha,\alpha')}}(x,x',k_y,k_y')$ are the retarded and advanced Green's functions in the representation (x,k_y) before impurity averaging.

The semiclassical (Boltzmann) conductivity is obtained by replacing the product $\langle G^RG^A\rangle_{\rm imp}$ by the product of the impurity averaged Green's functions. The impurity averaged single-particle Green's functions $\langle G_\epsilon^{R,A^{(\alpha,\alpha')}}(x,x',k_y,k_y')\rangle_{\rm imp}=\delta_{\alpha,\alpha'}\delta_{k_y,k_y'}G_\epsilon^{R,A^{(\alpha)}}(x,x',k_y)$ are directly obtained from the definition of the operators $\psi^{(\alpha)}(x,k_y)$, $\psi^{(\alpha)\dagger}(x,k_y)$ and from the impurity averaged Green's function $\tilde{\mathcal{G}}^{(\alpha)}(\mathbf{k},\epsilon_n)$. The retarded Green's function is written as

$$G_\epsilon^{R^{(\alpha)}}(x,x',k_y)=e^{i\varphi^{(\alpha)}(x,x',k_y)}\tilde{G}_\epsilon^{R^{(\alpha)}}(x-x'), \tag{26}$$

$$\varphi^{(\alpha)}(x,x',k_y)=\alpha\frac{t}{\omega_c}\left[\sin(k_yb-Gx)-\sin(k_yb-Gx')\right]. \tag{27}$$

We have introduced the one-dimensional Green's function $\tilde{G}_\epsilon^{R^{(\alpha)}}(x'-x)$ defined by

$$\begin{aligned}\tilde{G}_\epsilon^{R^{(\alpha)}}(x-x')&=\sum_{k_x}e^{ik_x(x-x')}\tilde{G}_\epsilon^{R^{(\alpha)}}(k_x)\\&=-\frac{i}{v}e^{i\alpha(k_F+\frac{\epsilon}{v}+\frac{i}{2l})(x-x')}\end{aligned} \tag{28}$$

if $\alpha(x-x')>0$, and 0 otherwise. Here $l=v\tau$ is the mean free path. Noting that the product $G^{R^{(\alpha)}}(x,x',k_y)G^{A^{(\alpha)}}(x',x,k_y)$ is equal to $\tilde{G}^{R^{(\alpha)}}(x-x')\tilde{G}^{A^{(\alpha)}}(x'-x)$, $\sigma_{xx}(\omega,H)$ is written as

$$\sigma_{xx}(\omega,H)=\frac{e^2v^2}{2\pi b}\sum_\alpha\int dx\,dx'\tilde{Q}_\omega^{\alpha,\alpha}(x-x'), \tag{29}$$

$$\tilde{Q}_\omega^{\alpha,\beta}(x-x')=\tilde{G}^{R^{(\alpha)}}_{E_F+\omega}(x-x')\tilde{G}^{A^{(\beta)}}_{E_F}(x'-x). \tag{30}$$

The integrals over x and x' yield $\tilde{Q}_\omega^{\alpha,\alpha}(q_x=0)$, where $\tilde{Q}_\omega^{\alpha,\beta}(q_x)$ is the Fourier transform of $\tilde{Q}_\omega^{\alpha,\beta}(x-x')$

$$\tilde{Q}_\omega^{\alpha,\beta}(q_x)=\delta_{\alpha,\beta}\frac{\tau/v}{1-i\omega\tau+i\alpha v\tau q_x}. \tag{31}$$

The dc conductivity σ_{xx} is then independent of H and is given by

$$\sigma_{xx}(H)=2e^2N(0)D_x. \tag{32}$$

The factor 2 comes from spin degeneracy and $D_x=v^2\tau$ is the diffusion coefficient along the x direction in zero field. The conductivity $\sigma_{yy}(\omega)$ is written as

$$\sigma_{yy}(\omega,H)=-\frac{e^2b^2t^2}{8\pi}\sum_{\alpha,\alpha'}\sum_{\beta,\beta'}\sum_{k_y}\int dx\,dx'\beta\beta' e^{ik_yb(\beta+\beta')-iG(\beta x+\beta'x')}\tilde{Q}_\omega^{\alpha,\alpha'}(x-x'). \tag{33}$$

Performing the sum over k_y and the integrals over x and x' leads to

$$\sigma_{yy}(\omega, H) = -\frac{e^2bt^2}{8\pi}\sum_{\alpha,\alpha'}\sum_{\beta}\tilde{Q}_\omega^{\alpha,\alpha'}(\beta G). \tag{34}$$

Eventually, introducing a factor 2 to take into account the spin degeneracy, the dc conductivity $\sigma_{yy}(H)$ can be written as

$$\sigma_{yy}(H) = 2e^2N(0)D_y(H), \tag{35}$$

where we have introduced the magnetic-field-dependent transverse diffusion coefficient $D_y(H) = D_y/(1+\omega_c^2\tau^2)$. $D_y = t^2b^2\tau/2$ is the diffusion coefficient along the y direction in zero field. $\sigma_{xy}(H)$ can be calculated in a similar way and is seen to be zero. There is no Hall effect because the curvature of the dispersion law at the Fermi level is zero. Thus, the effect of the magnetic field on the transport properties can be taken into account by renormalizing the transverse diffusion coefficient. The coefficient $D_y(H)$ defines a magnetic-field-dependent mean free path in the y direction $l_y(H) = l_y/(1+\omega_c^2\tau^2)^{1/2}$, where $l_y = tb\tau/\sqrt{2}$ is the mean free path in zero magnetic field. In the incoherent limit ($t\tau \ll 1$), $l_y \ll b$. The hopping of an electron to the neighboring chain is diffusive for every value of the magnetic field. In the coherent limit ($t\tau \gg 1$), $l_y \gg b$. In this case the localization on the chains of the electrons in high field leads to a decrease of the mean free path $l_y(H)$, which becomes of the order of bt/ω_c when $\omega_c\tau \gg 1$. Therefore, the hopping to the neighboring chain becomes diffusive when $t \sim \omega_c$.

III. WEAK LOCALIZATION

We now consider the first quantum correction to the conductivity. This correction is obtained by summing the maximally crossed diagrams which give rise to a divergent correction to the conductivity (in 2D and 1D) at low temperature in zero field. As pointed out in the Introduction, these diagrams have to be considered in both the low-field limit $\omega_c\tau \ll 1$ and the high-field limit $\omega_c \gg t, 1/\tau$. The maximally crossed diagrams define a propagator P (the cooperon), which becomes a ladder diagram in the particle-particle channel (Fig. 2). In the representation (x, k_y), P is determined by the following integral equation:

$$\begin{aligned} P_\omega(x_1,x_2,q_y) = {} & n_iV^2\delta(x_1-x_2) \\ & + n_iV^2\int dx_3 Q_\omega(x_1,x_3,q_y) \\ & \times P_\omega(x_3,x_2,q_y), \end{aligned} \tag{36}$$

where the kernel

$$\begin{aligned} Q_\omega(x_1,x_3,q_y) = \sum_{\alpha,\alpha'}\sum_{p_y} & G_{E_F+\omega}^{R(\alpha)}(x_1,x_3,p_y) \\ & \times G_{E_F}^{A(\alpha')}(x_1,x_3,q_y-p_y) \end{aligned} \tag{37}$$

is the usual pair propagator. The cooperon is calculated at finite frequency, and the inelastic processes are taken into account by replacing $-i\omega$ by $1/\tau_{\text{in}}$, where τ_{in} is the shortest inelastic relaxation time in the system.[17–21] The temperature is assumed to be low enough so that $\tau \ll \tau_{\text{in}}$. Using expressions (26)–(28) of the Green's functions, the kernel Q is given by

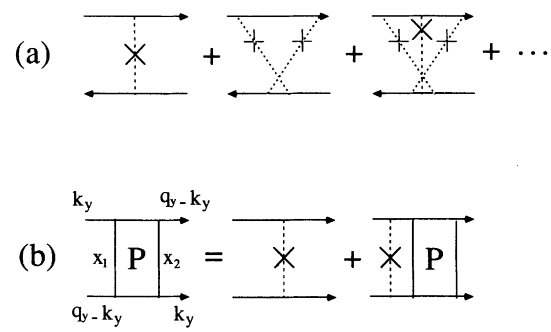

FIG. 2. (a) Maximally crossed diagrams. (b) Dyson equation for the cooperon $P(x_1, x_2, q_y)$ in the particle-particle channel.

$$\begin{aligned} Q_\omega(x_1,x_2,q_y) = {} & \frac{1}{bv^2}e^{-|x_2-x_1|/l}\cos\left(\frac{\omega}{v}(x_2-x_1)\right) \\ & \times J_0\left[\frac{4t}{\omega_c}\sin\left(\frac{G}{2}(x_2-x_1)\right)\right. \\ & \left.\times\sin\left(q_y\frac{b}{2}-\frac{G}{2}(x_1+x_2)\right)\right], \end{aligned} \tag{38}$$

where J_0 is the zeroth-order Bessel function. In order to solve the integral equation (36), it is necessary to solve the eigenvalue problem defined by the kernel Q

$$\int dx_2 Q_\omega(x_1,x_2,q_y)\psi_{\nu,q_y}(x_2) = \lambda_{\nu,q_y}\psi_{\nu,q_y}(x_1). \tag{39}$$

Here, ν is a quantum number which indexes the eigenfunctions ψ_{ν,q_y} and the eigenvalues λ_{ν,q_y}. The propagator is then given by

$$P_\omega(x_1,x_2,q_y) = n_iV^2\sum_\nu\frac{\psi^*_{\nu,q_y}(x_2)\psi_{\nu,q_y}(x_1)}{1-n_iV^2\lambda_{\nu,q_y}}. \tag{40}$$

