Magnetic-Field-Induced Anderson Localization in a Strongly Anisotropic Conductor

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The weak-localization correction to the conductivity for a strongly anisotropic 2D electron gas is studied in the presence of a magnetic field. We find that the low-field regime with usual negative magnetoresistance is followed by an *increase* in the localization with a crossover to a 1D regime. In a 3D anisotropic conductor, the magnetic field will induce a transition from a diffusive 3D regime to a localized 2D or 1D regime.

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For the past decade, the study of the disordered two-dimensional electron gas in a magnetic field has been a major topic in condensed matter physics. An important aspect of this subject has been the physics of the so-called "weak localization," which describes localization as due to constructive interferences between electronic paths and their time-reversed counterparts [1]. This picture holds at finite temperature in the limit of small disorder when the mean free path l is large so that $k_F l \gg 1$. A magnetic field, breaking the time-reversal invariance, destroys the interference effect and suppresses the weak-localization correction (in the absence of spin-orbit coupling, the only case which will be studied throughout this paper) [2,3]. In a strong magnetic field, the interplay between localization and Landau quantization plays a crucial role and leads to the quantization of the Hall effect [4].

On the other hand, the two-dimensional anisotropic electron gas such as can be found experimentally in weakly coupled chains systems also has spectacular properties in a magnetic field, which have also been studied intensively during the past years. The quasi-1D conductors of the Bechgaard salt family present a spectacular phase diagram which results from an interplay between the quasi-1D and -2D aspects [5]. It presents a cascade of spin-density-wave phases appearing for increasing field. Although a lot of theoretical work has been devoted to the thermodynamics of the quasi-1D electron gas in a field [5], transport properties have received much less attention. On the experimental side, many results are still unexplained. For example, the metallic phase of the Bechgaard salts exhibits an extremely large positive magnetoresistance [6]. On the other hand, another quasi-1D conductor with similar architecture shows a negative magnetoresistance in small field [7].

In this Letter, we show that the quasi-1D aspect also leads to new developments in the physics of localization. To be more specific, the magnetic field acting on an *open Fermi surface* makes the electronic motion more and more one dimensional in the sense that it confines the wave functions along the chains. When increasing the field, it is thus natural to expect a crossover to a strongly localized 1D regime. We have calculated the conductivity and the first quantum correction in a strongly anisotropic conductor (weakly coupled chains), where the transverse motion (perpendicular to the chains) is assumed to be coherent: $1/\tau \ll t$, where t is the hopping rate between chains. An interesting theoretical aspect of the quasi-1D problem is that the Green's functions have a very simple form. We can go beyond the usual semiclassical phase integral (also called eikonal) approximation, originally introduced by Gor'kov [8]. Starting from the exact Green's functions, we derive the eikonal approximation in the low-field limit $\omega_c \tau \ll 1$. We show that a weak magnetic field destroys the weak-localization effects, as in the isotropic case [2,3]. In the high-field regime where the eikonal approximation breaks down, the field localizes the electrons on the chains. As a result, the transverse conductivity (perpendicular to the chains) is strongly reduced and the first quantum correction to the longitudinal conductivity again diverges at low temperature. Although our perturbative calculation breaks down when the system becomes 1D, it shows that the magnetic field induces a transition from a weakly localized 2D regime towards a strongly localized 1D regime. Note that the other limit $1/\tau \gg t$ has already been studied by Nakhmedov *et al.* [9]. However, because they used the eikonal approximation, their calculation cannot be extended in the high-field regime to the coherent limit we are considering here.

We consider a strongly anisotropic 2D gas with an open Fermi surface, described by the dispersion law ($\hbar = 1$) $E_{\mathbf{k}} = v(|k_\parallel| - k_F) + t\cos(k_\perp b)$, where v is the velocity at the Fermi level. In the presence of a magnetic field H along the third direction, the semiclassical equations of motion lead to the following trajectory in real space: $y = b(t/\omega_c)\cos(Gx)$, where $G = eHb$ and $\omega_c = Gv$ is the frequency of the electronic motion. When the field is such that $\omega_c \sim t$, the electronic motion becomes 1D. Therefore, we expect that the (Boltzmann) conductivity and the first quantum correction will be strongly affected in the high-field limit $\omega_c \geq t$. On the other hand, since the motion perpendicular to the chains is coherent $1/\tau \ll t$, the gas is really 2D in zero field. Thus, we expect no essential difference with the isotropic case for very weak field ($\omega_c \tau \ll 1$).

