Spin fluctuations and pseudogap in the two-dimensional half-filled Hubbard model at weak coupling

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Starting from the Hubbard model in the weak-coupling limit, we derive a spin-fermion model where the collective spin excitations are described by a nonlinear sigma model. This result is used to compute the fermion spectral function $A(\mathbf{k},\omega)$ in the low-temperature regime where the antiferromagnetic (AF) coherence length is exponentially large ("renormalized classical" regime). At the Fermi level, $A(\mathbf{k}_F,\omega)$ exhibits two peaks around $\pm\Delta_0$ (with Δ_0 the mean-field gap), which are precursors of the zero-temperature AF bands, separated by a pseudogap.

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I. INTRODUCTION

In the last two decades, the discovery of heavy-fermion compounds, high-T_c superconductors, and organic conductors has revived interest in strongly correlated electron systems. Of particular interest are metallic phases which, although conducting, are not described by Landau's Fermi-liquid theory because of the absence of well-defined quasiparticle excitations. A well-known example is given by the normal phase of high-T_c superconductors. Instead of quasiparticles, these systems exhibit a pseudogap at low energy as shown by many experiments.[1] Although the origin of the pseudogap is still under debate, it is generally believed that antiferromagnetic (AF) fluctuations play a crucial role.

In this paper, we consider the pseudogap issue on the basis of the half-filled two-dimensional (2D) Hubbard model. We consider only the weak-coupling limit $U \ll t$ (U is the local Coulomb repulsion and t the intersite hopping amplitude). [In the strong-coupling limit at half-filling, the finite temperature paramagnetic phase is a Mott-Hubbard insulator with a (charge) gap of order U. At $T=0$, there is a transition to a Néel antiferromagnetic state.[2]] Although the ground state is AF, long-range order is destroyed by classical fluctuations at any finite temperature, in agreement with the Mermin-Wagner theorem. Nevertheless, below a crossover temperature T_X (of the order of the mean-field transition temperature), the system enters a renormalized classical regime where AF correlations start to grow exponentially. Contrary to the 3D case, at the zero-temperature 2D phase transition the system goes directly into the (Néel) ordered state where the fermion spectral function $A(\mathbf{k},\omega)$ exhibits two well-defined quasiparticle (QP) peaks corresponding to the Bogoliubov QP's. By continuity, the two-peak structure in $A(\mathbf{k},\omega)$ cannot disappear as soon as we raise the temperature. As pointed out in Ref. 3, the only possible scenario is that at finite but low temperature the fermion spectral function exhibits two (broadened) peaks which are precursors of the $T=0$ Bogoliubov QP's, separated by a pseudogap. We therefore expect the presence of a pseudogap at finite temperature, due to the strong (classical) AF fluctuations.

Clearly, traditional mean-field techniques fail to describe these phenomena. For instance, the random-phase approximation (RPA) predicts a finite temperature phase transition which is forbidden in two dimensional by the Mermin-Wagner theorem. More sophisticated approaches are therefore required. In the weak-coupling limit, the pseudogap formation has been considered within the fluctuation exchange (FLEX) approximation[4] and the two-particle–self-consistent (TPSC) theory[3,5,6] which both satisfy the Mermin-Wagner theorem. Only the TPSC theory predicts the formation of a pseudogap in the fermion spectral function $A(\mathbf{k},\omega)$ at low temperature.

The aim of this paper is to describe an alternative approach to the 2D half-filled Hubbard model in the weak-coupling limit. We first derive a spin-fermion model where the collective spin excitations are described by a nonlinear sigma model (NLσM). The spin-wave velocity and the coupling constant of the NLσM are expressed in terms of the ground-state properties of the system. Solving the NLσM in a "large-$\mathcal{N}$" limit, we then compute the fermion spectral function $A(\mathbf{k},\omega)$ to lowest order in the spin-fermion interaction. At the Fermi level, $A(\mathbf{k}_F,\omega)$ exhibits two peaks around $\pm\Delta_0$ (with Δ_0 the mean-field gap) which are precursors of the zero-temperature AF bands, separated by a pseudogap. We compare our results with those of the TPSC theory.