When the propagator $P_\omega(x_1,x_2,q_y)$ is known, the first quantum correction $\delta\sigma_{xx}$ to σ_{xx} is given by

$$\begin{aligned} \delta\sigma_{xx}(H) = \frac{e^2v^2}{2\pi}\sum_{q_y}\int & dx_1dx_2F(x_1,x_2,q_y) \\ & \times P_{\omega=0}(x_1,x_2,q_y), \end{aligned} \tag{41}$$

$$\begin{aligned} F(x_1,x_2,q_y) = \sum_{\alpha,\alpha'}\alpha\alpha'\sum_{k_y}\int & dx\,dx' G_{E_F}^{R(\alpha)}(x,x_1,k_y) \\ & \times G_{E_F}^{R(\alpha')}(x_2,x',q_y-k_y) \\ & \times G_{E_F}^{A(\alpha')}(x',x_1,q_y-k_y) \\ & \times G_{E_F}^{A(\alpha)}(x_2,x,k_y). \end{aligned} \tag{42}$$

In Eq. (41), $P_{\omega=0}$ is obtained from P_ω by replacing $-i\omega$ by $1/\tau_{\rm in}$. The function $F(x_1,x_2,q_y)$ is calculated in Appendix B and turns out to be proportional to the kernel $Q_{\omega=0}(x_1,x_2,q_y)$

$$F(x_1,x_2,q_y) = 2\tau^2 Q_{\omega=0}(x_1,x_2,q_y). \tag{43}$$

Using the preceding result, the integration over x_1 and x_2 in Eq. (41) can be done. $\delta\sigma_{xx}(H)$ is given by

$$\delta\sigma_{xx}(H) = -\frac{e^2v^2}{2\pi} lb \sum_{\nu,q_y} \frac{\lambda_{\nu,q_y}}{1-n_iV^2\lambda_{\nu,q_y}}. \tag{44}$$

A very interesting feature is that we only need to know the eigenvalues λ_{ν,q_y} in order to calculate $\delta\sigma_{xx}(H)$. A similar expression has been obtained by Kawabata for the 2D isotropic electron gas.[22]

In order to explain qualitatively the effect of the magnetic field on the electronic motion, we start considering the Green's function in real space

$$\begin{aligned} G_\epsilon^{R(\alpha)}(x,x',y=nb) &= \frac{1}{b} e^{in\frac{G}{2}(x+x')-in\frac{\pi}{2}} \\ &\times J_n\left[-2\alpha\frac{t}{\omega_c}\sin\left(\frac{G}{2}(x'-x)\right)\right] \\ &\times \tilde{G}_\epsilon^{R(\alpha)}(x-x'), \end{aligned} \tag{45}$$

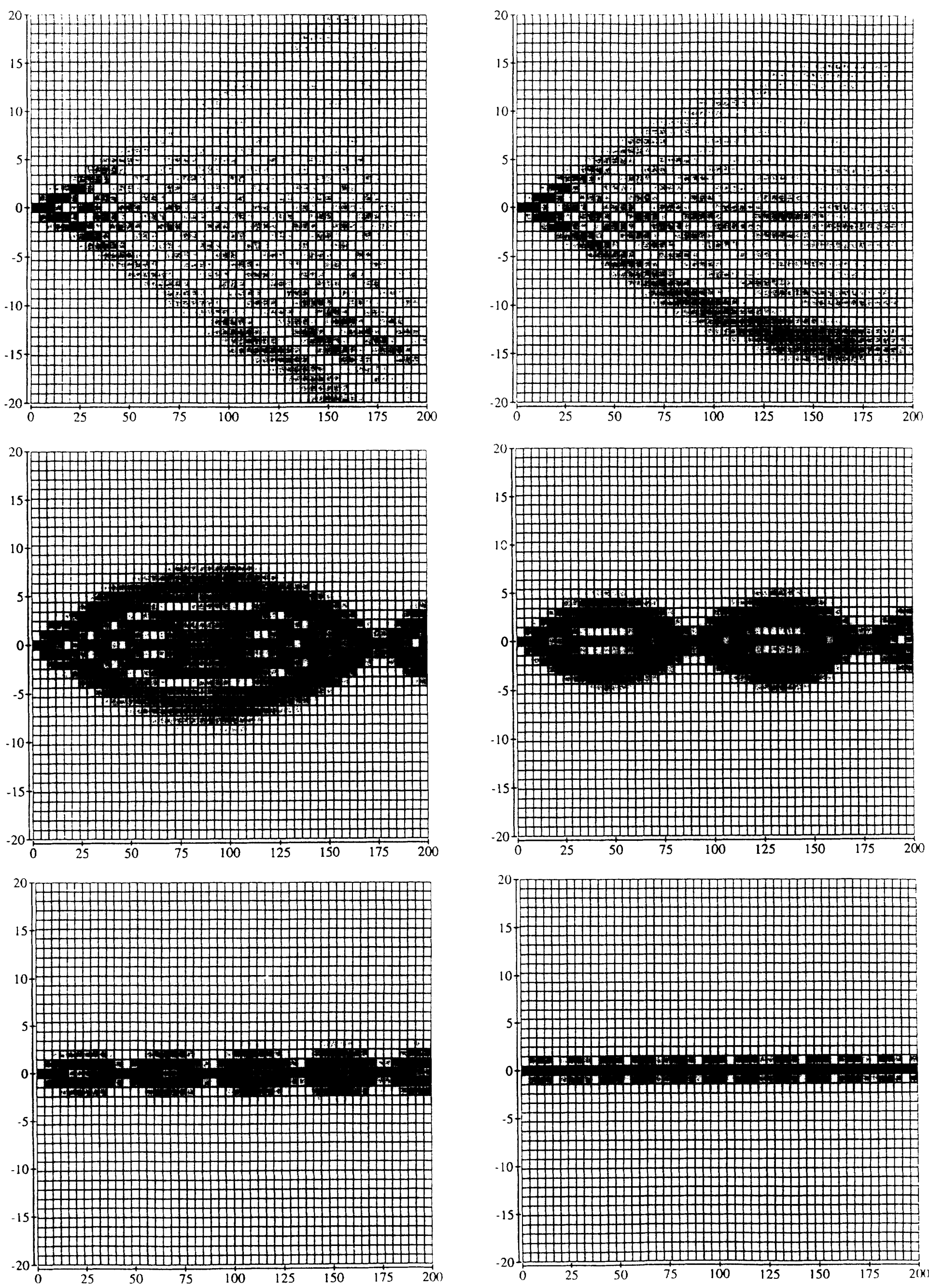

FIG. 3. Amplitude of the Green's function in real space. Each horizontal line corresponds to a given chain. The magnetic field is defined by $\omega_c/t = 0$, 0.125, 0.25, 0.5, 1, and 2.5. For $\omega_c \gg t$, the electronic motion is one dimensional.

where $\tilde{G}_\epsilon^{R(\alpha)}$ is given by (28) and does not depend on the magnetic field. $G_\epsilon^{R(\alpha)}(x,x',y=nb)$ describes the propagation of an electron from the point $(x',0)$ to the point $(x,y=nb)$. The field has two different effects. On the one hand, it adds a phase factor to the Green's function. It can be verified that this phase factor is equal to

$$n\frac{G}{2}(x+x') = e\int_{(x',0)}^{(x,y)} d\mathbf{s}\cdot\mathbf{A}(\mathbf{s}), \tag{46}$$

where the path of integration is a straight line between the two points $(x',0)$ and $(x,y=nb)$. This phase factor is responsible for the breakdown of time-reversal symmetry. On the other hand, the field modifies the argument of the Bessel function J_n. Clearly, this latter effect is related to the one dimensionalization induced by the magnetic field. For infinite magnetic field, the argument of the Bessel function vanishes and the Green's function is equal to zero unless $n=0$: the electronic motion is one dimensional. The phase factor (46) vanishes and time-reversal symmetry is restored. Since the Green's function $\tilde{G}_\epsilon^{R(\alpha)}(x-x')$ introduces a cutoff $|x-x'|\sim l$, in the limit $\omega_c\tau = Gl \ll 1$ it is possible to neglect the second effect. Consequently, the field effect reduces to the addition of the phase factor (46) in the Green's function. Thus, we recover the semiclassical phase integral (also called eikonal) approximation.[23] Note that in the limit $\omega_c\tau = Gl \ll 1$, it is always possible to neglect the field dependence of the argument of the Bessel function, even if $\omega_c \gg t$. Therefore, the crossover to the 1D regime is defined by the two conditions $\omega_c\tau \gg 1$ and $\omega_c \gg t$. In the semiclassical picture described at the beginning of this paper, these conditions can be understood as follows. When the condition $\omega_c \gg t$ is fullfilled, the semiclassical orbits become 1D. However, for the electron to be sensible to the 1D aspect of the orbits, it has to cover more than one spatial period between two elastic diffusions. This latter condition leads to $\omega_c\tau \gg 1$. Figure 3 shows the amplitude of the Green's function for different values of the magnetic field. In high field, the electronic motion is localized on a single chain.

A. Low-field limit

We first consider the low field limit $\omega_c\tau \ll 1$, where the eikonal approximation is valid. The kernel Q is then given by

$$Q_\omega^{(eik)}(x_1,x_2,q_y) = \frac{1}{bv^2}e^{-|x_2-x_1|/l}\cos\!\left(\frac{\omega}{v}(x_2-x_1)\right)J_0\left[\frac{2t}{v}(x_2-x_1)\sin\!\left(q_y\frac{b}{2}-\frac{G}{2}(x_1+x_2)\right)\right]. \tag{47}$$

The integral equation (39) can be rewritten as

$$\frac{1}{b}\int dx_2 J_0\left[\frac{2t}{v}x_2\sin\!\left(q_y\frac{b}{2}-Gx-\frac{G}{2}x_2\right)\right] \times\sum_{\alpha,q_x} e^{iq_xx_2}\tilde{Q}_\omega^{\alpha,-\alpha}(q_x)\psi(x+x_2) = \lambda\psi(x), \tag{48}$$

where x_2 is now the distance over which the particle-particle pair propagates. We have introduced the 1D part of the kernel and its Fourier transform

$$\tilde{Q}_\omega^{\alpha,\alpha'}(x) = \tilde{G}_{E_F+\omega}^{R(\alpha)}(x)\tilde{G}_{E_F}^{A(\alpha')}(x), \tag{49}$$

$$\tilde{Q}_\omega^{\alpha,\alpha'}(q_x) = \delta_{\alpha,-\alpha'}\frac{\tau/v}{1-i\omega\tau+i\alpha v\tau q_x}. \tag{50}$$

