

## Unified Picture of Superfluidity: From Bogoliubov's Approximation to Popov's Hydrodynamic Theory

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Using a nonperturbative renormalization-group technique, we compute the momentum and frequency dependence of the anomalous self-energy and the one-particle spectral function of two-dimensional interacting bosons at zero temperature. Below a characteristic momentum scale  $k_G$ , where the Bogoliubov approximation breaks down, the anomalous self-energy develops a square-root singularity and the Goldstone mode of the superfluid phase (Bogoliubov sound mode) coexists with a continuum of excitations, in agreement with the predictions of Popov's hydrodynamic theory. Thus our results provide a unified picture of superfluidity in interacting boson systems and connect Bogoliubov's theory (valid for momenta larger than  $k_G$ ) to Popov's hydrodynamic approach.

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Following the success of the Bogoliubov theory [1] in providing a microscopic explanation of superfluidity, much theoretical work has been devoted to the calculation of the one-particle Green function of Bose superfluids [2]. Early attempts to improve the Bogoliubov approximation however encountered difficulties due to a singular perturbation theory plagued by infrared divergences [3]. In the 1970s, Nepomnyashchii and Nepomnyashchii proved that the anomalous self-energy (the main quantity determining the one-particle Green function) vanishes at zero frequency and momentum in dimension  $d \leq 3$  [4,5]. This exact result shows that the Bogoliubov approximation, where the linear spectrum and the superfluidity rely on a finite value of the anomalous self-energy, breaks down at low energy. The vanishing of the anomalous self-energy in the infrared limit has a definite physical origin; it is due to the coupling between transverse (phase) and longitudinal fluctuations and the resulting divergence of the longitudinal susceptibility—a general phenomenon in systems with a continuous broken symmetry [6].

An alternative approach to superfluidity, based on a phase-amplitude representation of the boson field, has been proposed by Popov [7]. This approach is free of infrared singularity, but restricted to the (low-momentum) hydrodynamic regime and therefore does not allow the study of the higher-momentum regime where the Bogoliubov approximation is valid. Nevertheless, Popov's theory agrees with the exact result and the asymptotic low-energy behavior obtained by Nepomnyashchii and co-workers [8,9]. Furthermore, the expression of the anomalous self-energy obtained by Nepomnyashchii and co-workers and Popov in the low-energy limit yields a continuum of (one-particle) excitations coexisting with the Bogoliubov sound mode [10], in marked contrast with the Bogoliubov theory where the sound mode is the sole excitation at low energies.

Although the breakdown of the Bogoliubov approximation in  $d \leq 3$  is now well understood within the renormalization-group approach [11–14], no theoretical framework has given a unified description, from high to low energies, of superfluidity in interacting boson systems. Taking advantage of recent progress in the nonperturbative renormalization group (NPRG), we have calculated the momentum and frequency dependence of the anomalous self-energy of interacting bosons. Our results provide a unified description of superfluidity that encompasses both Bogoliubov's theory and Popov's hydrodynamic approach.

In this Letter, we focus on a two-dimensional system [15]. We show that the Bogoliubov approximation breaks down at a characteristic momentum scale  $k_G$  which, for weak boson-boson interactions, is much smaller than the inverse healing length  $k_h$  [16]. Moreover, the anomalous self-energy  $\Sigma_{\text{an}}(\mathbf{p}, \omega)$  becomes singular below  $k_G$  and induces a continuum of excitations which coexists with the Bogoliubov sound mode, in agreement with the predictions of Popov's hydrodynamic approach. We compute the longitudinal (one-particle) spectral function  $A_{\parallel}(\mathbf{p}, \omega)$  and discuss its dependence on  $|\mathbf{p}|/k_G$  [17].

We consider two-dimensional interacting bosons at zero temperature, with the action

$$S = \int dx \left[ \psi^*(x) \left( \partial_\tau - \mu - \frac{\nabla^2}{2m} \right) \psi(x) + \frac{g}{2} |\psi(x)|^4 \right] \quad (1)$$

( $\hbar = k_B = 1$  throughout the Letter), where  $\psi(x)$  is a bosonic (complex) field,  $x = (\mathbf{r}, \tau)$ , and  $\int dx = \int_0^\beta d\tau \int d^2r$ .  $\tau \in [0, \beta]$  is an imaginary time,  $\beta \rightarrow \infty$  the inverse temperature, and  $\mu$  denotes the chemical potential. The interaction is assumed to be local in space, and the model is regularized by a momentum cutoff  $\Lambda$ . It is convenient to write the boson field as  $\psi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)$  with  $\psi_1$  and  $\psi_2$  real. We assume the dimensionless coupling parameter  $2gm$  to be much smaller than unity (weak-coupling limit).