II. MODEL

The two-dimensional Hubbard model is defined by the Hamiltonian

$$H=-t\sum_{\langle \mathbf{r},\mathbf{r}'\rangle,\sigma}(c^\dagger_{\mathbf{r}\sigma}c_{\mathbf{r}'\sigma}+\text{H.c.})+U\sum_{\mathbf{r}} n_{\mathbf{r}\uparrow}n_{\mathbf{r}\downarrow}, \quad (2.1)$$

where t is the intersite hopping amplitude and U the on-site Coulomb repulsion. $c_{\mathbf{r}\sigma}$ is a fermionic operator for a σ-spin particle at site $\mathbf{r}$ ($\sigma=\uparrow,\downarrow$), and $n_{\mathbf{r}\sigma}=c^\dagger_{\mathbf{r}\sigma}c_{\mathbf{r}\sigma}$. $\langle \mathbf{r},\mathbf{r}'\rangle$ denotes nearest-neighbor sites. We take the lattice spacing equal to unity and $\hbar=k_B=1$ throughout the paper.

Since spin fluctuations play a crucial role in the Hubbard model at half-filling, it is convenient to introduce auxiliary fields describing these collective excitations. The standard approach is to write the interaction part of the Hamiltonian in terms of charge and spin fluctuations, i.e., $n_{\mathbf{r}\uparrow}n_{\mathbf{r}\downarrow}=[(c^\dagger_{\mathbf{r}}c_{\mathbf{r}})^2-(c^\dagger_{\mathbf{r}}\sigma_z c_{\mathbf{r}})^2]/4$, and then perform a Hubbard-Stratonovich transformation by means of two (real) auxiliary fields Δ_c and Δ_s $[c_{\mathbf{r}}=(c_{\mathbf{r}\uparrow},c_{\mathbf{r}\downarrow})^T]$. Although this procedure recovers the standard mean-field (or Hartree-Fock) theory of the Néel state within a saddle-point approximation, it leads to a loss of spin-rotation invariance and does not allow to obtain the spin-wave excitations. Alternatively, one could write $n_{\mathbf{r}\uparrow}n_{\mathbf{r}\downarrow}$ in a spin-rotation invariant form, e.g., $n_{\mathbf{r}\uparrow}n_{\mathbf{r}\downarrow}=-(c_{\mathbf{r}}^\dagger\boldsymbol{\sigma}c_{\mathbf{r}})^2/6$ where $\boldsymbol{\sigma}$ denotes the Pauli matrices, and use a vector Hubbard-Stratonovich field. Such decompositions, however, do not reproduce the mean-field results at the saddle-point level.[7]

As noted earlier,[7,8] this difficulty can be circumvented by writing $n_{\mathbf{r}\uparrow}n_{\mathbf{r}\downarrow}=[(c_{\mathbf{r}}^\dagger c_{\mathbf{r}})^2-(c_{\mathbf{r}}^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}_{\mathbf{r}}c_{\mathbf{r}})^2]/4$ where $\boldsymbol{\Omega}_{\mathrm{r}}$ is an arbitrary unit vector. Spin-rotation invariance is maintained by averaging the partition function over all directions of $\boldsymbol{\Omega}_{\mathbf{r}}$. In a path-integral formalism, $\boldsymbol{\Omega}_{\mathbf{r}}$ becomes a time-dependent variable. After the Hubbard-Stratonovich transformation, the partition function is given by $Z=\int\mathcal{D}[c^\dagger,c]\int\mathcal{D}[\Delta_c,\Delta_s,\boldsymbol{\Omega}]e^{-S}$ with the action

$$S=S_0+\sum_{\mathbf{r}}\int_0^\beta d\tau\left\{\frac{1}{U}(\Delta_{c\mathbf{r}}^2+\Delta_{s\mathbf{r}}^2)-c_{\mathbf{r}}^\dagger(i\Delta_{c\mathbf{r}}+\Delta_{s\mathbf{r}}\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}_{\mathbf{r}})c_{\mathbf{r}}\right\}. \tag{2.2}$$