As can be seen from Eq. (44), the main contribution to $\delta\sigma_{xx}(H)$ comes from the eigenvalues λ_{ν,q_y} which verify $1-n_iV^2\lambda_{\nu,q_y} \ll 1$. Thus, it is sufficient to take into account the contribution of the states ψ_{ν,q_y}, $\nu<\nu_0$, where the cut off ν_0 is defined by $1-n_iV^2\lambda_{\nu_0,q_y}\sim 1$. As will be verified latter, for weak enough magnetic field $\omega_c t\tau^2 \ll 1$, the two terms $\psi(x+x_2)$ and J_0 appearing in Eq. (48) are slowly varying for $|x_2|\le l$. The slow variation of J_0 when $t\tau \gg 1$ is due to the fact that $2t\tau\sin(q_yb/2-Gx)\ll 1$ in the region where $\psi_{\nu,q_y}(x)$ $(\nu<\nu_0)$ has a nonzero value. Expanding $\psi(x+x_2)$ and J_0 in second order in x_2 leads to the equation

$$\frac{1}{b}\sum_\alpha\left[\tilde{Q}_\omega^{\alpha,-\alpha}(0)+\frac{t^2}{v^2}\sin^2\!\left(\frac{q_yb}{2}-Gx\right)\frac{\partial^2\tilde{Q}_\omega^{\alpha,-\alpha}}{\partial q_x^2}(0) +\frac{\partial\tilde{Q}_\omega^{\alpha,-\alpha}}{\partial q_x}(0)i\partial_x-\frac{1}{2}\frac{\partial^2\tilde{Q}_\omega^{\alpha,-\alpha}}{\partial q_x^2}(0)\partial_x^2\right]\psi(x)=\lambda\psi(x). \tag{51}$$

Using Eq. (50), we come eventually to the following diffusion equation for the particle-particle pair:

$$-v^2\frac{\partial^2\psi}{\partial x^2}(x)+t^2[1-\cos(q_yb-2Gx)]\psi(x) = \frac{1}{\tau^2}(1-n_iV^2\lambda+i\omega\tau)\psi(x). \tag{52}$$

The preceding equation was used as starting point by Nakhmedov, Prigodin, and Firsov.[14] We have shown that this equation is correct for weak magnetic field $\omega_c t\tau^2 \ll 1$. As in the isotropic case,[4] the effect of the magnetic field is to replace the operator $-i\boldsymbol{\nabla}$ by $-i\boldsymbol{\nabla}-2e\mathbf{A}$, where $2e$ is the charge of the particle-particle pair. Equation (52) is the well known Mathieu equation. The spectrum $1-n_iV^2\lambda_{\nu,q_y}+i\omega\tau$ is well approximated by a continuous spectrum above the sinusoidal barrier and a discrete spectrum below the barrier. The eigenvalues and eigenstates are then approximated by (replacing $-i\omega$ by $1/\tau_{\rm in}$)

$$1 - n_i V^2 \lambda_{q_x,q_y} = D_x \tau q_x^2 + t^2\tau^2 + \tau/\tau_{\rm in} \tag{53}$$

$$\psi_{q_x,q_y}(x) = e^{iq_x x} \tag{54}$$

above the barrier, and

$$1 - n_i V^2 \lambda_{n,m,q_y} = 4eD_{\rm eff} H\tau(m+\tfrac{1}{2}) + \tau/\tau_{\rm in}, \tag{55}$$

$$\psi_{n,m,q_y}(x) = f_m(x - x_n) \tag{56}$$

below the barrier. Here $x_n = q_y b/2G - n\pi/G$ and $f_m(x)$ is the mth level of the harmonic oscillator with "frequency" $2\sqrt{2}\omega_c t$ and "mass" $1/2v^2$. We have introduced the effective diffusion coefficient $D_{\rm eff} = (D_x D_y)^{1/2}$. According to the relative values of $1/t$, τ, and $\tau_{\rm in}$, we distinguish the three different limits: (a) coherent limit, $1/t \ll \tau$; (b) incoherent limit (low temperature), $\tau \ll 1/t$ and $\tau \ll \Delta t_0 \ll \tau_{\rm in}$; (c) incoherent limit (high temperature), $\tau \ll 1/t$ and $\tau \ll \tau_{\rm in} \ll \Delta t_0$. In the coherent limit, the electronic motion from one chain to the neighboring chain is ballistic (the mean free path l_y is much larger than the spacing between chains b). In the incoherent limits (b) and (c), the electron diffuses to the neighboring chain in a time $\Delta t_0 = 1/2t^2\tau \sim b^2/D_y$. The spectra corresponding to these three different limits are shown in Fig. 4. In the coherent limit, all the levels of the spectrum which have to be considered ($1 - n_i V^2 \lambda_{\nu,q_y} < 1$) are discrete [Fig. 4(a)]. In the incoherent limit, both the discrete part and the continuous part of the spectrum contribute to $\delta\sigma_{xx}(H)$ [Figs. 4(b) and 4(c)]. However, in the high-temperature limit (c), the discrete part of the spectrum is cut off by the inelastic processes and does not contribute to $\delta\sigma_{xx}(H)$.

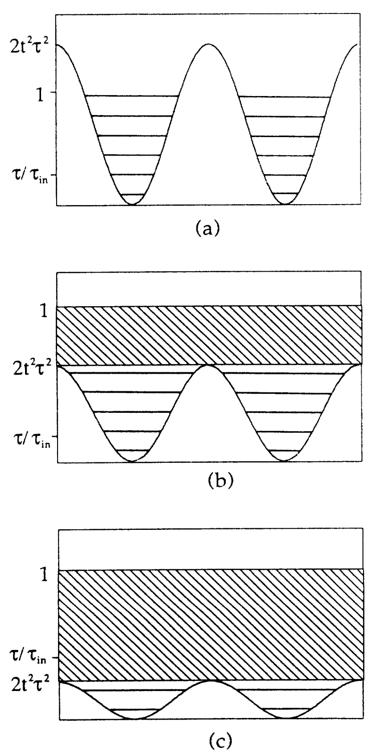

FIG. 4. Schematic spectrum $1 - n_i V^2 \lambda_{\nu,q_y} + i\omega\tau$ at $\omega = 0$. The shaded regions correspond to the continuous part of the spectrum. (a) Coherent limit: $t\tau \gg 1$. (b) Incoherent limit (low temperature): $t\tau \ll 1$ and $2t^2\tau^2 \gg \tau/\tau_{\rm in}$. (c) Incoherent limit (high temperature): $t\tau \ll 1$ and $2t^2\tau^2 \ll \tau/\tau_{\rm in}$.

We consider first the coherent limit (a). In this case, the sum over the discrete levels has to be cut off at $m_0 \sim 1/4eD_{\rm eff}H\tau$. When $\omega_c t\tau^2 \ll 1$, $m_0 \gg 1$ and the state $\psi_{n,m_0,q_y}(x)$ centered at point x_n has a width of the order of $\langle \psi_{n,m_0,q_y} | (x-x_n)^2 | \psi_{n,m_0,q_y} \rangle^{1/2} \sim 1/Gt\tau \ll 1/G$. For all the states ψ_{n,m,q_y}, $m < m_0$, we then have $t\tau \sin(q_y b/2 - Gx) < 1$. Moreover, it can be easily verified [also in limits (b) and (c)] that the states $\psi_{\nu,q_y}(x)$, $\nu < \nu_0$, are slowly varying on a length scale equal to the mean free path l. Therefore, the assumptions leading to Eq. (52) are justified when $\omega_c t\tau^2 \ll 1$.[24] Since $m_0 \gg 1$, the pole of the cooperon is not completely suppressed and the eigenvalues λ_{ν,q_y} appearing in the numerator of (44) can be replaced by $1/n_i V^2$. Taking into account the degeneracy G/π of the level λ_{n,m,q_y}, the correction to the conductivity is given by (introducing a factor 2 for spin degeneracy)

$$\delta\sigma_{xx}^{(a)}(H) = -\frac{e^2}{2\pi^2}\left(\frac{D_x}{D_y}\right)^{\frac{1}{2}} \left[\Psi\left(\frac{1}{2} + \frac{\Delta t_H}{\tau}\right) - \Psi\left(\frac{1}{2} + \frac{\Delta t_H}{\tau_{\rm in}}\right)\right], \tag{57}$$

where Ψ is the digamma function. We have introduced the time $\Delta t_H = 1/4eD_{\rm eff}H$. Equation (57) is analogous to the one obtained for a gas with anisotropic masses.[25,21] Therefore, in the limit $t\tau \gg 1$, the fact that the Fermi surface is open does not play an essential role. The system behaves like a gas with an elliptic Fermi surface.

In the incoherent limits (b) and (c), the cutoff m_0 is determined by $m_0 \sim \Delta t_H/\Delta t_0$. The levels λ_{q_x,q_y} which contribute to $\delta\sigma_{xx}(H)$ are such that $t/v < |q_x| < q_0$, where $q_0 \sim 1/l$. As in the preceding case, the eigenvalues λ_{ν,q_y} appearing in the numerator of (44) are replaced by $1/n_i V^2$. In the incoherent limit (b), the correction to the conductivity is given by

$$\delta\sigma_{xx}^{(b)}(H) = -\frac{e^2}{2\pi^2}\left(\frac{D_x}{D_y}\right)^{\frac{1}{2}} \left[\Psi\left(\frac{1}{2} + \frac{\Delta t_H}{\Delta t_0}\right) - \Psi\left(\frac{1}{2} + \frac{\Delta t_H}{\tau_{\rm in}}\right)\right] - \frac{e^2}{\pi b\sqrt{2}}\sqrt{D_x}(\sqrt{\Delta t_0} - \sqrt{\tau}). \tag{58}$$

In the incoherent limit (c), the contribution of the discrete levels vanishes. $\delta\sigma_{xx}^{(c)}(H)$ does not depend on the magnetic field and is given by the 1D result

$$\delta\sigma_{xx}^{(c)}(H) = \delta\sigma^{(1D)} = -\frac{e^2}{\pi b}\sqrt{D_x}(\sqrt{\tau_{\rm in}} - \sqrt{\tau}). \tag{59}$$

The preceding expressions for $\delta\sigma_{xx}(H)$ agree with those of Nakhmedov, Prigodin, and Firsov in the limits (a) and (b), but differ in the limit (c).[14] For each of the limits (a) and (b), it is possible to define a characteristic field H_0 by $\Delta t_{H_0} \sim \tau_{\text{in}}$.