In a quantum picture, the magnetic field is taken into

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account by the usual Peierls substitution. Using the Landau gauge $\mathbf{A}(0,Hx,0)$, the Hamiltonian is written as

$$\mathcal{H}=v(|k_x|-k_F)+t\cos(k_y b-Gx)+\sum_l V(\mathbf{r}-\mathbf{R}_l)\,, \quad (1)$$

where the last term describes the interaction with impurities located at points $\mathbf{R}_l$. We assume this interaction to be pointlike $V(\mathbf{r}-\mathbf{R}_l)=V\delta(\mathbf{r}-\mathbf{R}_l)$ and no correlation between the positions of the different impurities. The Green's functions for the clean system have been previously calculated in other contexts [10,11]. Calculating the self-energy to lowest order in n_i (impurity concentration) and V (Born approximation), the retarded Green's function is written as

$$G_\epsilon^{R(\alpha)}(x,x',k_\perp)=e^{i\Phi^{(\alpha)}(k_\perp,x,x')}\tilde{G}_\epsilon^{R(\alpha)}(x-x')\,, \quad (2)$$

where

$$\Phi^{(\alpha)}(k_\perp,x,x')=\alpha(t/\omega_c)\times[\sin(k_\perp b-Gx)-\sin(k_\perp b-Gx')]\,.$$

We have introduced the one-dimensional Green's function $\tilde{G}_\epsilon^{R(\alpha)}(x'-x)$ defined by

$$\tilde{G}_\epsilon^{R(\alpha)}(x-x')=-(i/v)e^{i\alpha(k_F+\epsilon/v+i/2l)(x-x')} \quad (3)$$

if $\alpha(x-x')>0$, and 0 otherwise. Here $l=v\tau$ is the mean free path and τ is the elastic scattering time given by $1/\tau=2\pi N(0)n_i V^2$. $N(0)$ is the density of states per spin at the Fermi level and $\alpha=+\ (-)$ refers to the right (left) sheet of the Fermi surface. To take advantage of the conservation of the transverse momentum in the Landau gauge, we have used a mixed representation $(x,k_\perp)$ for the Green's function.

Neglecting any localization effect, we obtain the longitudinal and transverse dc conductivities $\sigma_\parallel(H)=2e^2\times N(0)D_\parallel$ and $\sigma_\perp(H)=2e^2N(0)D_\perp(H)$. The factor 2 comes from spin degeneracy. The effect of the field can be taken into account by renormalizing the transverse diffusion coefficient $D_\perp(H)=D_\perp/(1+\omega_c^2\tau^2)$. $D_\parallel=v^2\tau$ and $D_\perp=t^2b^2\tau/2$ are the anisotropic diffusion coefficients in zero field. As could be expected from the preceding semiclassical arguments, $\sigma_\parallel(H)$ does not depend on H, but $\sigma_\perp(H)$ is strongly affected by the magnetic field in the quantum regime $\omega_c\tau\gg 1$. This reflects the one dimensionalization induced by the magnetic field.

We now consider the first quantum correction to the longitudinal conductivity in the weak-scattering limit. This correction is obtained by summing the maximally crossed diagrams, which give rise to a logarithmic divergence of the conductivity at low temperature in zero field [12]. In the particle-particle channel, the propagator P (the so-called Cooperon) becomes a ladder diagram and is determined by the following integral equation:

$$P(x_1,x_2,q_\perp)=n_iV^2\delta(x_1-x_2)+n_iV^2\int dx_3\,Q(x_1,x_3,q_\perp)P(x_3,x_2,q_\perp)\,, \quad (4)$$

where $Q(x_1,x_3,q_\perp)$ is the usual pair propagator. The conductivity is calculated at finite frequency, and we adopt the interpretation that at finite temperature, $-i\omega$ has to be replaced by the inelastic scattering rate $1/\tau_{\rm in}$. Using expressions (2) and (3) of the Green's functions, the kernel Q of the preceding equation is given by

$$Q(x,y,q_\perp)=\frac{1}{bv^2}e^{-|y-x|/l}\cos\left(\frac{\omega}{v}(y-x)\right)J_0\left(\frac{4t}{\omega_c}\sin\left(\frac{G}{2}(y-x)\right)\sin\left(q_\perp\frac{b}{2}-\frac{G}{2}(x+y)\right)\right), \quad (5)$$