S_0 is the action in the absence of interaction. Since charge fluctuations are not critical (even when $T\to 0$), they can be treated at the saddle point (i.e., Hartree-Fock) level. Their effect is to renormalize the chemical potential μ from $U/2$ to 0. Equation (2.2) then corresponds to a spin-fermion model where the fermions interact with their collective spin degrees of freedom $(\Delta_{\mathbf{r}}\boldsymbol{\Omega}_{\mathbf{r}})$. (We now denote $\Delta_{s\mathbf{r}}$ by $\Delta_{\mathbf{r}}$.) Below the crossover temperature T_X, i.e., when $T\ll T_X$, low-energy excitations correspond to orientational spin fluctuations described by the unit vector field $\boldsymbol{\Omega}_{\mathbf{r}}$. We can then consider $\Delta_{\mathbf{r}}$ within a saddle-point approximation, i.e., $\Delta_{\mathbf{r}}=\Delta_0(-1)^{\mathbf{r}}$, where the fluctuations of Δ_0 are ignored. In order to compute the fermion spectral function $A(\mathbf{k},\omega)$, one should first determine the effective action $S[\boldsymbol{\Omega}]$ of the unit vector field $\boldsymbol{\Omega}$.

III. SPIN FLUCTUATIONS

The effective action $S[\boldsymbol{\Omega}]$ is obtained by expanding around the Néel state. We first introduce a new field ϕ defined by $\phi_{\mathbf{r}}=R_{\mathbf{r}}^\dagger c_{\mathbf{r}}$, where $R_{\mathbf{r}}$ is a SU(2)/U(1) matrix which rotates the spin-quantization axis from $\hat{\mathbf{z}}$ to $\boldsymbol{\Omega}_{\mathbf{r}}$ $(R_{\mathbf{r}}\sigma_z R_{\mathbf{r}}^\dagger=\boldsymbol{\Omega}_{\mathbf{r}}\cdot\boldsymbol{\sigma})$. In terms of this new field, the action becomes

$$S=S_{\mathrm{MF}}+\sum_{\mathbf{r}}\int_0^\beta d\tau\phi_{\mathbf{r}}^\dagger R_{\mathbf{r}}^\dagger\partial_\tau R_{\mathbf{r}}\phi_{\mathbf{r}} -t\sum_{\langle\mathbf{r},\mathbf{r}'\rangle}\int_0^\beta d\tau[\phi_{\mathbf{r}}^\dagger(R_{\mathbf{r}}^\dagger R_{\mathbf{r}'}-1)\phi_{\mathbf{r}'}+\mathrm{c.c.}], \tag{3.1}$$

where $S_{\mathrm{MF}}=S_0+\Sigma_{\mathbf{r}}\int d\tau[\Delta_0^2/U-\Delta_0(-1)^{\mathbf{r}}\phi_{\mathbf{r}}^\dagger\sigma_z\phi_{\mathbf{r}}]$. Within a saddle-point approximation with $\boldsymbol{\Omega}_{\mathbf{r}}=\hat{\mathbf{z}}$ $(R_{\mathbf{r}}=1)$, i.e., ignoring spin fluctuations, one recovers the mean-field action S_{MF} of the Néel state. The value of the order parameter, $\Delta_0=(U/2)(-1)^{\mathbf{r}}\langle\phi_{\mathbf{r}}^\dagger\sigma_z\phi_{\mathbf{r}}\rangle$, is obtained by minimizing the free energy. In the weak-coupling limit, this gives $\Delta_0\sim te^{-2\pi\sqrt{t/U}}$.[9]

Low-energy spin excitations correspond to fluctuations of the unit vector field $\boldsymbol{\Omega}_{\mathbf{r}}$ around its saddle-point value. The standard procedure[10,8,7] is then to assume at least local AF order and write $\boldsymbol{\Omega}_{\mathbf{r}}=\mathbf{n}_{\mathbf{r}}(1-\mathbf{L}_{\mathbf{r}}^2)^{1/2}+(-1)^{\mathbf{r}}\mathbf{L}_{\mathbf{r}}$, where the (Néel) order parameter field $\mathbf{n}_{\mathbf{r}}$ is slowly varying in space and time and $\mathbf{L}_{\mathbf{r}}$ is a small canting field ($|\mathbf{n}_{\mathbf{r}}|=1$, $\mathbf{L}_{\mathbf{r}}\cdot\mathbf{n}_{\mathbf{r}}=0$, and $|\mathbf{L}_{\mathbf{r}}|\ll 1$). Integrating out both ϕ and $\mathbf{L}$ yields the action of the NLσM.[7,11] In the strong-coupling limit $U\gg t$, one recovers the action derived from the Heisenberg model.