We now adopt the physical interpretation of the weak localization as introduced originally by Altshuler, Aronov, and Bergmann.[21,20] The propagation of an electron from $\mathbf{r}$ to $\mathbf{r}'$ can be described by a Feynmann path integral over all paths connecting $\mathbf{r}$ to $\mathbf{r}'$. Each closed path $\mathbf{r} = \mathbf{r}'$ can be covered clockwise (with amplitude A_1) or counterclockwise (with amplitude A_2). When time-reversal symmetry holds, the two amplitudes A_1 and A_2 are coherent and lead to constructive interferences. Therefore, the probability for an electron to return to its starting point during its diffusive motion is enhanced. The divergence of the maximally crossed diagrams originates in these interferences. In presence of a magnetic field, the constructive interferences are destroyed when the magnetic flux across the area of the closed path is of the order of the flux quantum ϕ_0.

Consider first the coherent limit (a). The paths leading to the weak localization in zero field correspond to a propagation time Δt between τ and τ_{in}. Since $1/t \ll \tau$ ($1/t$ is of the order of the hopping time to the neighboring chain in the clean system), all these paths are two dimensional. A given closed path of area $S \sim D_{\text{eff}}\Delta t$ will contribute to the weak localization if $SH < \phi_0$. This condition can be rewritten as $\Delta t < \Delta t_H$. Therefore, the closed paths which contribute to the weak localization in presence of the magnetic field correspond to a propagation time Δt between τ and Δt_H.

Consider now the incoherent limits (b) and (c). Since $\tau \ll 1/t$, the electronic motion to the neighboring chain is diffusive. The diffusive time is given by $b^2/D_y \sim \Delta t_0$. In the limit (b), the paths are 1D if $\tau < \Delta t < \Delta t_0$ and 2D if $\Delta t_0 < \Delta t < \tau_{\text{in}}$. The one-dimensional paths are not affected by the magnetic field and therefore contribute to the weak localization for every value of the magnetic field. On the other hand, the two-dimensional paths which contribute to the weak localization correspond to a propagation time $\Delta t_0 < \Delta t < \Delta t_H$. Clearly, the 2D and 1D paths are associated with the discrete and the continuous parts of the spectrum $1-n_i V^2 \lambda_{\nu,q_y}$. In both the limits (a) and (b), we have $\delta\sigma_{xx}(H) - \delta\sigma_{xx}(0) \sim e^2 \ln(eHD_{\text{eff}}\tau_{\text{in}})$ for $\Delta t_H \leq \tau_{\text{in}}$.

In the incoherent limit (c), the diffusion time Δt_0 to the neighboring chain is larger than the inelastic scattering time τ_{in}. Therefore, all the paths are one dimensional and are not affected by the magnetic field. As a result, the correction to the conductivity does not depend on the magnetic field.

B. High-field limit

We now consider the other limit of interest: the high-field regime $\omega_c \gg t, 1/\tau$. In this limit, the magnetic field localizes the electrons on the chains of highest conductivity. The term in the Hamiltonian (3) which breaks time-reversal symmetry oscillates too fast and can be ignored. The gas becomes 1D and time-reversal symmetry is restored. As a result, the maximally crossed diagrams diverge at low temperature and lead to a strong correction to the conductivity.

In the limit $\omega_c \gg t, 1/\tau$, the eikonal approximation does not hold any more and we have to consider the exact kernel Q defined by (38). Noting that $\psi(x+x_2) = e^{x_2\partial_x}\psi(x)$, the integral equation for the functions $\psi(x)$ can be written as

$$\int dx_2 Q_\omega(x,x_2,q_y)\psi(x) = \lambda\psi(x) \tag{60}$$

with

$$Q_\omega(x,x_2,q_y) = \frac{1}{b} J_0\left[\frac{4t}{\omega_c}\sin\left(\frac{G}{2}x_2\right)\sin\left(q_y\frac{b}{2} - \frac{G}{2}(2x+x_2)\right)\right]\sum_{q_x,\alpha} e^{iq_x x_2}\tilde{Q}_\omega^{\alpha,-\alpha}(q_x)e^{x_2\partial_x}. \tag{61}$$

In the limit $\omega_c \gg t$, the argument of the Bessel function in Eq. (61) is small. The kernel of the integral equation can be written as

$$Q_\omega(x,x_2,q_y) = Q_\omega^\infty(x,x_2,q_y) + \delta Q_\omega(x,x_2,q_y) + O\left(\frac{t^4}{\omega_c^4}\right), \tag{62}$$

$$\begin{aligned}\delta Q_\omega(x,x_2,q_y) = -\frac{t^2}{b\omega_c^2}\{&1-\cos(Gx_2) - \cos[q_y b - G(2x+x_2)] \\ &+ \cos(Gx_2)\cos[q_y b - G(2x+x_2)]\}\sum_{q_x,\alpha} e^{iq_x x_2}\tilde{Q}_\omega^{\alpha,-\alpha}(q_x)e^{x_2\partial_x}.\end{aligned} \tag{63}$$

Here $Q_\omega^\infty(x,x_2,q_y)$ is the kernel for infinite magnetic field. It has the usual 1D expression. The corresponding eigenvalues and eigenfunctions are given by

$$1 - n_i V^2 \lambda^\infty_{q_x,q_y} = -i\omega\tau + D_x\tau q_x^2, \tag{64}$$

$$\psi^\infty_{q_x,q_y}(x) = e^{iq_x x}. \tag{65}$$

In first order in $\delta Q_\omega(x,x_2,q_y)$, the eigenvalues are given by

$$\lambda_{q_x,q_y} = \lambda^\infty_{q_x,q_y} + \left\langle \psi^\infty_{q_x,q_y} \middle| \int dx_2 \delta Q_\omega(x,x_2,q_y) \middle| \psi^\infty_{q_x,q_y} \right\rangle. \tag{66}$$

Retaining only the first correction in t/ω_c, the eigenvalues λ_{q_x,q_y} are given by (Appendix C)

$$1 - n_i V^2 \lambda_{q_x,q_y} = -i\omega\tau + D_x \tau q_x^2 + \frac{t^2}{\omega_c^2} \tag{67}$$

in the limits (a) and (b) and (c) ($\omega_c\tau \gg 1$), and

$$1 - n_i V^2 \lambda_{q_x,q_y} = -i\omega\tau + D_x \tau q_x^2 + t^2\tau^2 \tag{68}$$

in the limits (b) and (c) ($\omega_c\tau \ll 1$). In this latter case, we recover the continuous spectrum derived in the eikonal approximation. Since in the limit $t \ll \omega_c \ll 1/\tau$, the discrete spectrum (55) does not contribute to the weak localization, the two approaches (eikonal approximation and limit $\omega_c \gg t$) are in agreement. In the limit $\omega_c \gg t, 1/\tau$, the conductivity is given by

$$\delta\sigma_{xx}(H) = \delta\sigma^{(1D)} = -\frac{e^2}{\pi b}(\sqrt{D_x \tau_{in}} - \sqrt{D_x \tau}) \tag{69}$$

if $\tau_{in} \ll \Delta t_0(H)$, and

$$\delta\sigma_{xx}(H) = -\frac{e^2}{\pi b}[\sqrt{D_x \Delta t_0(H)} - \sqrt{D_x \tau}] \tag{70}$$

if $\Delta t_0(H) \ll \tau_{in}$. We have introduced the time $\Delta t_0(H) = \tau\omega_c^2/t^2$. Since Eqs. (69) and (70) are valid only in the limit $\omega_c\tau \gg 1$, $\delta\sigma_{xx}(H) \approx \delta\sigma^{(1D)} = \delta\sigma_{xx}^{(c)}$ in the incoherent limit (c), for every value $\omega_c \gg t, 1/\tau$ of the magnetic field. This result agrees with the preceding considerations from which it follows that the correction to the conductivity does not depend on the magnetic field. Indeed, in this limit, all the closed paths leading to the weak localization are 1D and are not affected by the magnetic field. As in the low-field case, the expression of $\delta\sigma_{xx}(H)$ can be interpreted with very simple arguments. In the limit $\omega_c \gg t, 1/\tau$, the hopping to the neighboring chain is diffusive [in both the limits (a) and (b)] with a diffusion coefficient $D_y(H) \sim b^2 t^2/\tau\omega_c^2$. Therefore, it takes a time of the order of $\Delta t_0(H)$ for an electron to diffuse to the neighboring chain. In this high-field limit, the weak localization is due to the 1D closed paths. These paths have a propagation time $\tau < \Delta t < \Delta t_0(H)$. As in the weak-field limit, we introduce a characteristic field H_1 defined by $\Delta t_0(H_1) \sim \tau_{in}$. For $H > H_1$, all the paths such that $\Delta t < \tau_{in}$ are 1D: $\delta\sigma_{xx} = \delta\sigma^{(1D)}$.

We have also calculated numerically the spectrum of the integral operator Q in the limit $t\tau \gg 1$. Figure 5 shows the eigenvalues $1 - n_i V^2 \lambda_{\nu,q_y}$ for $q_y = 0$ and $\omega = 0$

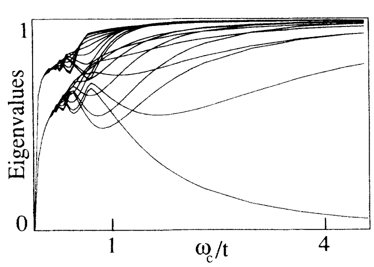

FIG. 5. Numerical calculation of the eigenvalues $1 - n_i V^2 \lambda_{\nu,q_y}$ versus magnetic field in the limit $t\tau \gg 1$.