where J_0 is the zeroth-order Bessel function. For any value of the magnetic field, the quantum correction to the conductivity is written as

$$\delta\sigma_\parallel(H)=-\frac{e^2}{2\pi}v^2lb\sum_{m,q_\perp}\frac{\lambda_{m,q_\perp}}{1-n_iV^2\lambda_{m,q_\perp}}\,, \quad (6)$$

where $\lambda_{m,q_\perp}$ are the eigenvalues of the integral operator $Q(x_1,x_3,q_\perp)$. A very interesting feature is that we only need to know the eigenvalues $\lambda_{m,q_\perp}$ in order to calculate $\delta\sigma_\parallel(H)$. We now calculate these eigenvalues in the two limits of interest: the low-field limit $\omega_c\tau\ll 1$ and the high-field limit $\omega_c\gg t$.

We first examine the effect of a weak magnetic field. Before we calculate the contribution of the maximally crossed diagrams to the conductivity, we consider the Green's function in real space,

$$G_\epsilon^{(\alpha)}(x,x',y=nb)=\frac{1}{b}e^{inG(x+x')/2-in\pi/2}J_n\left(-2\alpha\frac{t}{\omega_c}\sin\left(\frac{G}{2}(x'-x)\right)\right)\tilde{G}_\epsilon^{(\alpha)}(x-x')\,, \quad (7)$$

where J_n is the nth-order Bessel function and $\tilde{G}^{(\alpha)}$ is given by (3) and does not depend on the magnetic field. The field has two different effects. On the one hand, it adds a phase factor to the Green's function. It can be verified that this phase factor is equal to

$$n\frac{G}{2}(x+x')=e\int_{(x,0)}^{(x',y)}d\mathbf{s}\cdot\mathbf{A}(\mathbf{s})\,, \quad (8)$$

where the path of integration is a straight line between the two points $(x,0)$ and $(x',y=nb)$. On the other hand, it

modifies the argument of the Bessel function J_n. Clearly, this latter effect is related to the one dimensionalization induced by the magnetic field. Since the Green's function $\tilde{G}_\epsilon^{(a)}(x-x')$ introduces a cutoff $|x-x'|\sim l$, in the limit $\omega_c\tau = Gl \ll 1$ it is possible to neglect the second effect. Consequently, the field effect reduces to the addition of the phase factor (8) in the Green's function. Thus, we recover the semiclassical (or eikonal) approximation [8]. The kernel Q is then given by

$$Q^{(\mathrm{eik})}(x,y,q_\perp) = \frac{1}{bv^2} e^{-|y-x|/l} \cos\left[\frac{\omega}{v}(y-x)\right] J_0\left[\frac{2t}{v}(y-x)\sin\left[q_\perp\frac{b}{2} - Gx\right]\right]. \tag{9}$$

As can be seen from the general expression of $\delta\sigma_\parallel(H)$ (6), the main contribution comes from the eigenvalues $\lambda_{m,q_\perp}$ where $1 - n_i V^2 \lambda_{m,q_\perp} < 1$. For the corresponding eigenstates $\psi_{m,q_\perp}$, the integral equation (4) reduces to a second-order differential equation,

$$-v^2\frac{\partial^2\psi}{\partial x^2}(x) + t^2[1-\cos(q_\perp b - 2Gx)]\psi(x) = \frac{1}{\tau^2}(1 - n_i V^2\lambda + i\omega\tau)\psi(x). \tag{10}$$

As in the isotropic case [2], the effect of the magnetic field is to replace the operator $-i\nabla$ by $-i\nabla - 2e\mathbf{A}$, where $2e$ is the charge of the particle-particle pair. Using again the condition $1 - n_i V^2\lambda_{m,q_\perp} < 1$, the sinusoidal potential appearing in (10) can be approximated by a set of uncoupled harmonic potentials centered at points $x_n = q_\perp b/2G - n\pi/G$. The eigenvalues are then given by $1 - n_i V^2\lambda_{m,q_\perp} = -i\omega\tau + 4eD_{\mathrm{eff}}H\tau(m+\frac{1}{2})$ where we have introduced the effective diffusion coefficient $D_{\mathrm{eff}} = (D_\parallel D_\perp)^{1/2}$. For sufficiently small magnetic field, the eigenvalues $\lambda_{m,q_\perp}$ appearing in the numerator of (6) can be replaced by $1/n_i V^2$. The correction to the conductivity is then given by (introducing a factor of 2 for spin degeneracy)