As we now verify explicitly, the small canting field $\mathbf{L}_{\mathbf{r}}$ gives negligible contributions to the parameters of the NLσM in the weak-coupling limit $U\ll t$. If we identify $\boldsymbol{\Omega}_{\mathbf{r}}$ with the slowly varying Néel field, $\boldsymbol{\Omega}_{\mathbf{r}}\approx\mathbf{n}_{\mathbf{r}}$, the effective action $S[\mathbf{n}]$ is readily obtained. Integrating out the fermions in Eq. (3.1) and taking the continuum limit in space, one obtains to lowest order in gradient (i.e., in $\partial_\tau R$ and $\boldsymbol{\nabla}_{\mathbf{r}}R$)[12]

$$S[\mathbf{n}]=\frac{1}{2}\int d^2r\,d\tau[\chi_\perp^0(\partial_\tau\mathbf{n})^2+\rho_s^0(\boldsymbol{\nabla}_{\mathbf{r}}\mathbf{n})^2], \tag{3.2}$$

where $\chi_\perp^0$ is the uniform transverse spin susceptibility in the mean-field state and $\rho_s^0=-(\langle K\rangle_{\mathrm{MF}}/2+\Pi_\perp^0)/4$ the spin stiffness. Here $\langle K\rangle_{\mathrm{MF}}$ is the mean value of the kinetic energy and $\Pi_\perp^0$ the correlation function of the transverse spin current (j^x or j^y). Equation (3.2) should be supplemented with a short-distance cutoff (in momentum space) $\Lambda\sim\xi_0^{-1}$, since short-range AF order cannot be defined at length scales smaller that the coherence length $\xi_0\sim t/\Delta_0$. Using the mean-field action S_{MF}, one obtains (N is the number of lattice sites)

$$\chi_\perp^0=\frac{\Delta_0^2}{4N}\sum_{\mathbf{k}}\frac{1}{E_{\mathbf{k}}^3}\sim\frac{1}{t}\sqrt{\frac{t}{U}}, \tag{3.3}$$

$$\rho_s^0=\frac{t^2\Delta_0^2}{N}\sum_{\mathbf{k}}\frac{\sin^2k_x}{E_{\mathbf{k}}^3}\sim t, \tag{3.4}$$

where $E_{\mathbf{k}}=(\epsilon_{\mathbf{k}}^2+\Delta_0^2)^{1/2}$ is the Bogoliubov quasiparticle excitation energy in the mean-field state $[\epsilon_{\mathbf{k}}=-2t(\cos k_x+\cos k_y)$ is the dispersion of the free fermions]. We can verify that Eqs. (3.2)–(3.4) can be directly obtained from the results of Refs. 7 and 11 in the weak-coupling limit $(U\ll t)$. The value of the spin-wave velocity $c=\sqrt{\rho_s^0/\chi_\perp^0}\sim t(U/t)^{1/4}$ also agrees with the weak-coupling limit of the RPA result.[13] The approximation $\boldsymbol{\Omega}_{\mathbf{r}}\approx\mathbf{n}_{\mathbf{r}}$ is therefore justified when $U\ll t$. While it restricts the validity of our approach to the weak-coupling limit, it makes the computation of fermionic correlation functions considerably simpler, since the fermions couple directly to the Néel field [see Eq. (2.2)].

We solve the NLσM within a "large-$\mathcal{N}$" approach by extending the number of components of the unit vector $\mathbf{n}_{\mathbf{r}}$ from 3 to $\mathcal{N}$. When $\mathcal{N}\to\infty$, the action (3.2) can be solved exactly by a saddle-point method.[14] Figure 1 shows the resulting crossover diagram as a function of the dimensionless coupling constant $\overline{g}=\Lambda g=\Lambda c\mathcal{N}/\rho_s^0$ of the NLσM. In the

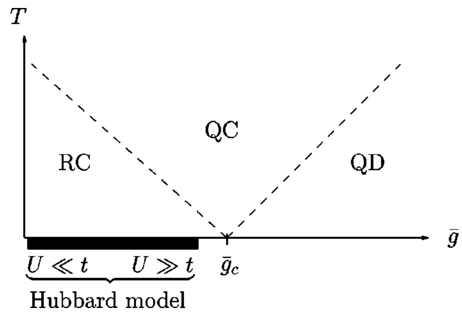

FIG. 1. Crossover diagram derived from the large-$\mathcal{N}$ limit of the 2D NLσM. At $T=0$, there is long-range order when the dimensionless coupling $\bar{g} \leqslant \bar{g}_c = 4\pi$. The three finite temperature regimes correspond to "renormalized classical" (RC), "quantum critical" (QC), and "quantum disordered" (QD). (Ref. 15). The ground state of the half-filled 2D Hubbard model on a square lattice is ordered for any value of the Coulomb repulsion U. At finite temperature, there are strong AF fluctuations with an exponentially large coherence length (RC regime).