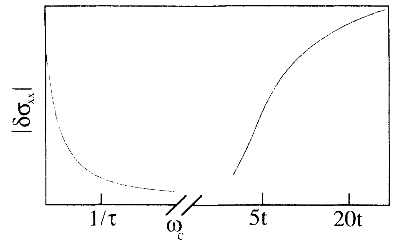

FIG. 6. Absolute value of the first quantum correction to the conductivity $|\delta\sigma_{xx}(H)|$ versus magnetic field in the limit $t\tau \gg 1$ ($L_{in} = 7.5l$). The critical field H_1 corresponds to $\omega_c \sim 7.5t$.

versus magnetic field. Writting the integral equation as $\hat{O}\psi_\nu = \lambda_\nu \psi_\nu$, where $\hat{O}$ is a differential operator of infinite order, it is easily seen that the operator $\hat{O}$ commutes with the translation operator of π/G. Consequently, the eigenstates can be chosen as Bloch functions $\psi_{\nu,q_y} = \psi_{n,q_x,q_y}$ where n is a band index and q_x is a vector between $-G$ and G. Figure 5 shows the two first bands (n=1 and n=2) for different values of q_x (note that since we have chosen values for q_x uniformely distributed between $-G$ and G, the value of each q_x increases with the magnetic field). For very weak field, each band is dispersionless and the Landau regime $1 - n_i V^2 \lambda_{n,q_x,q_y} = 4eD_{\text{eff}} H\tau(n + 1/2)$ is clearly visible. When the field is increased, the pole of the cooperon is suppressed and the degeneracy of each band is lifted. In the high-field regime, the pole is restored. The eigenvalues of the lowest band are distributed between 0 and 1 and we recover the continuous 1D spectrum. The eigenvalues of the higher bands tend to 1. Figure 6 shows $|\delta\sigma_{xx}(H)|$ versus magnetic field. In the low-field regime, the conductivity is given by the analytical result (57). In the high-field regime, the conductivity has been obtained from the eigenvalues of the kernel (38) shown in Fig. 5.

It should also be noted that both effects (negative magnetoresistance in low field and positive magnetoresistance in high field) bear similarities with the disappearance and the reentrance of the superconducting phase in strongly anisotropic 3D superconductors in a magnetic field.[26,27] In each case, the effect of the field can be understood as the suppression and the restoration of the Cooper pole for increasing magnetic field.

IV. SCALING

Up to now, we have calculated the first quantum correction to the conductivity, without considering the validity of this pertubative calculation. In the weakly localized regime, where the pertubative approach is valid, the weak-localization correction gives the temperature (via τ_{in}) and magnetic-field dependence of the conductivity. In order to obtain more information on the ground state of the system, we apply a scaling procedure following the approach of Apel and Rice.[28] We first note that the quantum correction at zero temperature for a system of finite size $L_x, L_y = [D_y(H)/D_x]^{1/2} L_x$ is obtained from the preceding calculation by replacing $L_{\rm in} = (D_x\tau_{\rm in})^{1/2}$ by L_x. We consider a system of microscopic size $l, l_y(H)$. From the form of $\delta\sigma_{xx}(H)$, it is possible to deduce the behavior of the system at length scales $l, l_y(H)$. We then increase the size of the system. Using the known results in 2D and 1D, we can deduce the ground state of a system of macroscopic size.

We first consider the case where $t\tau \ll 1$ in zero magnetic field. We start the scaling procedure with a system of size l, l_y. According to Eq. (59), the system has a 1D behavior as long as L_x is smaller than $L_0 = (D_x\Delta t_0)^{1/2}$. L_0 is the length along which the electron has to diffuse on a given chain before it diffuses to the neighboring chain. Therefore, for $L_x \ll L_0$, the chains can be considered as decoupled. When increasing the length of the chains, the dimensionless conductance g_x will evoluate according to the 1D scaling function $\beta^{\rm 1D}(g_x) = d\ln(g_x)/d\ln(L_x)$. Since $t\tau \ll 1$ implies $l \ll L_0$, the system will reach a 1D insulating state for $L_x \sim \xi^{\rm 1D} \sim l$, where $\xi^{\rm 1D}$ is the localization length for a 1D system. This result agrees with the one of Firsov and Prigodin,[29] who have calculated the localization lengths in a strongly anisotropic conductor, using the self consistent diagrammatic treatment originally introduced by Vollhard and Wölfe.[30] Consequently, the magnetic field will have no effect in the limit $t\tau \ll 1$.

Consider now the case $t\tau \gg 1$ in zero field. From Eq. (57), it is clear that a system of microscopic size l, l_y will have a 2D behavior. Since $\delta\sigma_{xx}/\sigma_{xx} = \delta\sigma_{yy}/\sigma_{yy}$,[29] it is possible to use a one parameter scaling procedure if we keep the ratio L_x/L_y constant. When increasing its size, the system will evoluate according to the 2D scaling function and will reach a 2D insulating state for $L_x/l = L_y/l_y \sim \xi_x^{2D}/l = \xi_y^{2D}/l_y$. The 2D localization lengths can be estimated by $\delta\sigma_{xx}(\xi_x^{2D}) = \sigma_{xx}$ and $\delta\sigma_{yy}(\xi_y^{2D}) = \sigma_{yy}$ where σ_{xx} and σ_{yy} are the Boltzmann conductivities. We then obtain $\xi_x^{2D}/l = \xi_y^{2D}/l_y \sim \exp(\alpha t\tau)$, where α is of the order of unity. This is in agreement with the result of Firsov and Prigodin.[29] As for a 2D gas, we expect that a small magnetic field $\omega_c \ll \tau^{-1}$ (which can be treated in the eikonal approximation) will not destroy the localization but will increase the localization lengths. When $\omega_c \gg \tau^{-1}$, the transverse diffusion coefficient $D_y(H)$ becomes magnetic-field dependent and the eikonal approximation breaks down. The effects associated with the one dimensionalization have to be taken into account. As long as $\tau^{-1} \ll \omega_c \ll t$, $\delta\sigma_{xx}(L_x)$ vanishes, so that it seems difficult to obtain some information from our calculation. However, since the mean free path in the transverse direction is still well defined, $l_y(H) \gg b$, we expect the gas to have a 2D (anisotropic) behavior. In the limit where $\omega_c \gg t$, Eq. (69) shows that the system has a 1D behavior when $L_x \ll L_0(H) = [D_x\Delta t_0(H)]^{1/2}$. $L_0(H)$ has the same physical meaning as the length L_0 previously introduced. For $L_x \ll L_0(H)$, the chains can be considered as decoupled. When increasing the size of the chains from l, the system will reach an insulating 1D state for $L_x \sim \xi^{\rm 1D} \ll L_0(H)$. Therefore, the magnetic field will induce a transition from a 2D regime towards a 1D regime, the crossover field being defined by $\omega_c \sim t$. It should be noted that the 1D regime is reached as soon as the pole of the cooperon is restored. This is related to the absence of diffusive regime ($\xi^{\rm 1D} \sim l$) in 1D. As will be shown in the next section, the 3D case can differ considerably.

V. 3D ANISOTROPIC CONDUCTOR

It is also possible to extend the preceding calculations to the case of a 3D anisotropic conductor described by the dispersion law

$$E(\mathbf{k}) = v(|k_x| - k_F) + t_y\cos(k_yb) + t_z\cos(k_zc) + \mu, \tag{71}$$

where t_y and t_z are the hopping rates in the y and z directions. A strong magnetic field $\mathbf{H}(0, H_y, H_z)$ will localize the electrons in the (x, y) planes ($H_z = 0$) or on the chains ($H_y \neq 0$ and $H_z \neq 0$). In the gauge $\mathbf{A}(0, H_zx, -H_yx)$, the Hamiltonian is given by

$$\begin{aligned}\mathcal{H} = {}& v(|-i\partial_x| - k_F) + t_y\cos(-ib\partial_y - G_yx)\\ &+t_z\cos(-ic\partial_z - G_zx) + \mu,\end{aligned} \tag{72}$$

where $G_y = eH_zb$ and $G_z = -eH_yc$. The Green's functions in the representation (x, k_y, k_z) are given by Eq. (26), where the phase $\varphi^{(\alpha)}(x, x', k_y)$ has to be replaced by

$$\begin{aligned}\varphi^{(\alpha)}(x, x', k_y, k_z) = {}& \alpha\frac{t_y}{\omega_{cy}}\big[\sin(k_yb - G_yx)\\ &-\sin(k_yb - G_yx')\big]\\ &+\alpha\frac{t_z}{\omega_{cz}}\big[\sin(k_zb - G_zx)\\ &-\sin(k_zb - G_zx')\big],\end{aligned} \tag{73}$$

where $\omega_{cy} = G_yv$ and $\omega_{cz} = G_zv$.

The Boltzmann conductivity can be calculated as in the 2D case and is given by

$$\sigma_{\mu\mu}(H) = 2e^2N(0)D_\mu(H). \tag{74}$$

Here $D_x(H) = D_x$, $D_y(H) = D_y/(1 + \omega_{cy}^2\tau^2)$, and $D_z(H) = D_z/(1 + \omega_{cz}^2\tau^2)$ are the anisotropic diffusion coefficients renormalized by the magnetic field. The coefficients D_μ are the diffusion coefficients in zero field. $N(0) = 1/\pi vbc$ is now the 3D density of states per spin. τ is the elastic scattering time calculated in the Born approximation and is given by Eq. (18). To obtain the first quantum correction to the conductivity, it is necessary to solve the integral equation for the cooperon. The kernel Q is now given by

$$Q_\omega(x_1,x_2,q_y,q_z) = \frac{1}{bcv^2}e^{-|x_2-x_1|/l}\cos\left(\frac{\omega}{v}(x_2-x_1)\right) \times J_0\left[\frac{4t_y}{\omega_{cy}}\sin\left(\frac{G_y}{2}(x_2-x_1)\right)\sin\left(q_y\frac{b}{2}-\frac{G_y}{2}(x_1+x_2)\right)\right] \times J_0\left[\frac{4t_z}{\omega_{cz}}\sin\left(\frac{G_z}{2}(x_2-x_1)\right)\sin\left(q_z\frac{c}{2}-\frac{G_z}{2}(x_1+x_2)\right)\right]. \quad (75)$$