The excitation spectrum can be obtained from the one-particle Green function

$$G_{ij}(p; \phi) = \frac{\phi_i \phi_j}{2n} G_{\text{ll}}(p; n) + \left( \delta_{ij} - \frac{\phi_i \phi_j}{2n} \right) G_{\text{tt}}(p; n) + \epsilon_{ij} G_{\text{lt}}(p; n) \quad (2)$$

or its inverse, the two-point vertex

$$\Gamma_{ij}^{(2)}(p; \phi) = \delta_{i,j} \Gamma_A(p; n) + \phi_i \phi_j \Gamma_B(p; n) + \epsilon_{ij} \Gamma_C(p; n), \quad (3)$$

where  $\epsilon_{ij}$  is the antisymmetric tensor and  $p = (\mathbf{p}, i\omega)$  (with  $\omega$  a Matsubara frequency).  $\phi_i = \langle \psi_i(x) \rangle$  ( $i = 1, 2$ ) is assumed to be space and time independent, and  $n = (\phi_1^2 + \phi_2^2)/2$  denotes the condensate density. Taking advantage of the (global) gauge invariance of the action (1), we have introduced different Green functions for transverse (phase) and longitudinal fluctuations with respect to the constant field  $(\phi_1, \phi_2)$ . Formally, the two-point vertex  $\Gamma_{ij}^{(2)}(p; \phi)$  can be defined as the second-order functional derivative of the effective action  $\Gamma[\phi]$  (the generating functional of one-particle irreducible vertices). The NPRG procedure is set up by adding to the action (1) an infrared regulator  $\Delta S_k = \sum_p \psi^*(p) R_k(\mathbf{p}) \psi(p)$  which suppresses fluctuations with momenta below a characteristic scale  $k$  but leaves the high-momenta modes unaffected (for a review, see Ref. [18]). The dependence of the effective action on  $k$  is given by Wetterich's equation [19]

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \{ \partial_k R_k (\Gamma_k^{(2)}[\phi] + R_k)^{-1} \}. \quad (4)$$

In Fourier space, the trace involves a sum over frequencies and momenta as well as a trace over the two components of the field  $\phi = (\phi_1, \phi_2)$ . For  $k \simeq \Lambda$ , the regulator  $\Delta S_k[\phi]$  suppresses fluctuations, and the mean-field theory, where the effective action  $\Gamma_\Lambda[\phi]$  reduces to the microscopic action  $S[\phi]$ , becomes exact thus reproducing the results of the Bogoliubov approximation. On the other hand for  $k = 0$ , provided that  $R_k(\mathbf{p})$  vanishes,  $\Gamma_k[\phi]$  gives the effective action of the original model (1) from which we can deduce  $\Gamma^{(2)}$  and the one-particle Green function.

We make the following two simplifications. (i) We neglect the field dependence of  $\Gamma_{\alpha,k}(p; n)$  ( $\alpha = A, B, C$ ) which we approximate by its value at the actual ( $k$ -dependent) condensate density  $n_{0,k}$ .  $n_{0,k}$  is obtained from the minimum of the effective potential  $U_k(n) = (\beta V)^{-1} \Gamma_k[\phi]$  with  $\phi = (\sqrt{2n}, 0)$  ( $V$  is the volume of the system). (ii) We approximate the effective potential by

$$U_k(n) = U_k(n_{0,k}) + \frac{\lambda_k}{2} (n - n_{0,k})^2, \quad (5)$$

where  $\lambda|_{k=\Lambda} = g$ . Our approach follows the NPRG scheme recently proposed by Blaizot, Méndez-Galain, and Wschebor and others [20–22] with a truncation in fields to lowest nontrivial order [23].