$$\delta\sigma_\parallel(H) = -\frac{e^2}{2\pi^2}\left(\frac{D_\parallel}{D_\perp}\right)^{1/2}\left\{\Psi\left(\frac{1}{2} + \frac{1}{4eD_{\mathrm{eff}}H\tau}\right) - \Psi\left(\frac{1}{2} + \frac{1}{4eD_{\mathrm{eff}}H\tau_{\mathrm{in}}}\right)\right\}, \tag{11}$$

where Ψ is the digamma function. $-i\omega\tau$ has been replaced by τ/τ_{in}. Equation (11) yields a characteristic field H_0 defined by $D_{\mathrm{eff}}\tau_{\mathrm{in}}H_0 \sim \phi_0$. Here $\phi_0 = 2\pi/e$ is the flux quantum. Equation (11) is analogous to the result for the isotropic 2D gas, except for the diffusion coefficients which take into account the anisotropy of the gas [if we set $D_\parallel = D_\perp$ we exactly recover the expression of $\delta\sigma(H)$ for an isotropic 2D gas]. Thus Eq. (11) is consistent with our assumption that the transverse motion is coherent ($1/\tau \ll t$).

We now consider the high-field regime ($\omega_c \gg t$), where the physics of localization presents new aspects due to the quasi-1D Fermi surface. In the limit $\omega_c \gg t$, the magnetic field localizes the electrons on the chains of highest conductivity. The term in the Hamiltonian (1) which breaks time-reversal symmetry oscillates too fast and can be ignored. The gas becomes 1D and time-reversal symmetry is restored. As a result, the maximally crossed diagrams diverge at low temperature and lead to a strong correction to the conductivity.

In the limit $\omega_c \gg t \gg 1/\tau$, the eikonal approximation does not hold any more and we have to consider the exact kernel Q as defined by (5). For an infinite field, Q^∞ has the usual 1D expression and the corresponding eigenvalues $\lambda^\infty_{q_\parallel,q_\perp}$ are given by $1 - n_i V^2\lambda^\infty_{q_\parallel,q_\perp} = -i\omega\tau + D_\parallel\tau q_\parallel^2$ for $\omega\tau, D_\parallel\tau q_\parallel^2 < 1$. In the limit $\omega_c \gg t$, the first correction to $\lambda^\infty_{q_\parallel,q_\perp}$ is given by $\delta\lambda^\infty_{q_\parallel,q_\perp} = -(t^2/\omega_c^2)\lambda^\infty_{q_\parallel,q_\perp}$. The conductivity is then given by

$$\delta\sigma_\parallel(H) = \delta\sigma^{(1\mathrm{D})} \Big/ \left[1 + \frac{\tau_{\mathrm{in}}}{\tau}\frac{t^2}{\omega_c^2}\right]^{1/2}. \tag{12}$$

Here $\delta\sigma^{(1\mathrm{D})} = -e^2L_{\mathrm{in}}/\pi b$ is the usual 1D result for the first quantum correction to the conductivity and $L_{\mathrm{in}} = (D_\parallel\tau_{\mathrm{in}})^{1/2}$ is the inelastic coherence length. Equation (12) yields a crossover field H_1 between a 2D regime and a 1D regime defined by $\omega_c \sim t(\tau_{\mathrm{in}}/\tau)^{1/2}$. The expression of H_1 can be interpreted as follows. Since the transverse motion is diffusive with a diffusion coefficient $D_\perp(H)$, it takes a time $\Delta t(H) \sim \tau(\omega_c/t)^2$ for an electron to hop to the nearest chain. When $\Delta t(H) > \tau_{\mathrm{in}}$, all the coherent orbits which lead to localization are 1D. In this case, the magnetic field does not destroy the localization. In the other limit where $\Delta t(H) < \tau_{\mathrm{in}}$, some of the orbits are 2D and the corresponding interference between an electronic path and its time-reversed counterpart is destroyed by the field.