As in the 2D case, $\delta\sigma_{xx}(H)$ is a function of the eigenvalues λ_{ν,q_y,q_z} of the integral operator defined by Q_ω

$$\delta\sigma_{xx}(H) = -\frac{e^2v^2}{2\pi}lbc\sum_{\nu,q_y,q_z}\frac{\lambda_{\nu,q_y,q_z}}{1-n_iV^2\lambda_{\nu,q_y,q_z}}. \quad (76)$$

In the following, we only consider the case of a magnetic field along the y direction [in strong field, the electrons are localized in the (x,y) planes]. Several cases are to be considered according to the relative values of t_y, t_z, $1/\tau$ and $1/\tau_{\rm in}$. We assume that $1/t_y \ll \tau$. According to Sec. IV, the behavior of the gas in the (x,y) planes is then 2D. We shall consider the three different limits (a) coherent limit, $1/t_z \ll \tau$; (b) incoherent limit (low temperature), $\tau \ll 1/t_z$ and $\tau \ll \Delta t_0 \ll \tau_{\rm in}$; (c) incoherent limit (high temperature), $\tau \ll 1/t_z$ and $\tau \ll \tau_{\rm in} \ll \Delta t_0$; according to the nature (diffusive or not) of the electronic motion along the z direction. $\Delta t_0 = 1/2t_z^2\tau$ is now the diffusive time between two neighboring (x,y) planes. In the limit $\omega_c\tau \ll 1$ (in the following we write $\omega_c = \omega_{cz}$ and $G = G_z$), the eikonal approximation is valid. The eigenvalues λ_{ν,q_y,q_z} are then determined by the following equation:

$$-v^2\frac{\partial^2\psi}{\partial x^2}(x) + t_z^2[1-\cos(q_zc-2Gx)]\psi(x) = \frac{1}{\tau^2}(1-n_iV^2\lambda+i\omega\tau-D_y\tau q_y^2)\psi(x) \quad (77)$$

for $1-n_iV^2\lambda_{\nu,q_y,q_z} < 1$. Except for the additional term $D_y\tau q_y^2$, this equation is identical to Eq. (52) obtained in the 2D case. The eigenvalues and eigenvectors are then directly deduced from Eqs. (53)–(56), where $D_{\rm eff}$ is now equal to $(D_xD_z)^{1/2}$. The correction to the conductivity (including spin degeneracy) will be obtained by adding the two contributions

$$\delta\sigma_{xx}^{(3)}(H) = -\frac{e^2}{2\pi^2}\left(\frac{D_x}{D_z}\right)^{\frac{1}{2}}\sum_{m=0}^{m_0}\int_{-q_0}^{q_0}\frac{dq_y}{2\pi}\left[m+\frac{1}{2}+\frac{1}{4eD_{\rm eff}H\tau_{\rm in}}+\frac{D_yq_y^2}{4eD_{\rm eff}H}\right]^{-1}, \quad (78)$$

$$\delta\sigma_{xx}^{(2)}(H) = -\frac{2e^2D_x\tau}{\pi c}\int\frac{dq_x}{2\pi}\frac{dq_y}{2\pi}\frac{1}{\tau/\tau_{\rm in}+t_z^2\tau^2+D_x\tau q_x^2+D_y\tau q_y^2}. \quad (79)$$

In Eq. (78), m_0 is equal to $\Delta t_H/\tau_c$, where $\Delta t_H = 1/4eD_{\rm eff}H$. $\tau_c = \tau$ in the limit (a) and $\tau_c = \Delta t_0$ in the limt (b). In Eq. (79), the sum over q_x, q_y is restricted to $D_x\tau q_x^2 + D_y\tau q_y^2 < 1$ and $|q_x| > t_z/v$. However, since the pole of the cooperon is cut off by $\tau/\tau_{\rm in}+t_z^2\tau^2$, the domain of integration can be extended to $0 < D_x\tau q_x^2+D_y\tau q_y^2 < 1$. The cutoff q_0 is of the order of $1/l_y$. After integration over q_y, $\delta\sigma_{xx}^{(3)}(H)$ is given by

$$\delta\sigma_{xx}^{(3)}(H) = -\frac{e^2}{\pi^3}C\sum_{m=0}^{m_0}\left(m+\frac{1}{2}+\frac{\Delta t_H}{\tau_{\rm in}}\right)^{-\frac{1}{2}} \times \arctan\left(\frac{\Delta t_H/\tau}{m+\frac{1}{2}+\frac{\Delta t_H}{\tau_{\rm in}}}\right)^{\frac{1}{2}}, \quad (80)$$

where $C = D_x^{3/4}/(D_y^{1/2}D_z^{1/4})$. Following Kawabata,[6] we can write

$$\delta\sigma_{xx}^{(3)}(H) - \delta\sigma_{xx}^{(3)}(0) = \frac{e^2}{2\pi^2}C\sqrt{eH}F(0) \quad (81)$$

for $\tau_c < \Delta t_H < \tau_{\rm in}$. The function $F(x)$ is defined by

$$F(x) = \sum_{m=0}^{\infty}\left[2(m+1+x)^{1/2} - 2(m+x)^{1/2} - (m+1/2+x)^{-1/2}\right] \quad (82)$$

and $F(0) = 0.605$. Taking into account the two contributions $\delta\sigma_{xx}^{(3)}$ and $\delta\sigma_{xx}^{(2)}$ leads to the following results in the different limits

$$\delta\sigma_{xx}^{(a)}(H) = \delta\sigma_{xx}^{(3,a)}(H), \quad (83)$$

$$\delta\sigma_{xx}^{(b)}(H) = \delta\sigma_{xx}^{(3,b)}(H) - \frac{e^2}{2\pi^2}\left(\frac{D_x}{D_y}\right)^{\frac{1}{2}}\ln\left(\frac{\Delta t_0}{\tau}\right), \quad (84)$$

$$\delta\sigma_{xx}^{(c)}(H) = \delta\sigma_{xx}^{(2D)} = -\frac{e^2}{2\pi^2}\left(\frac{D_x}{D_y}\right)^{\frac{1}{2}}\ln\left(\frac{\tau_{\rm in}}{\tau}\right), \quad (85)$$

where $\delta\sigma_{xx}^{(3)}(H)$ is given by Eq. (81). The expression of $\delta\sigma_{xx}(H)$ in the limit (a) is similar to the one obtained for a gas with anisotropic masses.[7,25,21] It is also similar to the one obtained by Szott, Jedrzejek, and Kirk in the case of a superlattice with an open Fermi surface with the assumption (although not said explicitly) that $t_z\tau \gg 1$.[31]

As in the 2D case, the preceding results can be simply understood by considering the nature (2D or 3D) of the closed paths which contribute to the weak localization.

We now consider the effect of a strong magnetic field. In the limit $\omega_c \gg t_z$, the electrons are localized in the planes (x, y) and the pole of the cooperon is restored. The calculation of the first quantum correction to the conductivity follows the same steps as in the 2D case. In the limit $\omega_c \gg t_z, 1/\tau$, the conductivity is given by

$$\delta\sigma_{xx}(H) = \frac{e^2}{2\pi^2 c}\left(\frac{D_x}{D_y}\right)^{\frac{1}{2}} \ln\left(\frac{\tau}{\tau_{\text{in}}} + \frac{\tau}{\Delta t_0(H)}\right), \tag{86}$$

where $\Delta t_0(H) = \tau\omega_c^2/t_z^2$ is the diffusive time between two neighboring (x, y) planes. When $\tau_{\text{in}} \ll \Delta t_0(H)$, all closed paths are 2D and are not affected by the magnetic field: we recover the 2D result. When $\Delta t_0(H) \ll \tau_{\text{in}}$, some of the closed paths are 3D and do not contribute to $\delta\sigma_{xx}(H)$. The first quantum correction is due to the 2D closed paths with a propagation time $\tau < \Delta t < \Delta t_0(H)$.

As in the 2D case, we apply a scaling procedure to the system. The quantum correction at zero temperature for a system of size $L_x, L_y = L_x l_y(H)/l, L_z = L_x l_z(H)/l$ is obtained from the preceding calculation by replacing $L_{\text{in}} = (D_x \tau_{\text{in}})^{1/2}$ by L_x. We first consider the zero-field case where $\tau \ll 1/t_z$. We start with a system of size l, l_y, l_z. It follows from Eq. (85) that the system has a 2D behavior as long as $L_x \ll L_0 = (D_x \Delta t_0)^{1/2}$. In this limit, the (x, y) planes can be considered as decoupled. Since $\delta\sigma_{xx}/\sigma_{xx} = \delta\sigma_{yy}/\sigma_{yy}$ ($\sigma_{\mu\mu}$ is the Boltzmann conductivity), we can use a one parameter scaling procedure. When increasing its size, the system will reach a 2D insulating state if $\xi_x^{2D}/l = \xi_y^{2D}/l_y \ll L_0/l$. This condition can be rewritten as $t_z \ll t_z^{(0)} \sim \tau^{-1}\exp(-\alpha t_y \tau)$, where α is of the order of unity. This value of $t_z^{(0)}$ agrees with the one obtained by Prigodin and Firsov[32] for the case of weakly coupled isotropic planes.

In the presence of a magnetic field along y, the only case of interest is the case where $t_z \gg t_z^{(0)}$. When the system is in the metallic state, the pertubative calculations are valid. According to Eqs. (83) and (84), a small field will give rise to a negative magnetoresistance. In the limit of strong field $\omega_c \gg 1/\tau, t_z$, Eq. (86) shows that the system has a 2D behavior as long as $L_x \ll L_0(H) = [D_x \Delta t_0(H)]^{1/2}$ and $L_y \ll L_0(H) l_y/l$. When increasing its size from l, l_y, the system will reach a 2D insulating state if $\xi_x^{2D}/l = \xi_y^{2D}/l_y \ll L_0(H)/l$. This condition can be rewritten as $\omega_c \gg \omega_c^{(0)} \sim t_z \exp(\alpha t_y \tau)$. Thus, the magnetic field will induce a metal-insulator transition (MIT), the critical field being of the order of $\omega_c^{(0)}$. It should be noted that the restoration of the pole of the cooperon appears well before the MIT. Therefore, there will be a regime of positive magnetoresistance for $1/\tau, t_z \ll \omega_c < \omega_c^{(0)}$.