weak-coupling limit of the Hubbard model ($U \ll t$), $\bar{g} = c\Delta_0 \mathcal{N}/(\rho_s^0 t) \propto e^{-2\pi\sqrt{t/U}}$ is exponentially small. This implies that the ground state has AF long-range order with very weak quantum fluctuations. This magnetic order persists in the strong-coupling regime ($U \gg t$) where[7] $\bar{g} \lesssim \bar{g}_c = 4\pi$ (see Fig. 1) in agreement with conclusions based on the Heisenberg model (for a square lattice). At finite temperature, magnetic long-range order is suppressed as required by the Mermin-Wagner theorem. The dominant fluctuations are classical since the gap m in the spin excitation spectrum (see below) is much smaller than the temperature (this regime is known as "renormalized classical" in the literature[15]).

Since we are primarily interested in the fermion spectral function $A(\mathbf{k},\omega)$ at finite temperature, we shall consider the action $S[\mathbf{n}]$ in this regime. In the large-$\mathcal{N}$ limit, it reads

$$S[\mathbf{n}] = \frac{\mathcal{N}}{2gc} \sum_{\mathbf{q},\omega_\nu} (\omega_\nu^2 + c^2 q^2 + m^2)|\mathbf{n}(\mathbf{q},i\omega_\nu)|^2, \quad (3.5)$$

where we have introduced the Fourier-transformed field $\mathbf{n}(\mathbf{q},i\omega_\nu)$ (ω_ν is a bosonic Matsubara frequency). The length of the vector $\mathbf{n}_\mathbf{r}$ is no longer fixed to unity. In the large-$\mathcal{N}$ solution, the constraint $|\mathbf{n}_\mathbf{r}|=1$ is imposed only on average (via the Lagrange multiplier m).[14] The mass m of the spin-fluctuation propagator ($\alpha = 1 \cdots \mathcal{N}$)

$$\chi(\mathbf{q},i\omega_\nu) = \langle \mathbf{n}_\alpha(\mathbf{q},i\omega_\nu)\mathbf{n}_\alpha(-\mathbf{q},-i\omega_\nu)\rangle = \frac{gc/\mathcal{N}}{\omega_\nu^2 + c^2q^2 + m^2} \quad (3.6)$$

is determined by the saddle-point equation

$$1 = gc\frac{T}{N}\sum_{\mathbf{q},\omega_\nu} \frac{1}{\omega_\nu^2 + c^2q^2 + m^2}. \quad (3.7)$$

In the renormalized classical regime, we can neglect quantum fluctuations. This approximation is excellent in the weak-coupling regime ($U \ll t$) since quantum fluctuations are

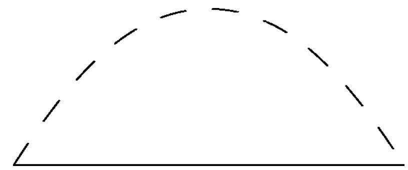

FIG. 2. Lowest-order contribution to the fermion self-energy Σ. The dashed line represents the spin propagator χ [Eq. (3.6)].

weak ($\bar{g} \ll \bar{g}_c$, see Fig. 1). From Eq. (3.7), we then obtain the AF coherence length $\xi = c/m \sim \Lambda^{-1}\exp(2\pi\rho_s^0/\mathcal{N}T)$.