Previous NPRG studies of interacting bosons assumed a simple form of the effective action  $\Gamma_k[\phi]$  with local

and  $\mathcal{O}(\nabla^2, \partial_\tau, \partial_\tau^2)$  terms [12–14]. In our formalism, this amounts to expanding the vertices  $\Gamma_A, \Gamma_B, \Gamma_C$  in powers of  $\mathbf{p}$  and  $\omega$ ,

$$\Gamma_A(p) = V_A \omega^2 + Z_A \epsilon_{\mathbf{p}}, \quad \Gamma_B(p) = \lambda, \quad \Gamma_C(p) = Z_C \omega \quad (6)$$

(we drop the  $k$  index to alleviate the notation), where  $\epsilon_{\mathbf{p}} = \mathbf{p}^2/2m$  is the dispersion of the free bosons. The initial conditions ( $k = \Lambda$ ),  $Z_A = Z_C = 1$ ,  $V_A = 0$ ,  $\lambda = g$ , and  $n_0 = \mu/g$ , reproduce the Green function  $G_{ij}(p)$  of the Bogoliubov approximation. Equations (2) and (6) yield a low-energy mode  $\omega = c|\mathbf{p}|$ , with ( $k$ -dependent) velocity

$$c = \left( \frac{Z_A/(2m)}{V_A + Z_C^2/(2\lambda n_0)} \right)^{1/2}, \quad (7)$$

and a superfluid density  $n_s = Z_A n_0 = \bar{n}$  where  $\bar{n}$  is the mean boson density [13,14]. In the weak-coupling limit,  $n_0$  and  $Z_A$  are weakly renormalized, and  $n_0^* \simeq \mu/g$ ,  $Z_A^* \simeq 1$  (with  $n_0^* = n_0|_{k=0}$ , etc.). On the other hand  $Z_C \sim k$  and  $\lambda \sim k$  vanish when  $k \rightarrow 0$ , while  $V_A$  takes a finite value  $V_A^*$  (Fig. 1) [12,13]. The anomalous self-energy  $\Sigma_{\text{an}}(0, 0) = n_0 \Gamma_B(0, 0) = n_0 \lambda$  therefore vanishes for  $k \rightarrow 0$  in agreement with the exact result [4]. The existence of a linear spectrum is then due to the relativistic form of the action which emerges at low energy ( $Z_C \rightarrow 0$  and  $V_A \rightarrow V_A^* > 0$ ) [12,13]. The characteristic momentum scale  $k_G$  (“Ginzburg” scale) at which the Bogoliubov approximation breaks down can be defined by the criterion  $V_A|_{k_G} = V_A^*/2$ . In the weak-coupling limit  $2gm \ll 1$ , it is found to be proportional to  $\sqrt{(gm)^3 \bar{n}} \sim gmk_h$  ( $k_h = \sqrt{2mg\bar{n}}$  is the inverse healing length below which the spectrum becomes linear) in agreement with a simple estimate based on the (one-loop) perturbative correction to the Bogoliubov ap-

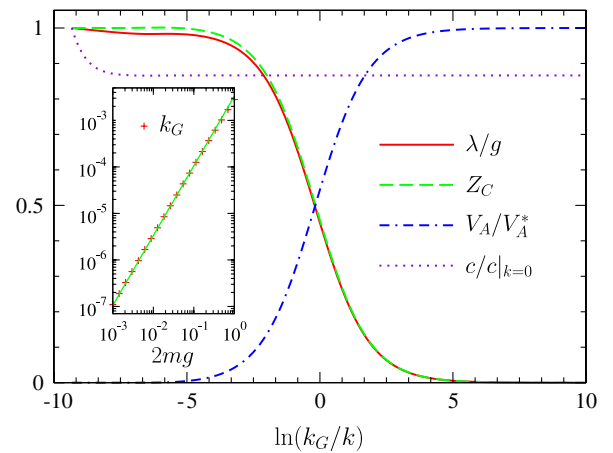


FIG. 1 (color online).  $\lambda/g$ ,  $Z_C$ ,  $V_A/V_A^*$ , and  $c/c|_{k=0}$  vs  $\ln(k_G/k)$ , where  $k_G = \sqrt{(gm)^3 \bar{n}}/4\pi$ , for  $\bar{n} = 0.01$  and  $2mg = 0.1$  [ $\ln(k_G/k_h) \simeq -5.87$ ]. The inset shows  $k_G$  vs  $2mg$  obtained from the criterion  $V_A|_{k_G} = V_A^*/2$  [the green solid line is a fit to  $k_G \propto (2mg)^{3/2}$ ]. All figures are obtained with  $\Lambda = 1$ ,  $2m = 1$ , and the regulator  $R_k(\mathbf{p}) = Z_{A,k} \epsilon_{\mathbf{p}} / (e^{\mathbf{p}^2/k^2} - 1)$ .

proximation [11] (see the inset in Fig. 1). In practice, we use the definition  $k_G = \sqrt{(gm)^3 \bar{n}} / (4\pi)$ .