It is also possible to consider the case where the magnetic field has nonzero components along the y and z directions ($H_y \neq 0$ and $H_z \neq 0$). The scaling procedure is based on the first quantum correction to the conductivity and on the zero-field diagrammatic calculations[33] applied to the case of weakly coupled chains with an isotropic coupling $t_y = t_z = t$. In zero field, the self-consistent diagrammatic treatment predicts a MIT when t becomes smaller than $t^{(0)} \sim 1/\tau$. We assume here that $t_y, t_z > t^{(0)}$: in zero field, the system is metallic. A weak magnetic field leads to a negative magnetoresistance. A strong magnetic field localizes the electrons on the chains of highest conductivity: the system has a 1D behavior when $\omega_{cy} \gg t_y$ and $\omega_{cz} \gg t_z$. Thus, there is a magnetic-field-induced MIT. The critical field is defined by $\omega_{cy} \sim t_y$ and $\omega_{cz} \sim t_z$. As in the 2D case, the gas becomes localized as soon as the pole of the cooperon is restored.

VI. CONCLUSION

We have studied the effect of impurities in a quasi-1D conductor in a magnetic field. First, we have calculated the semiclassical (Boltzmann) conductivity and the first quantum correction. Then, using a scaling procedure, we have determined the ground state at zero temperature. Different cases have been examined according to the dimensionality (2D or 3D) and the relative values of the elastic scattering time and the hopping rate between chains.

In 2D, a strongly anisotropic conductor behaves as a set of uncoupled 1D chains if $t\tau \ll 1$. In the limit $t\tau \gg 1$, the gas shows a 2D (anisotropic) behavior in zero field. It exhibits a negative magnetoresistance at finite temperature in a weak magnetic field. A strong magnetic field induces a crossover from a 2D regime (where the localization lengths are exponentially large compared to the mean free paths) towards a 1D regime where the wave functions are one-dimensional and localized on a length of the order of the mean free path along the chain. The crossover field is defined by $\omega_c \sim t$.

In 3D, we have studied the case where $t_y \tau \gg 1$. The system can be seen as a set of weakly coupled parallel anisotropic 2D planes. The magnetic field is assumed to be along the y axis and tends to localize the electrons in the (x,y) planes. If $t_z \ll t_z^{(0)} = \tau^{-1}\exp(-\alpha t_y \tau)$, the gas behaves as a set of uncoupled anisotropic 2D planes. Since the magnetic field is parallel to the (x,y) planes, it has no effect in this case. If $t_z > t_z^{(0)}$, the gas shows a 3D (anisotropic) behavior in zero field and exhibits a negative magnetoresistance in a weak field. In strong field, a regime of positive magnetoresistance is reached for $\omega_c \gg 1/\tau, t_z$. In higher field, the gas undergoes a MIT. The critical field associated with this MIT is of the order of $\omega_c^{(0)} \sim t_z \exp(\alpha t_y \tau)$. In very high field $\omega_c \gg \omega_c^{(0)}$, the gas has a 2D behavior. The high-field regime has also been examined when the field has nonzero components along the y and z axis and $t_y, t_z > 1/\tau$. In this case, there is a MIT with a critical field defined by $\omega_{cy} \sim t_y$ and $\omega_{cz} \sim t_z$. In very high field $\omega_{cy} \gg t_y$ and $\omega_{cz} \gg t_z$, the gas has a 1D behavior.

For the quasi-1D conductors of the Bechgaard salts family, we have $t_y/2 \sim 300$ K, $t_z/2 \sim 10$ K, $1/\tau \ll 1$ K, and $b, c \sim 10$ Å. The gas has to be described by a 3D model. The high-field limit is accessible only in the configuration where the field is along the y direction. In this case, the regime with positive magnetoresistance ($\omega_c > t_z$) should be reached for $H \sim$ 20–30 T. In the weak-field limit, our model predicts a negative magnetoresistance for any direction of the magnetic field. Most of the experimental results on the Bechgaard salts seem not to present weak-localization effects. They show a very large positive magneto resistance on a large-field scale, which has to be explained by other mechanisms. The absence of weak-localization effects can be explained by the very large value of the elastic scattering time $\tau \sim \tau_{\text{in}}$. Nevertheless, a negative magnetoresistance was observed in a quasi-1D conductor with an architecture similar to the one of the Bechgaard salts.[34]

The high-field regime is difficult to reach in quasi-1D organic conductors because of the small value of the interspacing between chains and the large value of the coupling. This difficulty could be avoided by considering weakly coupled quantum wires. For example, the 2D gas described in Ref. 35 has a period between wires b = 102 Å and the coupling between wires t_y is equal to 150 K. This gas exibits quantum Hall caracteristics because the Fermi level lies in a region where the semiclassical orbits are closed (the Fermi surface is closed). In order to observe a magnetic-field-induced Anderson localization, the Fermi surface has to be open. This could be achieved by increasing the spacing between wires (t_y would then decrease) and increasing the electron density.[36]

ACKNOWLEDGMENTS

This work was supported by ESPRIT Basic Research Contract No. 3121. We thank H. Stormer for a useful comment.

APPENDIX A

The current operators are obtained from the velocity operator

$$v_x = v\,\text{sgn}(-i\partial_x), \tag{A1}$$

$$v_y = -bt\sin(-ib\partial_y - Gx). \tag{A2}$$

Since $|\phi_{\mathbf{k}}^{(\alpha)}\rangle$ is built from the plane-wave states $|k'_x, k_y\rangle$, where k'_x is positive (negative) if $\alpha = +(-)$, it is an eigenstate of the operator v_x with the eigenvalue αv. Equation (21) follows straightforwardly. In second quantization, the operator $j_y(x, q_y)$ is written as

$$j_y(\mathbf{r}) = -\frac{ebt}{2}\sum_{\alpha,\alpha'}\sum_{k_y,k'_y}\int d^2r'\psi^{(\alpha)^\dagger}(x',k_y)e^{-ik_y y'}[\,\delta(\mathbf{r}-\mathbf{r}')\sin(-ib\partial_{y'} - Gx') + \sin(-ib\partial_{y'} - Gx')\delta(\mathbf{r}-\mathbf{r}')]e^{ik'_y y'}\psi^{(\alpha')}(x',k'_y). \tag{A3}$$

Noting that $e^{ik'_y y'}$ is an eigenfunction of the operator $\sin(-ib\partial_{y'} - Gx')$ with the eigenvalue $\sin(k'_y b - Gx')$, the first term in the right-hand side of (A3) gives the contribution (after integration over x' and y')

$$-\frac{ebt}{2}\sum_{\alpha,\alpha'}\sum_{k_y,k'_y} e^{i(k'_y-k_y)y}\sin(k'_y b - Gx)\psi^{(\alpha)^\dagger}(x,k_y)\psi^{(\alpha')}(x,k_{y'}). \tag{A4}$$

In order to calculate the contribution due to the second term of the right-hand side of Eq. (A3), we write the operator $\sin(-ib\partial_{y'} - Gx')$ as

$$\sin(-ib\partial_{y'} - Gx') = \sum_{n=0}^{\infty} a_n \sum_{j=0}^{2n+1} C_{2n+1}^j(-Gx')^{2n+1-j}(k'_y b)^j, \tag{A5}$$

where the coefficients a_n come from the series expansion of the function $\sin(z)$. Letting the operator $\sin(-ib\partial_{y'} - Gx')$ act on the function $\delta(\mathbf{r}-\mathbf{r}')e^{ik_{y'}y'}$ yields

$$\sum_{n=0}^{\infty} a_n \sum_{j=0}^{2n+1} C_{2n+1}^j(-Gx')^{2n+1-j}(-ib)^j\sum_{k=0}^{j} C_j^k\delta(x-x')\delta^{(k)}(y-y')(ik'_y)^{(j-k)}e^{ik'_y y'}. \tag{A6}$$

Here $\delta^{(k)}(y-y')$ is the k^{th} derivative of the Dirac distribution $\delta(y-y')$. Summing over x' and y' yields the contribution

$$-\frac{ebt}{2}\sum_{\alpha,\alpha'}\sum_{k_y,k'_y} e^{i(k'_y-k_y)y}\sin(k_y b - Gx)\psi^{(\alpha)^\dagger}(x,k_y)\psi^{(\alpha')}(x,k_{y'}). \tag{A7}$$

Adding the two contributions and taking the Fourier transform with respect to y lead to Eq. (22).

APPENDIX B

In this appendix, we calculate the function $F(x_1,x_2,q_y)$ defined by Eq. (42). Using expressions (26) and (27) of the Green's functions, F can be written as

$$F(x_1,x_2,q_y)=\sum_{\alpha,\alpha'}\alpha\alpha'\sum_{k_y}\int dx\,dx' e^{i\Phi}\tilde{G}^{R(\alpha)}_{E_F}(x-x_1)\tilde{G}^{R(\alpha')}_{E_F}(x_2-x')\tilde{G}^{A(\alpha')}_{E_F}(x'-x_1)\tilde{G}^{A(\alpha)}_{E_F}(x_2-x), \tag{B1}$$

$$\Phi=\varphi^{(\alpha)}(x,x_1,k_y)+\varphi^{(\alpha')}(x_2,x',q_y-k_y)+\varphi^{(\alpha')}(x',x_1,q_y-k_y)+\varphi^{(\alpha)}(x_2,x,k_y), \tag{B2}$$

where the function $\varphi^{(\alpha)}(x,x',k_y)$ is defined by (27). Since the singularity of the maximally crossed diagrams is associated with the backscattering of the electrons, we only evaluate F for $\alpha=-\alpha'$. The phase Φ is then equal to

$$\Phi=-4\alpha\frac{t}{\omega_c}\sin\left(\frac{G}{2}(x_1-x_2)\right)\sin\left(\frac{q_yb}{2}-\frac{G}{2}(x_1+x_2)\right)\sin\left(\frac{q_yb}{2}-k_yb\right) \tag{B3}$$

and the sum over k_y yields

$$\sum_{k_y}e^{i\Phi}=\frac{1}{b}J_0\left[\frac{4t}{\omega_c}\sin\left(\frac{G}{2}(x_1-x_2)\right)\sin\left(q_y\frac{b}{2}-\frac{G}{2}(x_1+x_2)\right)\right]. \tag{B4}$$

The product of the four functions $\tilde{G}$ can be written as

$$\tilde{G}^{R(\alpha)}_{E_F}(x-x_1)\ \tilde{G}^{R(-\alpha)}_{E_F}(x_2-x')\tilde{G}^{A(-\alpha)}_{E_F}(x'-x_1)\tilde{G}^{A(\alpha)}_{E_F}(x_2-x) \tag{B5}$$

$$=\begin{cases}\frac{1}{v^4}e^{-\alpha\frac{x_1+x_2}{l}+\alpha\frac{x+x'}{l}} & \text{if } \alpha x,\alpha x'<\alpha x_1,\alpha x_2\\ 0 & \text{otherwise.}\end{cases} \tag{B6}$$

(B7)

The integration over x and x' yields

$$\int dx\,dx'\tilde{G}^{R(\alpha)}_{E_F}(x-x_1)\tilde{G}^{R(-\alpha)}_{E_F}(x_2-x')\tilde{G}^{A(-\alpha)}_{E_F}(x'-x_1)\tilde{G}^{A(\alpha)}_{E_F}(x_2-x)=l^2e^{-\frac{|x_1-x_2|}{l}}. \tag{B8}$$

Equation (43) follows from (B4) and (B8).