Note that we expect also a term $m^2|\omega_\nu|/\omega_{\rm sf}$ in the denominator in Eq. (3.6). This term comes from the damping of spin fluctuations by gapless fermion excitations.[16] It is missed in our approach since we expand around the zero-temperature AF state which has only gapped quasiparticle excitations. Fluctuations are classical when $m \ll T$ and $\omega_{\rm sf} \ll T$. Both conditions are satisfied in the renormalized classical regime ($T \ll T_X$) since $\omega_{\rm sf} \sim \xi^{-2} \to 0$ (critical slowing down).[3,6,16]

IV. SPECTRAL FUNCTION

Knowing the effective action $S[\mathbf{n}]$ of the spin excitations [Eq. (3.5)], we are now in a position to compute the spectral function $A(\mathbf{k},\omega) = -\pi^{-1}\mathrm{Im}G(\mathbf{k},\omega)$ from the spin-fermion model (2.2). Here $G(\mathbf{k},\omega)$ denotes the retarded part of the fermionic Green's function. By integrating first the fermions and then the spin fluctuations, we can write the Green's function as

$$G(\mathbf{r}-\mathbf{r}',\tau-\tau') = \frac{1}{Z}\int \mathcal{D}[\mathbf{n}]e^{-S[\mathbf{n}]}G(\mathbf{r},\tau;\mathbf{r}',\tau'|\mathbf{n}). \quad (4.1)$$

$G(\mathbf{r},\tau;\mathbf{r}',\tau'|\mathbf{n})$ is the Green's function for a given configuration of $\mathbf{n}$: $G^{-1}[\mathbf{n}] = G_0^{-1} + \Delta_0(-1)^{\mathbf{r}}\boldsymbol{\sigma}\cdot\mathbf{n}_\mathbf{r}$, where G_0 is the Green's function of the free fermions. Since $S[\mathbf{n}]$ is Gaussian in the large-$\mathcal{N}$ limit, the averaging in Eq. (4.1) is easily done. The result can be written as $G^{-1}(\mathbf{k},i\omega_n) = G_0^{-1}(\mathbf{k},i\omega_n) - \Sigma(\mathbf{k},i\omega_n)$ (ω_n is a fermionic Matsubara frequency).

We consider the lowest-order contribution to the self-energy Σ (Fig. 2):

$$\begin{aligned}\Sigma(\mathbf{k},i\omega_n) &= \Delta_0^2 \frac{T}{N}\sum_{\mathbf{q},\omega_\nu} \mathcal{N}\chi(\mathbf{q},i\omega_\nu) \\ &\quad \times G_0(\mathbf{k}-\mathbf{Q}-\mathbf{q},i\omega_n - i\omega_\nu) \\ &\simeq \Delta_0^2\frac{gT}{cN}\sum_{\mathbf{q}} \frac{1}{q^2+\xi^{-2}}\frac{1}{i\omega_n - \epsilon_{\mathbf{k}-\mathbf{Q}-\mathbf{q}}}, \end{aligned} \quad (4.2)$$

where the last line has been obtained in the classical limit ($\omega_\nu = 0$) and $\mathbf{Q} = (\pi,\pi)$. At low temperature when $\xi \to \infty$, the sum over $\mathbf{q}$ in Eq. (4.2) diverges in two-dimensions due to the contribution of long wavelengths ($\mathbf{q} \sim 0$). We can therefore expand $-\epsilon_{\mathbf{k}-\mathbf{Q}-\mathbf{q}} = \epsilon_{\mathbf{k}-\mathbf{q}} \simeq \epsilon_\mathbf{k} - \mathbf{v}_\mathbf{k}\cdot\mathbf{q}$ around $\mathbf{q}=0$ ($\mathbf{v}_\mathbf{k}$ is the velocity of the free fermions). Let us first consider a particle at the Fermi level. One easily finds that the imaginary part of the retarded self-energy ($i\omega_n \to \omega + i0^+$) takes the form

$$\Sigma''(\mathbf{k}_F,\omega=0) \approx -\frac{\Delta_0^2 \xi}{\rho_s^0 \xi_{\rm th}} \propto -T\xi, \tag{4.3}$$

where $\xi_{\rm th} = |\mathbf{v}_\mathbf{k}|/T$ is the de Broglie thermal wavelength. Since ξ grows exponentially below T_X, it quickly becomes larger than $\xi_{\rm th}$. As a result, $\lim_{T\to 0}\xi/\xi_{\rm th} = \infty$ and $\Sigma''(\mathbf{k}_F,\omega=0)$ diverges at low temperature in contradiction with the Fermi-liquid theory hypothesis. Thus, the lowest-order perturbation result shows that quasiparticles are suppressed by spin fluctuations when $T \ll T_X$. This phenomenon is accompanied by the formation of a pseudogap. For $|\omega + \epsilon_\mathbf{k}| \gg |\mathbf{v}_\mathbf{k}|/\xi$, the real and imaginary parts of the self-energy are given by[17]

$$\Sigma'(\mathbf{k},\omega) \simeq \frac{\Delta_0^2}{\omega + \epsilon_\mathbf{k}}, \quad \Sigma''(\mathbf{k},\omega) \simeq -\frac{3\Delta_0^2 T}{4\pi\rho_s^0|\omega + \epsilon_\mathbf{k}|}. \tag{4.4}$$