Now we discuss the momentum and frequency dependence of the vertices beyond their derivative expansion (6). To simplify the numerical evaluation of  $\partial_k \Gamma_{\alpha,k}(p)$  ( $\alpha = A, B, C$ ), we approximate the propagators entering the flow equations using (6). This type of approximation has been shown to be very accurate for classical systems [24]. It is sufficient to obtain the anomalous self-energy  $\Sigma_{\text{an}}(p) = n_0 \Gamma_B(p)$  but is less reliable for the calculation of the (small) damping terms arising from  $\Gamma_A$  and  $\Gamma_C$  [25]. In practice, we compute the vertices  $\Gamma_\alpha(\mathbf{p}, i\omega)$  for typically 100 frequency points and then use a Padé approximant to obtain the retarded part  $\Gamma_\alpha^R(\mathbf{p}, \omega) = \Gamma_\alpha(\mathbf{p}, \omega + i0^+)$  [26]. While the Bogoliubov result  $\Gamma_B^R(\mathbf{p}, \omega) = g$  is a good approximation when  $|\mathbf{p}| \gg k_G$ ,  $\Gamma_B^R(\mathbf{p}, \omega)$  develops a strong frequency dependence for  $|\mathbf{p}| \lesssim k_G$  (Fig. 2). In the limit  $|\mathbf{p}| \ll k_G$  and  $|\omega| \ll c^* k_G$ , the vertices  $\Gamma_A$  and  $\Gamma_C$  are very well approximated by their low-energy limit (6), while  $\Gamma_B(p)$  is well fitted by a square-root singularity,

$$\Gamma_B(\mathbf{p}, i\omega) \simeq C \sqrt{\omega^2 + (c^* \mathbf{p})^2}, \quad (8)$$

with  $C$  a  $\mathbf{p}$ -independent constant [the Bogoliubov result  $\Gamma_B^R(\mathbf{p}, \omega) \simeq g$  is nevertheless reproduced at high energies  $|\omega| \gg c^* k_G$ ] (Fig. 3). For the retarded part, Eq. (8) implies

$$\Gamma_B^R(\mathbf{p}, \omega) \simeq C \theta(c^* |\mathbf{p}| - \omega) \sqrt{(c^* \mathbf{p})^2 - \omega^2} - iC \theta(\omega - c^* |\mathbf{p}|) \sqrt{\omega^2 - (c^* \mathbf{p})^2} \quad (9)$$

[ $\theta(x)$  is the step function] when  $|\mathbf{p}|$ ,  $\omega/c^* \ll k_G$  (we discuss only  $\omega \geq 0$ ). The square-root singularity (9) of the anomalous self-energy  $n_0 \Gamma_B^R(\mathbf{p}, \omega)$  agrees with the result obtained from diagrammatic resummations [5] or the predictions of the hydrodynamic approach [8] in the limit  $(\mathbf{p}, \omega) \rightarrow 0$ . As shown in Fig. 3, this singularity is very well reproduced by the result deduced from the Padé approximant. Thus our results interpolate between Bogoliubov's

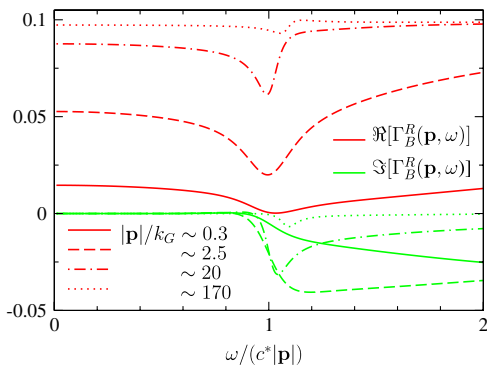


FIG. 2 (color online). Real and imaginary parts of the retarded vertex  $\Gamma_B^R(\mathbf{p}, \omega) = \Sigma_{\text{an}}^R(\mathbf{p}, \omega)/n_0$  for various values of  $|\mathbf{p}|$  ranging from  $0.3k_G$  up to  $170k_G \sim k_h/2$  ( $\bar{n} = 0.01$ ,  $2mg = 0.1$ , and  $k = 0$ ). The Bogoliubov approximation corresponds to  $\Gamma_B^R(\mathbf{p}, \omega) = g = 0.1$ . ( $\Re[\Gamma_B^R] \geq 0$  and  $\Im[\Gamma_B^R] \leq 0$ .)

approximation ( $|\mathbf{p}| \gg k_G$ ) and Popov's hydrodynamic theory ( $|\mathbf{p}| \ll k_G$ ).