As pointed out in the introduction, our perturbative approach breaks down in the high-field limit due to the absence of a diffusive regime in 1D. Although it is useful to understand qualitatively the expression (12) of the first quantum correction to the conductivity, the preceding explanation, based on the assumption that the motion is diffusive on a given chain, is not valid. However, it is possible to obtain more information on the ground state of the system from the following argument. The result (12) has been explained by the fact that the electron has to diffuse on a length $L(H) = [D_\parallel\Delta t(H)]^{1/2}$ on a given chain before it diffuses to the neighboring chain. This picture remains meaningful as long as $L(H)$ is smaller than the 1D localization length $\xi^{1\mathrm{D}} \sim l$. If $\xi^{1\mathrm{D}} \ll L(H)$, the electron becomes localized on the chain before it diffuses to the neighboring chain: The diffusion in the

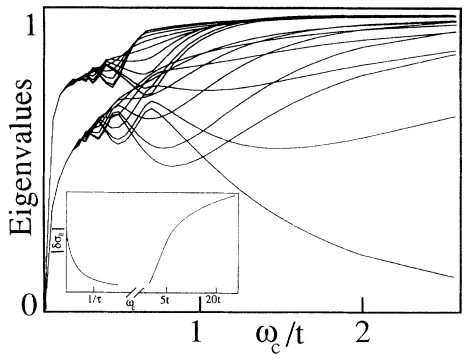

FIG. 1. Eigenvalues $1-n_iV^2\lambda_{m,q_\perp}$ vs magnetic field. Inset: First quantum correction to the conductivity $|\delta\sigma_\parallel(H)|$ vs magnetic field.

transverse direction is frozen. At zero temperature, we therefore expect a crossover between a 2D and a 1D behavior for $L(H)\sim\xi^{1D}$ $(\omega_c\sim t)$.

We have also calculated numerically the spectrum of the integral operator Q. Figure 1 shows the eigenvalues $1-n_iV^2\lambda_{m,q_\perp}$ for $q_\perp=0$ and $\omega=0$ versus magnetic field. The eigenstates can be chosen as Bloch functions $\psi_{m,q_\perp}=\psi_{n,q_\parallel,q_\perp}$, where n is a band index and $q_\parallel$ is a vector between $-G$ and G. Figure 1 shows the first two bands ($n=1$ and $n=2$) for different values of $q_\parallel$ (note that since we have chosen values for $q_\parallel$ uniformly distributed between $-G$ and G, the value of each $q_\parallel$ increases with the magnetic field). For very weak field, each band is dispersionless and the Landau regime $1-n_iV^2\lambda_{n,q_\parallel,q_\perp}=4eD_{\rm eff}\times H\tau(n+\frac{1}{2})$ is clearly visible. When the field is increased, the pole in the Cooperon is suppressed and the degeneracy of each band is lifted. In the high-field regime, the pole is restored. The eigenvalues of the lowest band are distributed between 0 and 1 and we recover the continuous 1D spectrum. The eigenvalues of the higher bands tend to 1. Figure 1 also shows the conductivity $\delta\sigma_\parallel(H)$ versus magnetic field. In the low-field regime, the conductivity is given by the analytical result (11). In the high-field regime, the conductivity has been obtained from the eigenvalues of the kernel (5) shown in the same figure.

In conclusion, the magnetic field has a new and remarkable effect on the electronic properties of disordered quasi-1D systems. Such systems can exhibit negative magnetoresistance in low field followed by strong positive magnetoresistance in large field. In the limit where the magnetic field localizes the electrons on the chains of highest conductivity, all the states are localized in the presence of an arbitrarily weak disorder. The strongly anisotropic 3D gas (weakly coupled chains with a hierarchy of hopping rates $t_z\ll t_y$) can be analyzed in the same way. In this case, the field localizes the electrons in planes or on chains, depending on its direction. Therefore, in the limit of weak disorder, the magnetic field may induce a transition from a diffusive 3D regime towards a localized 2D or 1D regime.

It should be noted that both effects (negative magnetoresistance in low field and positive magnetoresistance in high field) are related to the disappearance and the reentrance of the superconducting phase in strongly anisotropic 3D superconductors in a magnetic field [13]. In each case, the effect of the field can be understood as the suppression and the restoration of the Cooper pole for increasing magnetic field.

Since $t_y\sim 300$ K and $t_z\sim 10$ K in Bechgaard salts, the high-field limit is accessible only in the configuration where the magnetic field is along the y direction. In this case, the crossover between 3D and 2D regimes should be reached for a reasonable value of the magnetic field ($H\sim 20$–30 T). Thus, we expect a strong decrease of the conductivity $\sigma_{zz}(H)$ along the z direction and also a decrease of the conductivity along the x direction due to an enhancement of the localization effect in the 2D regime.

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