Note that the condition $|\omega + \epsilon_\mathbf{k}| \gg |\mathbf{v}_\mathbf{k}|/\xi$ is satisfied for any value of ω except in an exponentially small window around $\omega = -\epsilon_\mathbf{k}$. From Eq. (4.4), we deduce the spectral function

$$A(\mathbf{k},\omega) = \frac{\gamma}{\pi} \frac{|\omega + \epsilon_\mathbf{k}|}{(\omega^2 - E_\mathbf{k}^2)^2 + \gamma^2}, \quad \gamma \approx \frac{3\Delta_0^2 T}{4\pi\rho_s^0}. \tag{4.5}$$

$A(\mathbf{k},\omega)$ exhibits two peaks at $\pm E_\mathbf{k}$ that are precursors of the AF bands that exist in the $T=0$ ordered state. The width of these peaks is given by $\gamma/\Delta_0 \sim T\Delta_0/\rho_s^0 \sim Te^{-2\pi\sqrt{t/U}}$. The precursors of the AF bands are separated by a pseudogap. In particular $A(\mathbf{k}_F,\omega)$ vanishes at $\omega = 0$.

When $T \to 0$ ($\gamma \to 0$),

$$A(\mathbf{k},\omega) \to \frac{1}{2}\left(1 + \frac{\epsilon_\mathbf{k}}{E_\mathbf{k}}\right)\delta(\omega - E_\mathbf{k}) + \frac{1}{2}\left(1 - \frac{\epsilon_\mathbf{k}}{E_\mathbf{k}}\right)\delta(\omega + E_\mathbf{k}), \tag{4.6}$$

which is the spectral function of the $T=0$ AF state. Thus, the simple self-energy (4.2) predicts that the pseudogap evolves smoothly into the gap of the ground state when $T \to 0$. It should be noted that neglecting quantum fluctuations is justified only at low energy $|\omega| < T$. In particular, the precise location of the peaks around $\pm\Delta_0$ should depend on quantum fluctuations since $\Delta_0 \sim T_X \gg T$.

The spectral function $A(\mathbf{k},\omega)$ [Eq. (4.5)] is similar to the result of the TPSC theory.[3,5,6] In the latter, the position of the maxima in $A(\mathbf{k}_F,\omega)$ scales with the zero-temperature gap,[18,19] and the width of these two peaks is proportional to T.[3,5,6] These two features agree with our conclusions. This similarity is not surprising since in both approaches a paramagnonlike self-energy [Eq. (4.2)] with a similar spin susceptibility [Eq. (3.6)] is used to obtain the spectral function. The main difference comes from the spin-fluctuation propagator χ. While χ comes from the NLσM (which is itself based on an expansion around the ordered AF state), it is obtained by considering the paramagnetic phase in the TPSC theory. As a result, the basic parameters entering the spectral function $A(\mathbf{k},\omega)$ [Eq. (4.5)], namely the $T=0$ order parameter Δ_0 and the $T=0$ spin stiffness ρ_s^0, do not appear in the TPSC theory. Instead, $A(\mathbf{k},\omega)$ is expressed only in terms of the paramagnetic properties of the system.

Two comments are in order here. The validity of Eq. (4.2), which does not include vertex correction, may be questioned.[20] The importance of these corrections is a longstanding problem which is still under debate. Vertex corrections are expected to play a crucial role when higher-order self-energy contributions are taken into account. The FLEX approximation, which sums up contributions to all order without vertex correction, does not predict the formation of a pseudogap in $A(\mathbf{k},\omega)$ at low temperature[4] (see Ref. 3 for a detailed discussion of the FLEX approximation).