In the low-energy limit  $\mathbf{p}, \omega \rightarrow 0$ ,  $\Gamma_B^R(\mathbf{p}, \omega)$  is of order  $|\mathbf{p}|, |\omega|$ , while  $\Gamma_A^R(\mathbf{p}, \omega)$  and  $\Gamma_C^R(\mathbf{p}, \omega)^2$  are of order  $\mathbf{p}^2, \omega^2$ . The one-particle Green function then becomes

$$\begin{aligned} G_{\text{tt}}^R(\mathbf{p}, \omega) &= -\frac{\Gamma_A^R(\mathbf{p}, \omega) + 2n_0^* \Gamma_B^R(\mathbf{p}, \omega)}{D^R(\mathbf{p}, \omega)} \simeq -\frac{1}{\Gamma_A^R(\mathbf{p}, \omega)}, \\ G_{\text{ll}}^R(\mathbf{p}, \omega) &= -\frac{\Gamma_A^R(\mathbf{p}, \omega)}{D^R(\mathbf{p}, \omega)} \simeq -\frac{1}{2n_0^* \Gamma_B^R(\mathbf{p}, \omega)}, \\ G_{\text{tt}}^R(\mathbf{p}, \omega) &= \frac{\Gamma_C^R(\mathbf{p}, \omega)}{D^R(\mathbf{p}, \omega)}, \end{aligned} \quad (10)$$

where  $D^R(\mathbf{p}, \omega) = \Gamma_A^R(\mathbf{p}, \omega)[\Gamma_A^R(\mathbf{p}, \omega) + 2n_0^* \Gamma_B^R(\mathbf{p}, \omega)] + \Gamma_C^R(\mathbf{p}, \omega)^2$ . From (6) and (10), one concludes that the transverse spectral function,

$$A_{\text{tt}}(\mathbf{p}, \omega) = -\frac{1}{\pi} \Im[G_{\text{tt}}^R(\mathbf{p}, \omega)] \simeq \frac{\delta(\omega - c^* |\mathbf{p}|)}{2V_A^* c^* |\mathbf{p}|} \quad (11)$$

(for  $\omega \geq 0$ ), exhibits a Dirac-like peak at the Bogoliubov mode frequency  $\omega = c^* |\mathbf{p}|$ , in very good agreement with the result obtained from the Padé approximant [25]. From (9) and (10), we deduce that the longitudinal spectral function

$$A_{\text{ll}}(\mathbf{p}, \omega) \simeq \frac{1}{2\pi n_0^* C} \frac{\theta(\omega - c^* |\mathbf{p}|)}{\sqrt{\omega^2 - (c^* \mathbf{p})^2}} \quad (12)$$

exhibits a continuum of excitations with a singularity at the Bogoliubov mode frequency  $\omega = c^* |\mathbf{p}|$ , again in agreement with the prediction of the hydrodynamic approach [10]. The analytic expression (12) gives a good approximation to the result obtained from the Padé approximant

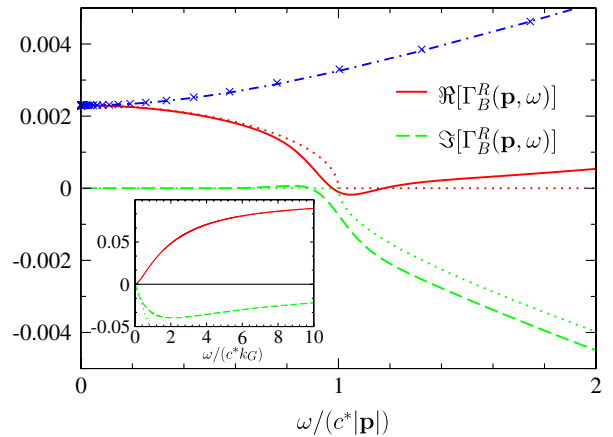


FIG. 3 (color online). Same as Fig. 2 but for  $|\mathbf{p}|/k_G \simeq 0.036$ . The dotted lines show the analytical result (9) obtained from the approximation (8). The latter is shown by the blue dash-dotted line, while the blue crosses give the numerical results for  $\Gamma_B(\mathbf{p}, i\omega)$  vs the rescaled Matsubara frequency  $\omega/(c^* |\mathbf{p}|)$ . The inset shows  $\Gamma_B^R(\mathbf{p}, \omega)$  on a much larger energy scale ( $\omega \leq 10c^* k_G \sim 280c^* |\mathbf{p}|$ ); the square-root singularity at  $\omega \ll c^* k_G$  is not visible on this plot.