APPENDIX C

According to Eq. (63), there are four contributions to $\delta Q_\omega(x,x_2,q_y)$. The first contribution is equal to

$$\delta Q^{(1)}_\omega(x,x_2,q_y)=-\frac{t^2}{\omega_c^2}Q^\infty_\omega(x,x_2,q_y). \tag{C1}$$

The other contributions are equal to

$$\delta Q^{(2)}_\omega(x,x_2,q_y)=\frac{t^2}{2b\omega_c^2}\sum_{\beta=+,-}e^{-i\beta Gx_2}\sum_{\alpha,q_x}e^{iq_xx_2}\tilde{Q}^{\alpha,-\alpha}(q_x)\sum_{n=0}^{\infty}\frac{x_2^n}{n!}\partial_x^n, \tag{C2}$$

$$\delta Q^{(3)}_\omega(x,x_2,q_y)=\frac{t^2}{2b\omega_c^2}\sum_{\beta=+,-}e^{-i\beta[q_yb-G(2x+x_2)]}\sum_{\alpha,q_x}e^{iq_xx_2}\tilde{Q}^{\alpha,-\alpha}(q_x)\sum_{n=0}^{\infty}\frac{x_2^n}{n!}\partial_x^n, \tag{C3}$$

$$\delta Q^{(4)}_\omega(x,x_2,q_y)=-\frac{t^2}{4b\omega_c^2}\sum_{\beta=+,-}e^{-i\beta Gx_2}\sum_{\beta'=+,-}e^{-i\beta'[q_yb-G(2x+x_2)]}\sum_{\alpha,q_x}e^{iq_xx_2}\tilde{Q}^{\alpha,-\alpha}(q_x)\sum_{n=0}^{\infty}\frac{x_2^n}{n!}\partial_x^n. \tag{C4}$$

Using the equality

$$\sum_{q_x}\int dx_2x_2^pe^{iq_xx_2}\tilde{Q}^{\alpha,-\alpha}_\omega(q_x)=i^p\frac{\partial^p\tilde{Q}^{\alpha,-\alpha}_\omega}{\partial q_x^p}(0) \tag{C5}$$

we have

$$\int dx_2 \delta Q_\omega^{(2)}(x,x_2,q_y) = \frac{t^2}{2b\omega_c^2} \sum_{\alpha,\beta} \sum_{n=0}^{\infty} \frac{i^n}{n!} \frac{\partial^n \tilde{Q}_\omega^{\alpha,-\alpha}}{\partial q_x^n} (\beta G) \partial_x^n, \tag{C6}$$

$$\int dx_2 \delta Q_\omega^{(3)}(x,x_2,q_y) = \frac{t^2}{2b\omega_c^2} \sum_{\alpha,\beta} \sum_{n=0}^{\infty} e^{i\beta(q_y b - 2Gx)} \frac{i^n}{n!} \frac{\partial^n \tilde{Q}_\omega^{\alpha,-\alpha}}{\partial q_x^n} (\beta G) \partial_x^n, \tag{C7}$$

$$\int dx_2 \delta Q_\omega^{(4)}(x,x_2,q_y) = -\frac{t^2}{4b\omega_c^2} \sum_{\alpha} \sum_{\beta,\beta'} \sum_{n=0}^{\infty} e^{i\beta'(q_y b - 2Gx)} \frac{i^n}{n!} \frac{\partial^n \tilde{Q}_\omega^{\alpha,-\alpha}}{\partial q_x^n} [(\beta - \beta')G] \partial_x^n. \tag{C8}$$

The contributions $\delta Q_\omega^{(3)}$ and $\delta Q_\omega^{(4)}$ vanish when we evaluate the averaged values in the state ψ_{q_x,q_y}^∞. Noting that

$$\frac{i^n}{n!} \frac{\partial^n \tilde{Q}_\omega^{\alpha,-\alpha}(q_x)}{\partial q_x^n} = \frac{\alpha}{v^2} \left(\frac{\alpha l}{1 - i\omega\tau + i\alpha l q_x} \right)^{n+1}, \tag{C9}$$

the contribution of $\delta Q_\omega^{(2)}$ is given by

$$n_i V^2 \left\langle \psi_{q_x,q_y}^\infty \middle| \int dx_2 \delta Q_\omega^{(2)}(x,x_2,q_y) \middle| \psi_{q_x,q_y}^\infty \right\rangle = \frac{t^2}{\omega_c^2} \frac{1}{1+\omega_c^2\tau^2} - \frac{t^2}{\omega_c^2} \frac{1 - 3\omega_c^2\tau^2}{(1+\omega_c^2\tau^2)^3} D_x \tau q_x^2 \tag{C10}$$

in the limit $\omega, q_x \to 0$. Equations (67) and (68) follow from Eqs. (C1) and (C10).

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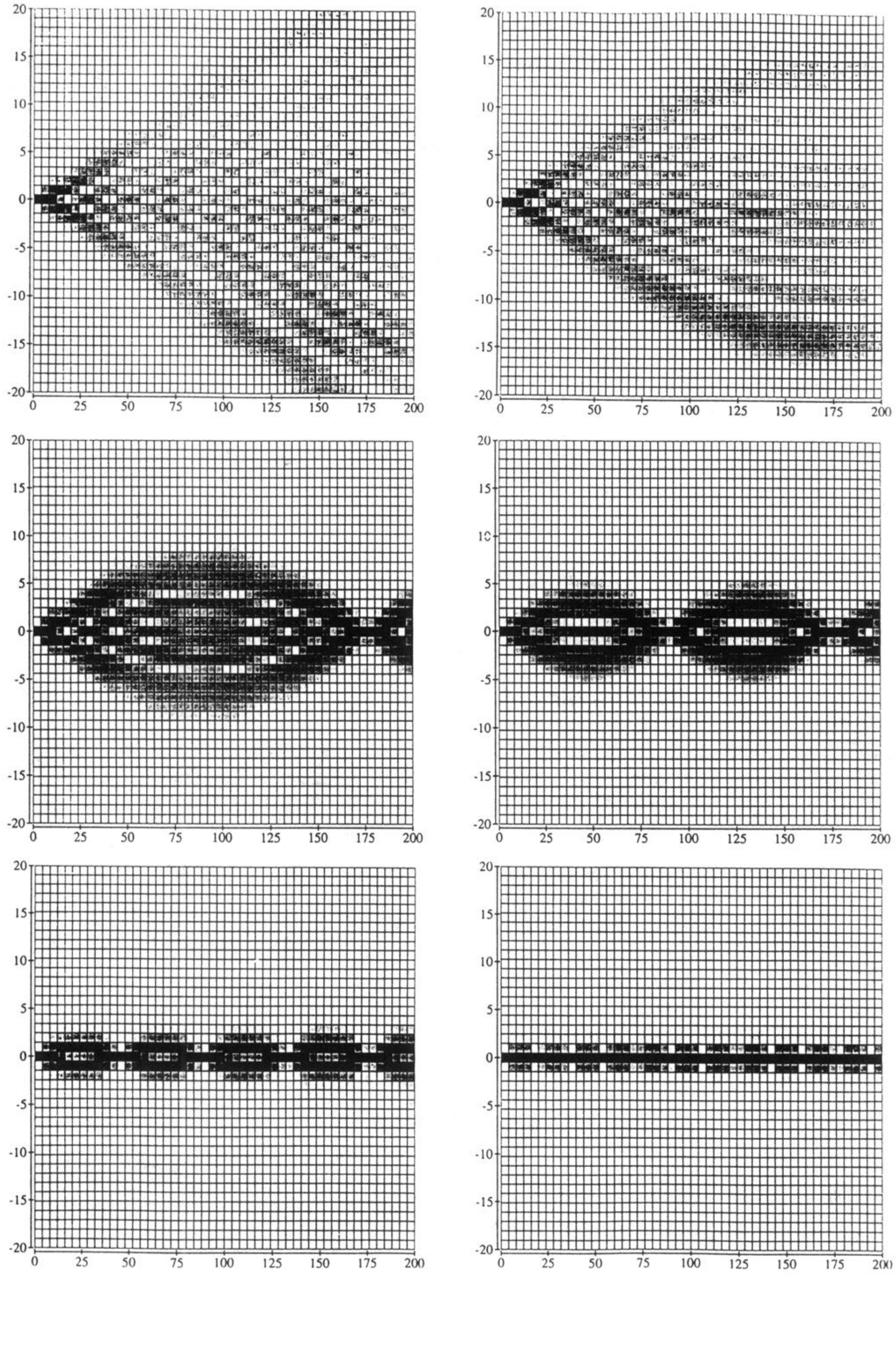

FIG. 3. Amplitude of the Green's function in real space. Each horizontal line corresponds to a given chain. The magnetic field is defined by $\omega_c/t = 0, 0.125, 0.25, 0.5, 1$, and 2.5. For $\omega_c \gg t$, the electronic motion is one dimensional.