In the spin-fermion model defined by Eqs. (2.2) and (3.2), there are only two (transverse) spin excitation modes, as expected when only orientational fluctuations are important ($T \ll T_X$). Unfortunately, this property is lost in the large-$\mathcal{N}$ limit of the NLσM [Eq. (3.5)], where both transverse and amplitude fluctuations are allowed. Following Ref. 21, $A(\mathbf{k},\omega)$ can be obtained exactly when $\xi \to \infty$ by summing all the self-energy diagrams. The result,

$$A(\mathbf{k},\omega) = \frac{3^{3/2}}{\sqrt{2\pi}\Delta_0^3}(\omega^2 - \epsilon_\mathbf{k}^2)^{1/2}(\omega + \epsilon_\mathbf{k})\exp\left(-\frac{3}{2}\frac{\omega^2 - \epsilon_\mathbf{k}^2}{\Delta_0^2}\right) \times [\theta(\omega - |\epsilon_\mathbf{k}|) - \theta(-\omega - |\epsilon_\mathbf{k}|)], \tag{4.7}$$

shows two broad incoherent features, located around $\pm\sqrt{2/3}\Delta_0$ for $\epsilon_\mathbf{k} = 0$, instead of the correct $T=0$ limit given by Eq. (4.6). The correct limit is obtained only when amplitude fluctuations are frozen in the limit $\xi \to \infty$.[22,23] We therefore conclude that our approach, which is based on the large-$\mathcal{N}$ solution of the NLσM, must break down at very low temperature. The fact that the spectral function $A(\mathbf{k},\omega)$ derived from the lowest-order self-energy contribution does reproduce the correct result when $T \to 0$ [Eq. (4.5)] appears somewhat accidental. A correct treatment of the $T \to 0$ limit must freeze the amplitude fluctuations of the Néel field $\mathbf{n}$.

V. CONCLUSION

We have described a new approach to the pseudogap in the half-filled 2D Hubbard model at weak coupling. Within this approach, only orientational spin fluctuations are considered, whereas fluctuations of the amplitude of the local spin density are ignored. This approximation is justified below a crossover temperature T_X (of the order of the mean-field AF transition temperature) where the AF correlation length starts to grow exponentially (renormalized classical regime). The effective action of spin fluctuations is then given by a NLσM. Solving the NLσM within a "large-$\mathcal{N}$" approach, we find that the ground state of the Hubbard model on a square lattice is antiferromagnetic (Néel order) for any value of the Coulomb interaction U (Fig. 1).[7]

We have obtained the fermion spectral function $A(\mathbf{k},\omega)$ in the weak-coupling limit by computing the self-energy $\Sigma(\mathbf{k},\omega)$ to lowest order in the spin-fermion interaction (Fig. 2). The QP peak which characterizes the Fermi-liquid state is suppressed by spin fluctuations when $T \ll T_X$. Instead, $A(\mathbf{k},\omega)$ exhibits a pseudogap separating two broadened peaks. These peaks are precursors of the Bogoliubov QP's that appear at the $T=0$ AF transition. Our results are in very good agreement with those obtained by the TPSC theory.[3,5,6] An important limitation of our analysis comes from the large-$\mathcal{N}$ solution of the NLσM. The latter introduces amplitude fluctuations of the Néel field which should be frozen at low temperature. As a result, when going beyond the lowest-order contribution to $\Sigma(\mathbf{k},\omega)$, we do not obtain the correct $T \to 0$ limit of the fermion spectral function. In Ref. 11, we show how this difficulty can be circumvented.

There are several directions in which this work could be further developed. Since the NLσM description is valid both at weak ($U \ll t$) and strong ($U \gg t$) coupling, our analysis of the fermion spectral function could be extended in the regime $U \gg t$. In the Mott-Hubbard insulator, we expect the pseudogap to transform into a (charge) gap of order U, the precursors of the $T=0$ AF bands becoming the upper and lower Hubbard bands.

It is also possible to consider variants of the square lattice Hubbard model [Eq. (2.1)] where antiferromagnetism becomes frustrated. This would be the case for the t-t' Hubbard model (t' is the hopping amplitude for next-nearest neighbors) or if the lattice is triangular instead of square. Doping may also induce some kind of magnetic frustration.[24] This opens up the possibility to reach the quantum disordered and quantum critical regimes of the NLσM (Fig. 1) and to study the corresponding fermion spectral functions.

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