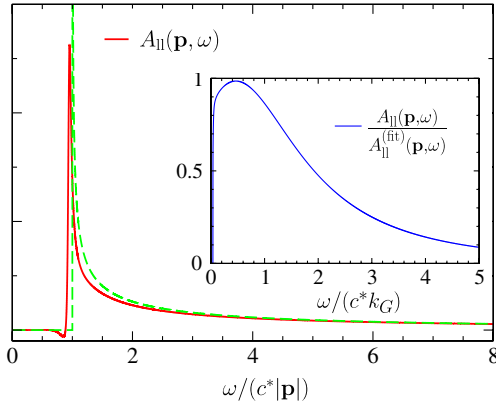


FIG. 4 (color online). Longitudinal spectral function  $A_{||}(\mathbf{p}, \omega)$  for  $|\mathbf{p}|/k_G \approx 0.036$  and the same parameters as in Fig. 3. The red solid line is the result obtained from the Padé approximant, while the green dashed line is obtained from the analytic expression (12). The inset shows the ratio between  $A_{||}(\mathbf{p}, \omega)$  and the approximate result (12) on a larger energy scale.

when  $|\mathbf{p}| \ll k_G$  and  $|\omega| \ll c^*k_G$  (Fig. 4). This defines the domain of validity of the Popov hydrodynamic approach. For  $|\mathbf{p}| \sim k_G$ , the continuum of excitations is suppressed and the longitudinal spectral function  $A_{||}(\mathbf{p}, \omega)$  reduces to a Dirac-like peak as in the Bogoliubov theory.

It should be noticed that we have retained only the leading term  $\Gamma_B(p=0) = \lambda$  in the derivative expansion of  $\Gamma_B(p)$  [Eq. (6)]. This is made possible by the fact that  $\Gamma_B(p=0) = \lambda = \mathcal{O}(k)$  is a very large energy scale with respect to  $\Gamma_A(p)$ ,  $\Gamma_C^2(p) \sim k^2$  for typical momentum and frequency  $|\mathbf{p}|$ ,  $\omega/c = \mathcal{O}(k)$ . The  $(\mathbf{p}, \omega)$  dependence of  $\Gamma_B(p) \sim \lambda + C\sqrt{\omega^2 + (c\mathbf{p})^2}$  does not change the leading behavior of  $\Gamma_B(p) = \mathcal{O}(k)$  which acts as a large mass term in the vertices. It should also be noticed that the singularity of  $\Gamma^{(2)}$  yields a similar singularity in higher-order vertices [4,5]:  $\Gamma^{(3)} \sim \sqrt{n_0}\Gamma_B$  and  $\Gamma^{(4)} \sim \Gamma_B$ . These singular terms are neglected in our approach, and we have approximated  $\Gamma^{(3)} \sim \sqrt{n_0}\lambda$  and  $\Gamma^{(4)} \sim \lambda$ . Again this is justified by the fact that  $\lambda = \mathcal{O}(k)$ .

To conclude, we have obtained a unified description of superfluidity which is valid at all energy scales and connects Bogoliubov's theory to Popov's hydrodynamic approach. Our results reveal the fundamental role of the Ginzburg momentum scale  $k_G$  in interacting boson systems.  $k_G$  sets the scale at which the Bogoliubov approximation breaks down and determines the region  $|\mathbf{p}|$ ,  $|\omega|/c^* \ll k_G$  where the longitudinal spectral function  $A_{||}(\mathbf{p}, \omega) \sim 1/\sqrt{\omega^2 - (c^*\mathbf{p})^2}$  [Eq. (12)], i.e., the domain of validity of the hydrodynamic approach. From a more general perspective, our results also show that the NPRG is a very efficient tool to study strongly correlated quantum systems and, in particular, to compute spectral functions.

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